TRANSNAV International Journal on Marine Navigation and Safety of Sea Transportation
Volume 1 Number 4 December 2007

Availability of the Certain Value of Position Error in the Navigational Systems – Model Application

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ABSTRACT: The distributions and parameters of the random variable (density functions, distribution functions, expected values, variances, moments etc.), which in navigation is represented by position error, are well defined. This approach is similar like in geodesy, where passing time hasn't any impact on statistical solution. Presented paper evaluates alternative approach to position error analyses, where the random values are: working and failures times of the system. Arbitrary accepted certain value of position error was assumed as a decision criterion of the system for working state definition. The general mathematical model of the availability of certain value of position error is presented and the special case – exponential distributions of lifetimes and failure times model – was also analysed. The model was adopted for positioning based on EGNOS system measurements and some results are present.

1 INTRODUCTION

The parametric assessment of navigation systems during the last decade has been the most common way of their classification with regard to their quality. Within the scope of this evaluation critical space is provided. This space is very closely related to the navigation requirements set for its various forms. The comparable criteria of navigation systems were often discussed in world literature as well as in Polish publications. The analysis of the criteria allows distinguishing three main groups, which are identical with particular phases of positioning systems development over the years and they are as follows: positioning, reliability and the safety of exploitation criteria.

1 Positioning criteria - system characteristics in quality of position fixing. They have in their scope 3 types of accuracy (predictable, repeatable, relative) as well as fix rate and dimension, system: capacity, ambiguity, and coverage.
2 Reliability criteria – they form a separate group of indicators with reference to characteristics of systems exploitation. Reliability, availability and continuity are among them.
3 The safety of exploitation criteria – their function is to give the user current information about the quality (status) of operating system allowing for the proper level of their utility. So far, integrity, the only criterion belonging to this group, has been characterised by a wide range of variables such as: time to alarm, the probability of false alarm etc. [Ober P. B., 1999].

So far perceiving of those two groups (positioning and reliability) has differentiated them due to the methods of statistical and probabilistic inference. The classic approach to analogous estimation of a system is characterized by the following assumptions: the determining error is a random variable, it does not respect playtimes of the system work and also the lack of estimation of the reliability of the system. In this case the new probability characteristic, which joins the accuracy and one of the reliability criterions – availability of the certain value of position error could be introduced. Let's define the availability of certain value of position error – as a probability that in any moment of time (t) the position error of determining coordinates (δ_n) is lower or equal then the arbitrary acceptable value (U), which mean than $\delta_n \le U$, $n = 1,2,...$. The suggested approach treats the lifetimes and the times of failure as the random variables being in relation in the fixed value of the position error and also it introduces the measures which making the reliability estimation possible.

2 WORKING PROCESS OF THE POSITIONING SYSTEM

Let the position error of any navigational system, in the function of time, be a variable taking its values from the given interval of errors $\langle 0, \infty)$. Assume that the process of determining the coordinates is alternating with the renew in general reliability theory sense. Then we can recognize two states: the working one – the state where the error $\delta_n \le U$ for $n = 1,2,...$ and the state of failure where $\delta_n > U$. Let $X_1, X_2,...$ be the working times while $Y_1, Y_2,...$ are the times of failures. Hence the moments: $Z_n^{'} = X_1 + Y_1 + X_2 + Y_2 + ... + Y_{n-1} + X_n$, $n = 1,2,...,$ are the moments of failures and $Z_n^{''} = Z_n^{'} + Y_n$, are the moments of renewal. Assume also that the random variables X_i, Y_i, $i = 1,2,...$ are independent and the working failure times have the same distributions. Let's define the analytical form of the distributions of the variables X_n and Y_n as

$$P(X_i \le x) = F(y), \tag{1}$$

$$P(Y_i \le y) = G(y) \quad \text{for} \quad i = 1,2,..., \tag{2}$$

where: $F(x)$, $G(y)$ means the distribution functions of X_n and Y_n.

Introduce also the notations of the expected value and the variance as follows

$$E(X_i) = E(X), \quad E(Y_i) = E(Y) \tag{3}$$

and

$$V(X_i) = \sigma_1^2, \quad V(Y_i) = \sigma_2^2, \quad i = 1,2,...., \tag{4}$$

where: $E(X_i)$, $E(Y_i)$ - the expected values of the working and failure times. We have also to admit that $\sigma_1^2 + \sigma_2^2 > 0$.

3 GENERAL MODEL

Let's define the reliability process in which the relation between the single measurement error δ_n and the parameter U decide about its state (work or failure). Let $\alpha(t)$ be the binary interpretation of the reliability state of the process as:

$$\alpha(t) = \begin{cases} 1, & Z_n^{''} \le t < Z_{n+1}^{'} \\ 0, & Z_{n+1}^{'} \le t < Z_{n+1}^{''} \end{cases} \quad \text{for} \quad n = 0,1,... \tag{5}$$

The state $\alpha(t) = 1$ means that in the moment t the error of the single measurement is less or equal than U. In the opposite case for $\delta_n > U$, the system is in the state of failure. The availability of a certain value of position error will be denoted as

$$D(t) = P[\delta(t) \le U]. \tag{6}$$

Let us consider the following sequence of events such that

$$V_n = \{Z_n^{''} \le t < Z_{n+1}^{'}\}, \quad n = 0,1,2... . \tag{7}$$

It means that at t moment $\delta(t) \le U$ and up to t moment exactly n renewals took place. As the events $V_0, V_1, ..., V_n$ are pairwise mutually exclusive then:

$$P[\delta(t) \le U] = P\left(\bigcup_{n=0}^{\infty} V_n\right) = \sum_{n=0}^{\infty} P(V_n). \tag{8}$$

To define the value of $\sum_{n=0}^{\infty} P(V_n)$ we introduce the additional notations:

$$S_n^{'} = X_1 + X_2 + ... + X_n, \quad P(S_n^{'} \le x) = F_n(x) \tag{9}$$

and

$$S_n^{''} = Y_1 + Y_2 + ... + Y_n, \quad P(S_n^{''} \le y) = G_n(y). \tag{10}$$

The variables $S_n^{'}$ and $S_n^{''}$ corresponds to cumulative times of work and failure of the process of determining the coordinates of position.

The distribution functions $F_n(x)$ and $G_n(y)$ could be found by the n-times convolution operation. For any n we get the final form [Barlow R. E., Proshan F. 1975] as

$$F_n(t)=\int_0^t F_{n-1}(t-x)\,dF(x) \tag{11}$$

$$G_n(t)=\int_0^t G_{n-1}(t-y)\,dG(y), \quad n=2,3..., \tag{12}$$

As $Z''_n = S'_n + S''_n$ so

$$\Phi_n(t)=P\left(Z''_n \le t\right)=\int_0^t F_n(t-u)\,dG_n(u), \quad n=1,2,... \tag{13}$$

where $\Phi_n(t)$ is the distribution function of Z''_n.

To determine $D(t)$ let's compute the probabilities $P(V_0)$ and $\sum_{n=1}^{\infty} P(V_n)$ separately. As for

$$t \ge 0 \; V_0 = \left\{Z''_0 \le t < Z'_{n+1}\right\}=\left\{0 \le t < X_1\right\} \tag{14}$$

hence

$$P(V_0)=P(X_1 > t)=1-P(X_1 \le t)=1-F(t), \tag{15}$$

however $P(V_n)$, $n=1,2...$ we find from the formula of total probability

$$\begin{aligned}P(V_n)&=P\left(Z''_n \le t < Z'_{n+1}\right)=P\left(Z''_n \le t < Z'_n + X_{n+1}\right)=\\&=\int_0^t P\left[Z''_n \in [x, x+dx)\right]\cdot P(X > t-x)\end{aligned} \tag{16}$$

Then

$$P(V_n)=\int_0^t d\Phi_n(x)\left[1-F(t-x)\right]=\int_0^t\left[1-F(t-x)\right]d\Phi_n(x) \tag{17}$$

The availability is calculated as a sum probabilities of the mutually exclusive events

$$\begin{aligned}D(t)&=P\left[\delta(t)\le U\right]=P\left[\alpha(t)=1\right]=\\&=\sum_{n=0}^{\infty}P(V_n)=P(V_0)+\sum_{n=1}^{\infty}P(V_n)\end{aligned} \tag{18}$$

Substituting (15) and (17) to (18) we obtain final form for availability of the certain value of position error as follows

$$D(t)=1-F(t)+\int_0^t\left[1-F(t-x)\right]dH_\Phi(x), \tag{19}$$

where

$$H_\Phi(x)=\sum_{n=1}^{\infty}\Phi_n(x) \tag{20}$$

is a renewal function of stream made of the renewal moments.

4 EXPONENTIAL MODEL

Typical realizations of the operating time in navigational systems are characterized by the exponential distributions of the lifetime and the time of failures due to the property called the "memoryless" property. Let define the exponential process where the distribution functions as

$$F(t)=\begin{cases}1-e^{-\lambda t} & \text{for} \quad t>0\\ 0 & \text{for} \quad t\le 0\end{cases}, \tag{21}$$

$$G(t)=\begin{cases}1-e^{-\mu t} & \text{for} \quad t>0\\ 0 & \text{for} \quad t\le 0\end{cases}, \tag{22}$$

where λ, μ are failure and renewal rates. When substituting (21) and (22) to (19) we obtain

$$\begin{aligned}D_{\exp}(t)&=1-F(t)+\int_0^t\left[1-F(t-x)\right]dH_\Phi(x)=\\&=e^{-\lambda t}+\int_0^t\left[1-\left(1-e^{-\lambda(t-x)}\right)\right]dH_\Phi(x),\end{aligned} \tag{23}$$

where $D_{\exp}(t)$ denotes the availability of the certain value of position error in the navigational system in the case of the exponential life and failure times distributions. After few simple transformations [Specht C., 2003] finally form could be find as

$$D_{\exp}(t)=\frac{\mu}{\lambda+\mu}+\frac{\lambda}{\lambda+\mu}e^{-(\lambda+\mu)t} \tag{24}$$

5 EXPERIMENT

The measurement campaign focuses on methodology verification. Two-weeks registration was done based on EGNOS augmentation system. More than 2 millions fixes were used for availability of the certain value of position error calculations (fig. 1).

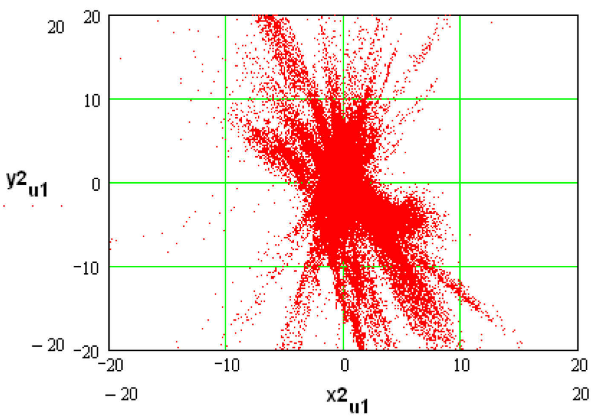

Fig. 1. Scatter plot for EGNOS system (500.000 fixes, values in meters)

For establishing availability functions, three decision limits were fixed: $\delta < 1\ m$, $\delta < 2\ m$, $\delta < 3\ m$.

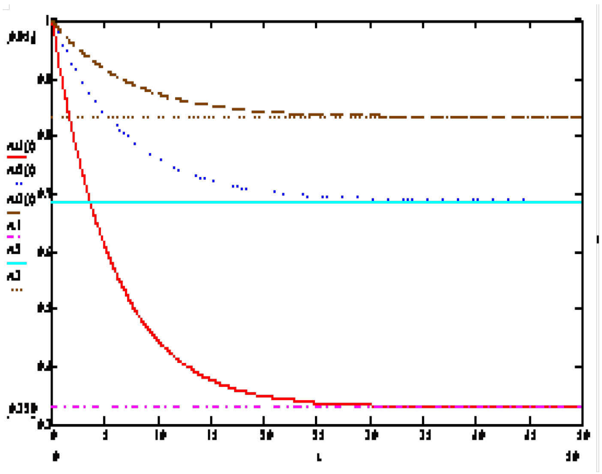

Fig. 2. Availability of the certain value of position error functions for: $\delta < 1\ m$ (red), $\delta < 2\ m$ (blue), $\delta < 3\ m$ (brown) with limited values

6 CONCLUSIONS

The article presents the mathematical model of availability of the certain value of position error calculation. The classic approach to analogous estimation is characterized by the following assumptions: the determining error is a random variable, it does not respect playtimes of the system work and new one suggested approach treats the lifetimes and the times of failure as the random variables being in relation with the fixed value of the position error. General and also exponential statistical models were presented. The model were also verified based on EGNOS system measurements.

ACKNOWLEDGEMENTS

Polish State Committee for Scientific Reseach has supported this work. Grant No 4 T12C 064 27.

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