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The Safe Control Sensitivity Functions in Matrix Game of Ships

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ABSTRACT: The paper describes the use of matrix game theory for the synthesis of safe control of a ship in collision situations. An analysis of the sensitivity of the ship control algorithm to the inaccuracy of process state information and changes in its parameters was presented. Sensitivity characteristics were compared on the example of the navigational situation in the Kattegat Strait for good and restricted visibility at sea.

1 BACKGROUND

One of the most important issues in marine navigation is safe control of the ship's movement. As a result of the relative movement of own ship with the speed V and the course ψ and the met j-th ship moving at the speed Vj and course ψj, a certain situation at sea is determined (Lisowski 2016).

The variables characterizing situation in the form of distance Dj and bearing Nj to j-th met ship are measured by Automatic Radar Plotting Aids ARPA anti-collision system (Lebkowski 2018).

The standard ARPA system performs automatic tracking of 20 encountered objects, determination of their speed and courses as well as parameter s approach to the own ship - Distance of the Closest Point of Approach $D_{j\min} = DCPA_j$ and Time to the Closest Point of Approach $T_{j\min} = TCPA_j$ (Fig. 1).

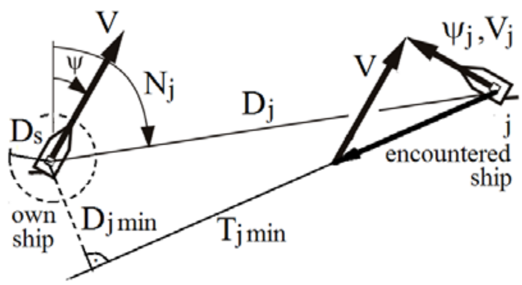

Figure 1. Passing own ship with the *j*-th encountered ship.

The proper use of the ARPA anti-collision system in order to achieve greater safety of navigation requires, in addition to the preparation of its operation and data interpretation, supplementing the system with appropriate algorithms of computer navigator maneuvering decision support, eliminating human characteristics and taking into account the uncertainty of the situation and the game properties of the control process (Lazarowska 2017, Lisowski 2014, Malecki 2013, Mohamed-Seghir 2016).

The necessity to take into account the strategies of the ships encountered and their kinematics and dynamics as control objects determines the application of the differential game model for analysis (Isaacs 1965, Lisowski 2012, 2014).

Apart from the ship's dynamics equation, the differential game model can be reduced to matrix game model and j-th participants (Osborne 2004).

Taking into account the practical application of algorithm for controlling your own ship in a collision situation, it is advisable to conduct the sensitivity analysis of safe control, on one hand, accuracy of information from ARPA anti-collision system, and on the other changes of kinematic and dynamic parameters of the control process.

2 GAME SHIP CONTROL

The game matrix $\boldsymbol{R}[r_j(\delta_j,\delta_0)]$ includes values of the collision risk r_j determined on basis of data obtained from the ARPA anti-collision system for the acceptable strategies δ_0 of the own ship and acceptable strategies δ_j of any particular number of J encountered ships (Fig. 2).

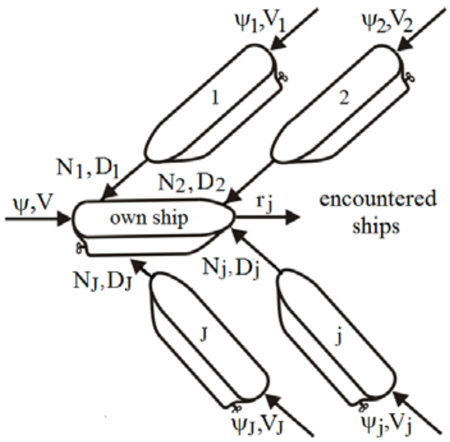

Figure 2. Block diagram of the matrix game of ships.

The risk value is defined by referring the current situation of approach, described by parameters D_{jmin} and T_{jmin}, to the assumed evaluation of the situation as safe, determined by a safe distance of approach D_s and a safe time T_s – which are necessary to execute a collision avoiding manoeuvre (Modarres 2006):

$$r_j = \left[\sqrt{c_1\left(\frac{D_{j\min}}{D_s}\right)^2 + c_2\left(\frac{T_{j\min}}{T_s}\right)^2 + c_3\left(\frac{D_j}{D_s}\right)^2} \right]^{-1} \quad (1)$$

The weight coefficients c_1, c_2 and c_3 are depended on the state visibility at sea, dynamic length L_d and dynamic beam B_d of the ship, and kind of water region - open waters or fairways (Fig. 3).

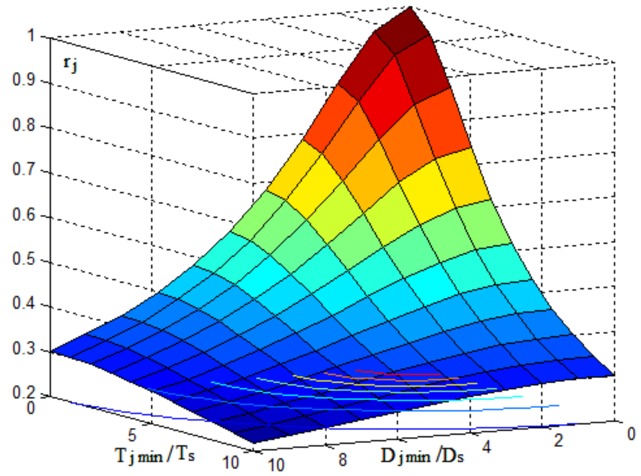

Figure 3. An example of the dependence of collision risk on the relative values of distance and time of approaching ships.

In a matrix game player 1 (own ship) has a possibility use δ_0 pure various strategies, and player 2 (encountered J ships) has δ_j various pure strategies:

$$R = [r_j(\delta_j, \delta_0)] = \begin{vmatrix} r_{11} & r_{12} & \cdots & r_{1,\delta_0-1} & r_{1\delta_n} \\ r_{21} & r_{22} & \cdots & r_{2,\delta_0-1} & r_{2\delta_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\delta_1 1} & r_{\delta_1 2} & \cdots & r_{\delta_1,\delta_0-1} & r_{\delta_1\delta_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\delta_j 1} & r_{\delta_j 2} & \cdots & r_{\delta_j,\delta_0-1} & r_{\delta_j\delta_n} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ r_{\delta_m 1} & r_{\delta_m 2} & \cdots & r_{\delta_m,\delta_0-1} & r_{\delta_m\delta_n} \end{vmatrix} \quad (2)$$

The constraints for the choice of a strategy (δ_0,δ_j) result from the recommendations of the COLREGs Rules (Szlapczynski & Szlapczynska 2016).

In a matrix game player 1 has a possibility to use δ_0 pure various strategies, and player 2 has ν_j various pure strategies. Constraints limiting the selection of a strategy result from COLREGs Rules. As most frequently the game does not have a saddle point, therefore the balance state is not guaranteed. In order to solve this task, we may use a dual linear programming (Nisan at al. 2007, Kula 2014).

In a dual problem player 1 aims to minimize the risk of collision and player 2 aims to minimize the collision risk. The components of the mixed strategy express the distribution of the probability of using by the players their pure strategies. As a result, for the optimal control quality index in the form:

$$I^* = \min_{\delta_0} \min_{\delta_j} r_j \quad (3)$$

matrix probability $\boldsymbol{P}$ of applying each one of the particular pure strategies is obtained:

$$P=[p_j(\delta_j,\delta_0)]=\begin{vmatrix} p_{11} & p_{12} & .... & p_{1,\delta_0-1} & p_{1\delta_n} \\ r_{21} & r_{22} & .... & r_{2,\delta_0-1} & r_{2\delta_n} \\ .... & .... & .... & .... & .... \\ p_{\delta_1 1} & p_{\delta_1 2} & .... & p_{\delta_1,\delta_0-1} & p_{\delta_1\delta_n} \\ .... & .... & .... & .... & .... \\ p_{\delta_j 1} & p_{\delta_j 2} & .... & p_{\delta_j,\delta_0-1} & p_{\delta_j\delta_n} \\ .... & .... & .... & .... & .... \\ p_{\delta_m 1} & p_{\delta_m 2} & .... & p_{\delta_m,\delta_0-1} & p_{\delta_m\delta_n} \end{vmatrix} \tag{4}$$

The solution for the control problem is the strategy representing the highest probability:

$$I^* = u_{0\delta_0}\left\{ \left[p_j\left(\delta_j, \delta_0\right)\right]_{\max} \right\} \tag{5}$$

The safe trajectory of own ship is treated as a sequence of successive changes in time of her course and speed. A safe passing distance is determined for the prevailing visibility conditions at sea D_s, advance time to the manoeuvre t_m and duration of one stage of the trajectory Δt_k as a calculation step.

At each one step the most dangerous object relative to the value of the collision risk r_j is determined. Then, on the basis of semantic interpretation of COLREGs Rules, the direction of the own ship turn relative to the most dangerous ship is selected. A collision risk matrix $\boldsymbol{R}$ is determined for the acceptable strategies of the own ship δ_0 and that for the *j-th* encountered ship δ_j.

By applying a principle of the dual linear programming for solving matrix games the optimal course of the own ship and that of the *j-th* ship is obtained at a level of the smallest deviations from their initial values (Zak 2013).

Fig. 4 presents the hyper-surface of the collision risk for values δ_0 and δ_j of the strategies.

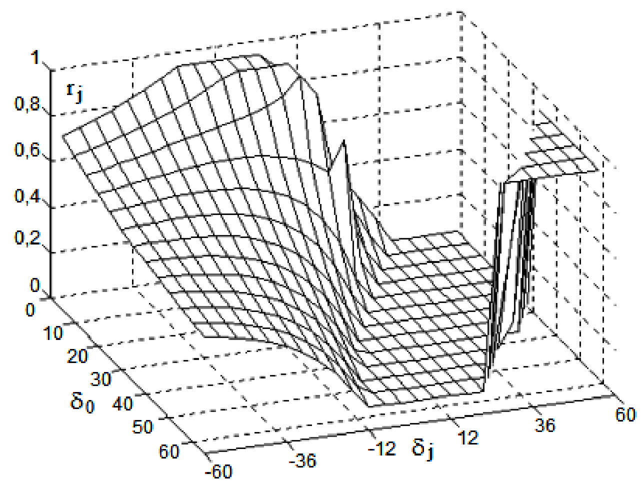

Figure 4. Dependence of the collision risk on the course strategies of the own ship δ_0 and those δ_j of the encountered ship.

If, at a given step, there is no solution at the own ship speed V, then the calculations are repeated for a speed decreased by 25%, until the game has been solved.

The calculations are repeated step by step until the moment when all elements of the matrix $\boldsymbol{R}$ are equal to zero and the own ship, after having passed encountered ships, returns to her initial course and speed (Tomera 2012).

By dual using function *linprog – linear programming* from Optimization Toolbox of MATLAB software the cooperative multi-step Matrix Game *MG* algorithm was developed to determine the safe game trajectory of ship in collision situation.

3 CONTROL SENSITIVITY ANALYSIS

The sensitivity theory methods were widely used for solving various theoretical and applied problems with analysis and synthesis, identification, adjustment, monitoring, testing, tolerance distribution (Eslami 1994, Wierzbicki 1977).

At the same distinction is made between the sensitivity of model control process for changing its parameters and process optimal control sensitivity to changes in its parameters and disturbance influence (Rosenwasser & Yusupov 2000).

In previous papers dealt with sensitivity of deterministic systems do not game systems.

At sea, land and air transport processes occur of own ship and many encountered ships. Control of such processes, due to the high proportion of human subjectivity in the decision-making maneuver, often takes the control as game character (Lisowski 2013).

Main investigation method in sensitivity theory consists in using so called sensitivity functions.

The first-order sensitivity functions f_x of optimal control u of game process described by state variables x can be presented as following partial derivate:

$$f_x = \frac{\partial I\left[x(u)\right]}{\partial x} \tag{6}$$

It is also possible consider of the sensitivity functions r-th order of optimal control $f_{r,x}$ in the following form:

$$f_{r,x} = \frac{\partial^r I\left[x(u)\right]}{\partial x_1^{r_1} \ldots \partial x_n^{r_m}} \qquad r_1 + r_2 + \ldots + r_m = r \tag{7}$$

Quality game control index $I_{0,j}$ takes the form of game payment, consisting of integral payments and final payment:

$$I = \int_{t_0}^{t_K} \left[x(t)\right]^2 + r_j\left(t_K\right) + d\left(t_K\right) \tag{8}$$

The integral payment represents loss of way by the ship while passing the encountered ships and the final payment determine the final risk of collision $r_j(t_K)$ relative to the j-th ship and the final deviation of own ship trajectory $d(t_K)$ from the reference trajectory.

The investigation of sensitivity of the game control makes, for sensitivity analysis of the final payment $d(t_K)$:

$$f_{xi} = \frac{\partial d(t_K)}{\partial x_i} \qquad (9)$$

Taking into consideration the practical application of the game control algorithm for the own ship in a collision situation it is recommended to perform the analysis of sensitivity of a safe control with regard to the accuracy degree of the information received from the anti-collision *ARPA* radar system in the current approach situation, from one side and also with regard to the changes in kinematical and dynamic parameters of the control process from the other side.

Admissible average errors, it can be contributed by sensors of anti-collision system can have following values for:

- radar,
  - bearing: ±0,22°,
  - form of cluster: ±0,05°,
  - form of impulse: ±20 m,
  - margin of antenna drive: ±0,5°,
  - sampling of bearing: ±0,01°,
  - sampling of distance: ±0,01 nm,
  - gyrocompass: ±0,5°,
- log: ±0,5 kn,
- GPS: ±15 m.

The algebraic sum of all errors, influent on picturing of navigational situation, cannot exceed ±5% or ±3°.

3.1 *Sensitivity of Safe Ship Control to Inaccuracy of Information from ARPA System*

Let *X* represent such a set of state process control information on the navigational situation that:

$$X = [V, \psi, V_j, \psi_j, D_j, N_j] \qquad (10)$$

Let then X_e represent a set of information from *ARPA* system containing errors of measurement and processing parameters:

$$X_e = [V \pm \delta V, \psi \pm \delta\psi, V_j \pm \delta V_j, \psi_j \pm \delta\psi_j, D_j \pm \delta D_j, N_j \pm \delta N_j] \qquad (11)$$

Relative measure of sensitivity of the final payment in the game f_x as a final deviation of the ship safe trajectory d_K from the reference trajectory will be:

$$f_x = \left| \frac{d_K(X_e) - d_K(X)}{d_K(X)} \right| 100\% \qquad (12)$$

$$f_x = [s_V, s_\psi, s_{Vj}, s_{\psi j}, s_{Dj}, s_{Nj}] \qquad (13)$$

3.2 *Sensitivity of Safe Ship Control to Process Parameters Alterations*

Let *P* represent a set of parameters of the state process control:

$$P = [t_m, D_s, \Delta t_k, \Delta V] \qquad (14)$$

Let then P_e represent a set of parameters containing errors of measurement and processing parameters:

$$P_e = [t_m \pm \delta t_m, D_s \pm \delta D_s, t_k \pm \delta t_k, \Delta V \pm \delta\Delta V] \qquad (15)$$

Relative measure of sensitivity of final payment in the game f_p as a final deflection of the ship safe trajectory d_K from the assumed trajectory will be:

$$f_p = \left| \frac{d_K(P_e) - d_K(P)}{d_K(P)} \right| 100\% \qquad (16)$$

$$f_p = [s_{tm}, s_{Ds}, s_{\Delta tk}, s_{\Delta V}] \qquad (17)$$

where:

t_m - advance time of the manoeuvre with respect to the dynamic properties of the own ship,

t_k - time of one stage of the ship's trajectory,

D_s – safe distance,

T_s – safe time of approach.

4 SENSITIVITY FUNCTIONS

Computer simulation of *MG* algorithm, as a computer software supporting the navigator manoeuvring decision, were carried out on an example of a real navigational situation of passing *J*=9 encountered ships (Fig. 5), (Tab. 1).

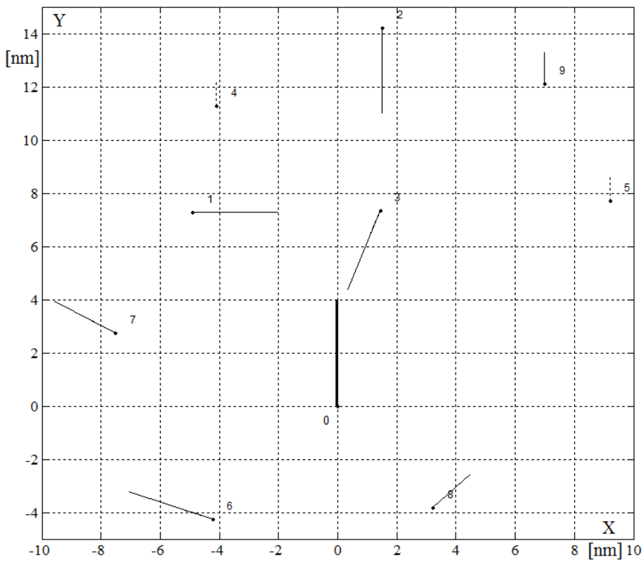

Figure 5. Nineteen-minute speed vectors of the own ship and nine encountered ships in a situation in the Kattegat Strait.

Table 1. Own and encountered ships movement parameters.

j	D_j [nm]	N_j [deg]	V_j [kn]	ψ_j [deg]
0	-	-	20.0	00.0
1	08.8	326	14.5	090
2	14.3	006	16.0	180
3	07.5	011	16.0	200

4	12.0	340	00.0	000
5	12.0	050	00.0	000
6	06.0	225	15.0	290
7	08.0	290	12.0	300
8	05.0	140	09.0	045
9	14.0	030	06.0	000

The situations were registered in Skagerrak Strait on board r/v HORYZONT II, a research and training vessel of Gdynia Maritime University, on the radar screen of ARPA anti-collision system Raytheon.

4.1 *Sensitivity Functions of Game Ship Control in Good Visibility at Sea for Ds=0.5 nm*

The safe trajectory of own ship and sensitivity functions determined by *MG* algorithm in Matlab software are presented in Fig. 6 and 7.

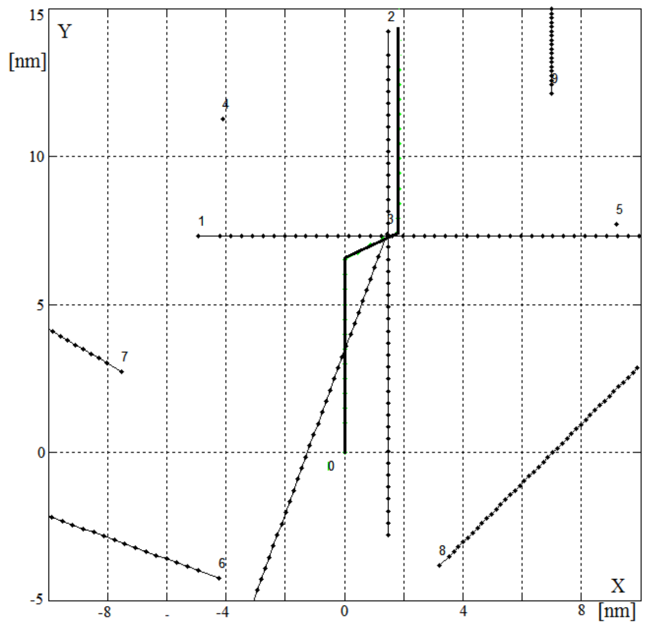

Figure 6. The safe trajectory of own ship for *MG_gv* algorithm in good visibility at sea D_S=0.5 *nm* in situation of passing *J*=9 encountered ships, $r(t_K)$=0, $d\ (t_K)$=2.15 *nm*.

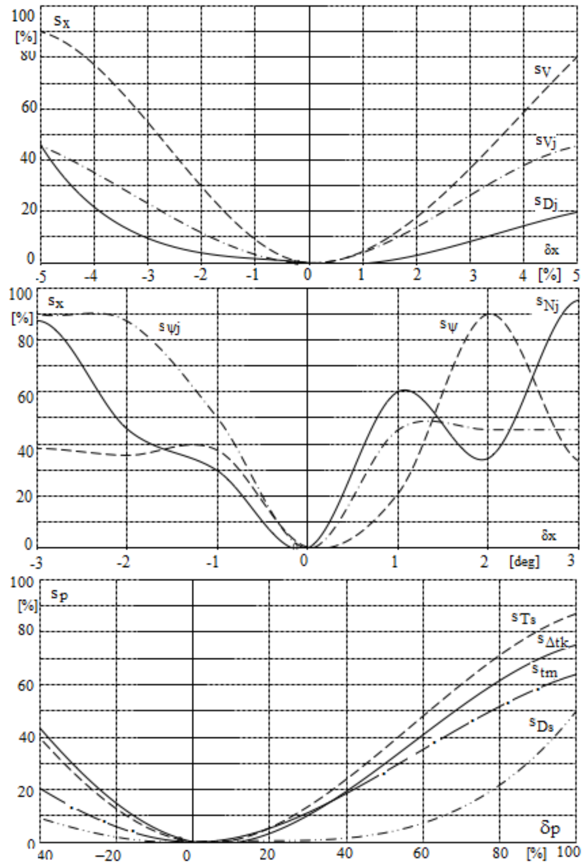

Figure 7. Sensitivity functions of the matrix game control of own ship in good visibility at sea according to *MG_gv* algorithm.

4.2 *Sensitivity Functions of Game Ship Control with Restricted Visibility at Sea for Ds=1.5 nm*

The safe trajectory of own ship and sensitivity functions determined by *MG* algorithm in Matlab software are presented in Fig. 8 and 9.

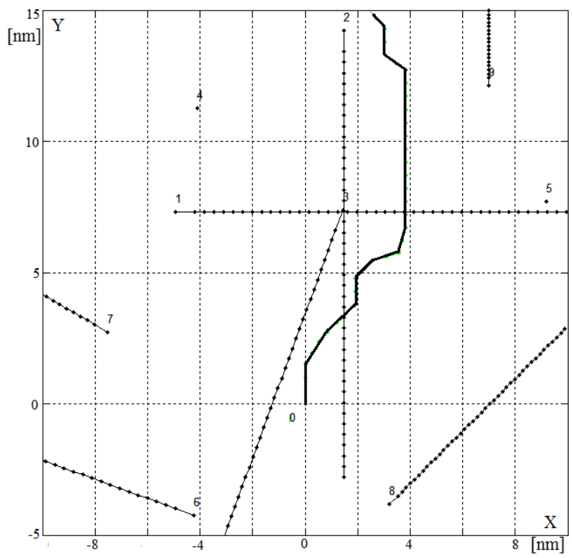

Figure 8. The safe trajectory of own ship for *MG_rv* algorithm in restricted visibility D_S=1.5 *nm* in situation of passing *J*=9 encountered ships, $r(t_K)=0$, $d(t_K)$=4.54 *nm*.

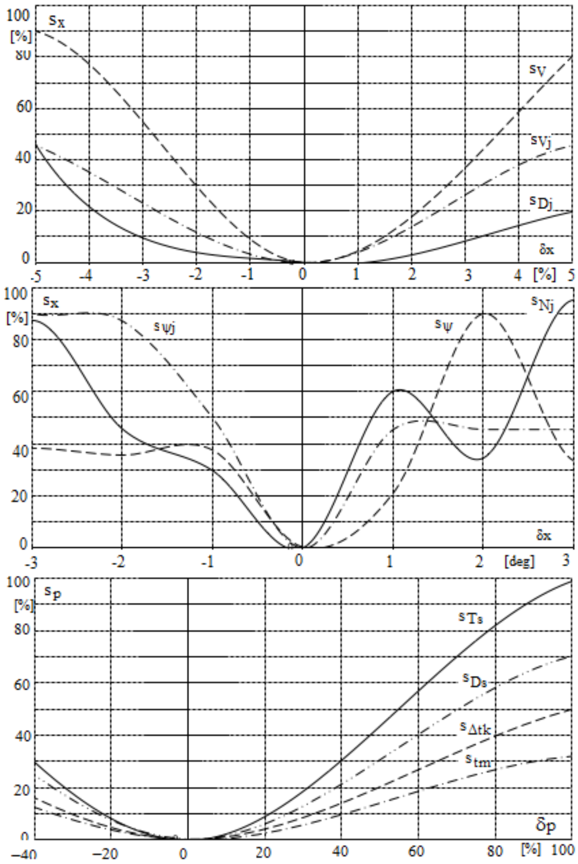

Figure 9. Sensitivity functions of the matrix game control of own ship in restricted visibility at sea according to *MG_rv* algorithm.

5 CONCLUSIONS

The algorithm of multistep cooperative matrix game takes into consideration the Rules of the COLREGs Rules and the advance time of the manoeuvre approximating ship's dynamic properties and evaluates the final deviation of the real trajectory from reference value.

Sensitivity of the final game payment:

- is the least for changes of the duration of one stage tis least relative to the sampling period of the trajectory and advance time manoeuvre,
- most is relative to changes of the own and met ships speed and course,
- it grows with the degree of playing character of the control process and with the quantity of admissible strategies.
- trajectory and for changes of the advance time manoeuvre.

The matrix game control algorithm is, in a certain sense, formal model of the thinking process of a navigator steering the ship's movement and making up manoeuvring decisions.

Therefore they may be applied in the construction of a new model of ARPA system containing a computer supporting the navigator's decision making.

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