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The use of estimation of position coordinates with constraints in navigation

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ABSTRACT: This article presents method of estimation of position coordinates with the use of conditions imposed on measurements. The application of two (or more) systems used to define position and taking into account the way they are related, makes it possible to improve both accuracy and reliability of the measured navigational parameters. The method of estimation with constraints is based on the method of determining extremes with constraints of many variables. In geodesy, this method is known as adjustment by conditions.

1 INTRODUCTION

In classical navigation the position of a vessel is defined as one chosen point. Most often it is the position of an antenna of the radio-navigational system or the measurements are treated as one common point, e.g. the centre of mass. Nowadays when the navigating techniques used for establishing ship's position are so easily accessible and relatively cheap, more than one device can be employed at the same time. In order to work out such measurements, a method which makes use not only the measured data but also mutual location of the antennas can be employed. The antennas can be positioned in a different configuration in relation to one another, i.e. they can be linear, angular or angular and linear. That is why in such a case estimation with constraints, known as estimation with constrained equations [7] or, in geodesy, as adjustment by conditions [2], [6] can be used. This method has its deterministic origin in constrained optimization which makes use of Lagrange multipliers [3], [4], and [5]. There are the following advantages in this method: fewer errors of systematic measurements of ship's coordinates and the increased accuracy and reliability of the calculated ship's trajectory.

This article presents method of estimation of position coordinates with the use of constraints imposed on measurements using true measurements of navigational satellite systems GPS (DGPS). The research has been carried out in static conditions, on an antenna platform located at Maritime University of Szczecin at places with known coordinates and mutual positions.

2 ESTIMATION WITH CONSTRAINTS

The core of the method of estimation with constraints is to find the minimum of a given function where the variables of that function are constrained in a defined way. In our case the sum of squares of deviations (the rule of the smallest squares) from the position of two average receivers antennas with known distance between them was minimized. This situation is illustrated by Figure1. Antenna '1' has the mean position $A_1(x_1, y_1)$ and antenna '2' – $A_2(x_2, y_2)$. Additionally, we know that the true distance (binds) equals d.

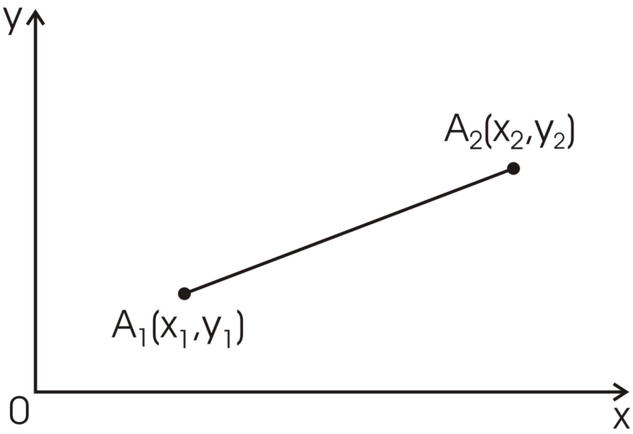

Fig. 1. Layout of two antennas GPS (DGPS)

We are searching the minimum of the following function:

$$f(\Delta x_1, \Delta x_2, \Delta y_1, \Delta y_2) =$$

$$\Delta x_2^2 + \Delta y_1^2 + \Delta y_2^2 + \Delta x_1^2 \to \min, \quad (1)$$

with constraint

$$g(\Delta x_1, \Delta x_2, \Delta y_1, \Delta y_2) = [(x_2 + \Delta x_2) - (x_1 + \Delta x_1)]^2 +$$

$$[(y_2 + \Delta y_2) - (y_1 + \Delta y_1)]^2 - d^2 = 0, \quad (2)$$

where:

Δx_1, Δx_1	–	deviation (correction) from the position of medium antenna A_1,
Δx_2, Δx_2	–	deviation (correction) from the position of medium antenna A_2,
d	–	distance between antennas '1' and '2' at the horizon plane.

In this case the Lagrange function will take the following form:

$$L(\Delta x_1, \Delta x_2, \Delta y_1, \Delta y_2, \lambda) = \Delta x_2^2 + \Delta y_1^2 + \Delta y_2^2 + \Delta x_1^2 +$$
$$\lambda\{[(x_2 + \Delta x_2) - (x_1 + \Delta x_1)]^2 +$$
$$[(y_2 + \Delta y_2) - (y_1 + \Delta y_1)]^2 - d^2\} =$$
$$= \Delta x_2^2 + \Delta y_1^2 + \Delta y_2^2 + \Delta x_1^2 + \lambda[(x_2 - x_1)^2 +$$
$$2(x_2 - x_1)(\Delta x_2 - \Delta x_1) + (\Delta x_2 - \Delta x_1)^2 + (y_2 - y_1)^2 +$$
$$2(y_2 - y_1)(\Delta y_2 - \Delta y_1) + (\Delta y_2 - \Delta y_1)^2 - d^2] \quad (3)$$

where: λ is Lagrange multiplier.

We introduce the following symbols to make the notation simpler:

$\Delta x = x_2 - x_1$ - the difference between mean positions X-axis (4)

$\Delta y = y_2 - y_1$ - the difference between mean positions Y-axis (5)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{\Delta x^2 + \Delta y^2}$$

- the distance between mean positions of the antennas (6)

Now the equation (3) has simpler form:

$$L(\Delta x_1, \Delta x_2, \Delta y_1, \Delta y_2, \lambda) = \quad \Delta x_2^2 + \Delta y_1^2 + \Delta y_2^2 + \Delta x_1^2 +$$
$$\lambda[\Delta x^2 + 2\Delta x(\Delta x_2 - \Delta x_1) +$$
$$(\Delta x_2 - \Delta x_1)^2 + \Delta y^2 + 2\Delta y(\Delta y_2 - \Delta y_1) +$$
$$(\Delta y_2 - \Delta y_1)^2 - d^2] \quad (7)$$

Now the values of the following corrections will be the solution to problem (1) with constraint (2)

$$\Delta x_1 = \frac{D-d}{2D}\Delta x, \ \Delta y_1 = \frac{D-d}{2D}\Delta y, \quad (8)$$

$$\Delta x_2 = -\frac{D-d}{2D}\Delta x, \ \Delta y_2 = -\frac{D-d}{2D}\Delta y. \quad (9)$$

The corrections calculated with functions (8) and (9) are added to the mean position of the antennas and in this way we obtain the corrected coordinates of the antennas:

$$x_1' = x_1 + \Delta x_1, \ y'_1 = y_1 + \Delta y_1, \quad (10)$$

$$x_2' = x_2 + \Delta x_2, \ y'_2 = y_2 + \Delta y_2. \quad (11)$$

Formulas (10) and (11) define the corrected mean positions of the antennas: $A_1'(x_1', y_1')$ and $A_2'(x_2', y_2')$, with constraint (2).

3 THE ANALYSIS OF LINEAR CONSTRAINTS CASE

In the analyzed case of linear constraints of the mean positions of the antennas, the following situations can be taken into consideration:

1 the estimated earlier mean positions of the antennas are not affected by relative systematic errors, i.e. $D = d$ and then according to formulas (8) and (9) corrections of the coordinates will equal zero;
2 the estimated earlier mean positions of the antennas are affected by relative systematic errors, i.e. $D \neq d$ and then one of the two cases will be noted:
   - $D > d$ and the corrected mean positions of the antennas are placed on a straight line joining points A_1, A_2 and within the segment $\overline{A_1 A_2}$,

– $D < d$ and the corrected mean positions of the antennas are placed on a straight line joining points A1, A_2 and outside the segment $A_1 A_2$.

Figure 2 illustrates the above described cases.

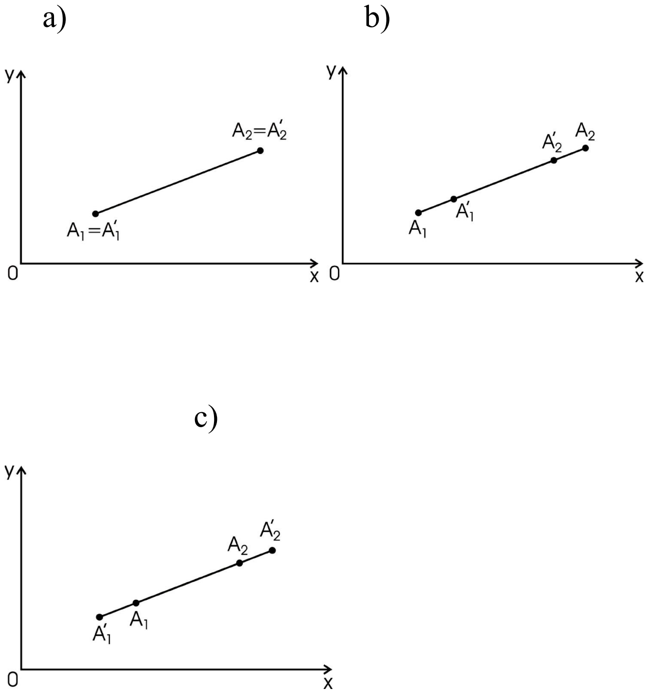

Fig. 2. The location of mean positions of the antennas in relation to each other and the corrected mean positions of the antennas

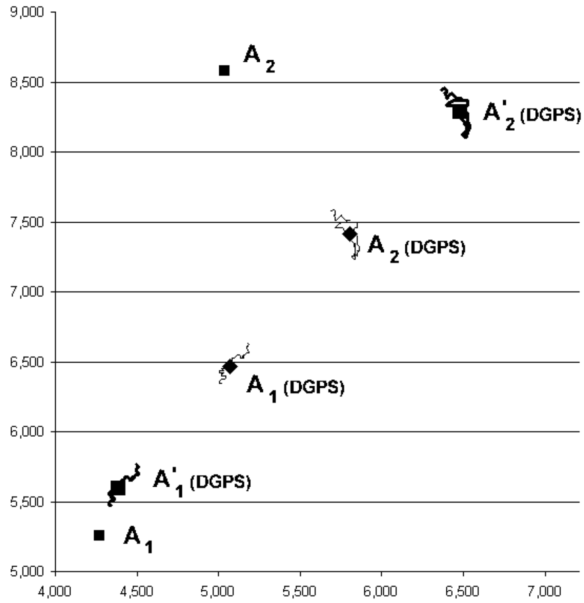

Fig. 3. Estimation with constraints with the use of true GPS/DGPS measurements

The deviations of the corrected positions from the true positions of the antennas may result from the errors of the systematic measurements of DGPS or also (in this case) from the systematic error of coordinates transformation from the ‘65’ system (the catalogue of points of geodesic matrix) into WGS-84 system.

4 ESTIMATION WITH CONSTRAINTS OF THE MEAN VALUES OF COORDINATES OF GPS/DGPS POSITIONS

True measurements will illustrate our investigations. The readings were taken on the antenna platform of the Maritime University of Szczecin on 26 April 2006. The positions of the antennas were established geodetically in ‘65’ system and then their coordinates were transformed into WGS-84 system- points A_1, A_2, whereas the measurements in the receiver were taken in WGS-84 system only- A_1(DGPS), A_2(DGPS). The positions of the antennas A'_1(DGPS) and A'_2(DGPS) were obtained once the corrections of the coordinates resulting from the imposed conditions on the distance between two antennas (in this case d = 3.406 m) were taken into consideration.

5 ESTIMATION WITH CONSTRAINS – MOVING VESSELS

The method of estimation with constrains can also be employed for dynamic measurements. So when the measurements are not the estimated values (e.g. from Kalman filter or from stochastic approximation) then we will have to do with a deterministic case – a conditioned extreme value. This situation is illustrated by the pictures below (Fig. 4. and Fig. 5.) The former presents combination of two circulations put together. The outer curves represent the original GPS measurements affected by great errors and the inner curves represent measurements which take into account constraints (1.897 m). The enlarged fragment of circulation is shown in Figure 5. Now, big systematic errors made in GPS measurements can clearly be seen.

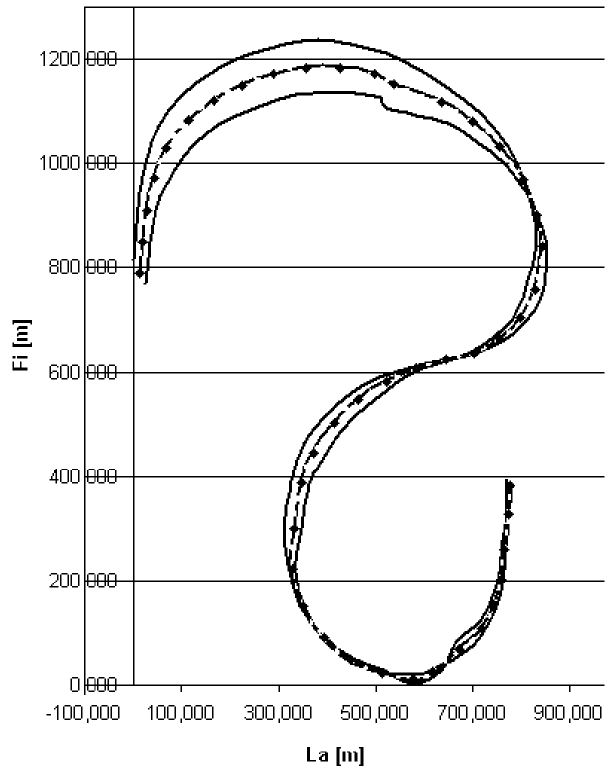

Fig. 4. Estimation with constraints with the use of true GPS measurements - circulating vessel ('eight'- shaped curve)

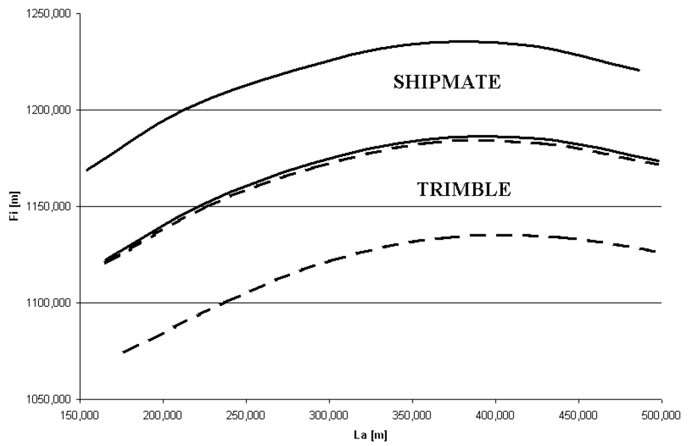

Fig. 5. The enlarged part of Figure 4- the outer curves represent mean trajectories of given receivers and the inner curves represent the trajectories of mean values with constraints

6 CONCLUSION

The presented method of estimation of the position of ship's coordinates with constraints can be used when at least two navigational receiving systems are employed. It can be spread to angular and linear-angular constraints (three or more receivers). The constraints can be defined either by direct linear measurements or by angular-linear ones or can be calculated analytically when the positions of antennas in relation to ship's construction elements are known.

In case when the coordinates of ends of the segment (two antennas) are measured and there is only one linear constraint, then the solution is represented by two points on one line with mean positions of direct measurements and placed symmetrically, in relation to them (Figures 2a-2c). The shape is represented correctly and the direction of the segment remains unchangeable. This means that the direction of the segment is marked by measuring error resulting not from one, but from two receivers which with their positive correlation (resulting from the constrains) may lead to the increased accuracy of the defined direction/course (true course) [1] and the spatial position of the vessel. It has significant importance both in hydrographic measurements and in navigation in restricted areas.

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