TRANSNAV International Journal on Marine Navigation and Safety of Sea Transportation
Volume 3 Number 4 December 2009

The H_2 and Robust H_{inf} Regulators Applied to Multivariable Ship Steering

W. Gierusz
Gdynia Maritime University, Gdynia, Poland

ABSTRACT: The main goal of this task was a calculation of the two multivariable regulators for precise steering of a real, floating, training ship. The first one minimized the H_2 norm of the closed-loop system. The second one was related to the H_{inf} norm. The robust control approach was applied in this controller with the usage of the structured singular value concept. Both controllers are described in the first part of the paper. Details of the training vessel and its simulation model then are presented. The state model of the control object obtained via identification process is described in the next section. This model with matrices weighting functions was the base for creation of 'the augmented state model' for the open-loop system. The calculation results of the multivariable controllers is also shown in this section. Several simulations were performed in order to verify the control quality of both regulators. Exemplary results are presented at the end of this paper together with final remarks.

1 INTRODUCTION

The process of the ship movement steering can be divided into several control subsystems, e.g. the ship's course and/or speed stabilization, damping of roll angle, dynamic ship positioning (DSP), guidance along trajectory etc. One of them is the control system for precise steering of the ship moving with the low and very low speed. Such kind of the vessel motion is also known as a crab movement. This regulation process means the full control of velocities during translation of the ship with any drift angle, e.g. motion ahead, astern and askew or rotation in place. No other help (tugs, anchors etc.) is required for this process.

In the beginning, the precise steering systems were installed as extensions of DSP units on research ships, drilling vessels, cable and pipe laying ships and similar ones. Nowadays these systems are mounted on ferries, passenger ships, shuttle tankers, FSO and dredging vessels (Fossen 2002).

The exemplary manoeuvres under such a steering are presented in Fig.1. It gives, among others, the following advantages:

– the increasing safety of the vessel, especially on constrained water with intensive traffic (harbours, navigation channels, closed or inner roads etc.), owing to ability to perform e.g. a fast anti-collision manoeuvre on very small area,
– the possibility of resignation of tugs cooperation for e.g. berthing or mooring manoeuvres,

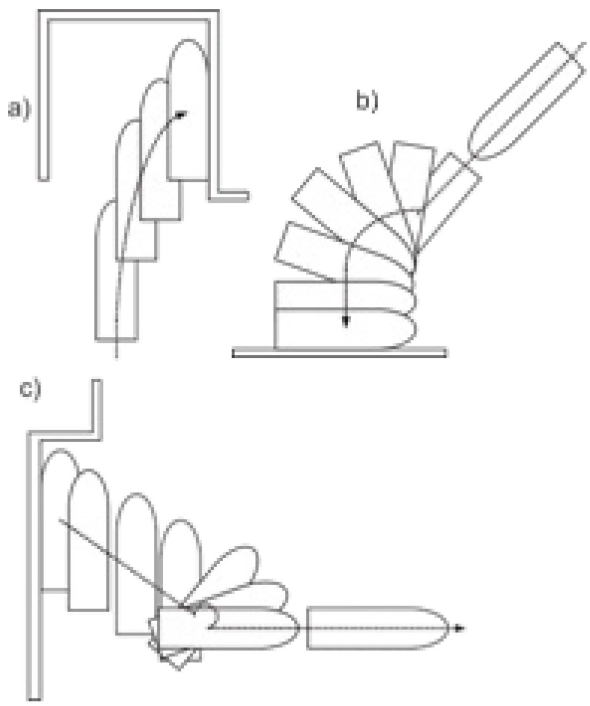

Figure 1: The exemplary situations when precise manoeuvres during berthing are needed and expected.

– the ability to pass along very shallow and tortuous navigation channels, inaccessible for ships with conventional drivers e.g. near attractive touristic places (islands, gulfs, fiords, etc.).

For this purpose the ship has to be equipped with at least a few driving devices like: main propellers, tunnel thrusters, jet-pump thrusters, or azipods (a blade rudder is useless in such operations). They allow to steer the ship in the manual manner, but it rarely leads to satisfying results - therefore the multivariable controller seems to be a reasonable solution.

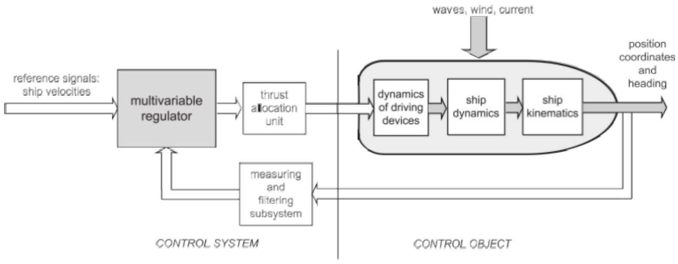

Figure 2: The block diagram of the multivariable ship control system.

The regulation of three ship's velocities: surge, sway and yaw often needs the 'usage' of only one velocity at a time (see Fig. 1), therefore the control system should ensure complete or almost complete decoupling steering of the ship.

The whole described system (see Fig. 2) consists of three elements:

- the measuring subsystem,
- the multivariable regulator,
- the thrust allocation unit.

As it was pointed out the precise steering of the vessel is performed with very slow velocities. The standard navigation devices for measuring of motion parameters have poor accuracy in these work conditions. Therefore ship's velocities have to be estimated (reconstructed) from position coordinates and a value of the heading. The Kalman filters are commonly used for this purpose (Anderson & Moore 2005).

The ship as a control object has very disadvantageous features:

- the characteristics of the ship strongly and in the nonlinear manner depend on operating conditions e.g. the ship's velocity, the direction of the motion, ship load, water depth, proximity of other ships, wharfs, etc.
- the allowance for all these factors in the model is very difficult and even after it has been done it leads to a badly complicated structure useless for synthesis,
- the linearization of the model in many working points gives a family of the models and the family of regulators. Next it generates another problem with the process of proper controllers shockingless switching.

A control system designer has two main ways to overcome these problems. One of them is matching regulator to the real plant during the control process i.e. adaptation of the control system - see for example Astrom and Wittenmark books or (Niederlinski, Moscinski & Ogonowski 1995). The second way is evaluation of the bounds of the plant (ship) changes and including them into the regulator synthesis process (Skogestad & Postlethwaite 2003), (Zhou 1998).

The last approach is often named H_{inf} robust control and requires a minimization of a process matrix norm called H_{inf} (Doyle, Glover, Khargonekar & Francis 1989).

The matrix norms are very convenient ways for formulation of performance criterions, especially in multivariable systems. One can use two norms: H_{inf} and H_2. Controllers related to each norm are commonly named 'H_{inf} regulator' and 'H_2 regulator'. The synthesis of both controllers for a ship is the objective of this paper.

2 THE H_{INF} AND H_2 REGULATORS

2.1 *Problem formulation*

The feedback controller design can be formulated for the general configuration of the MIMO system shown in Fig.3 (note opposite directions of signals - from right to left hand side, more convenient for matrix operations used in multivariable systems).

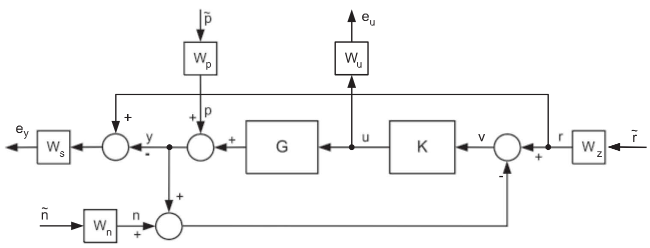

Figure 3: The block diagram of the closed-loop system with weighting functions for selected signals. The meaning of the particular signals is as follows: ˜r = references vector, ˜p - vector of disturbances, n˜ - noises vector, ˜ey - weighted control errors, ˜eu - weighted control signals.

The concept of weighting functions is a convenient way of introducing different signal specifications into a MIMO process:

- the signals scaling operation is easy to perform by means of this functions,
- one can distinguish between more and less important components of the signals vectors (e.g. in errors vector) by proper gain coefficients, introduced into these functions,
- the designer requirements related to the particular signals can be formulated for specified frequency ranges in a natural way.

Note the different sense of functions $\mathbf{W_u}$, $\mathbf{W_s}$ on the one hand and $\mathbf{W_p}$, $\mathbf{W_n}$, $\mathbf{W_z}$ on the other one. Functions matrices $\mathbf{W_s}$ and $\mathbf{W_u}$ define designer requirements for steering quality in the system while functions matrices $\mathbf{W_p}$, $\mathbf{W_n}$ and $\mathbf{W_z}$ form input signals in frequency domain. One can write the following equations based on the Fig.3:

$$\mathbf{e_y} = \mathbf{W_S W_z \tilde{r}} - \mathbf{W_s W_p \tilde{p}} - \mathbf{W_s G u} \tag{1}$$

$$\mathbf{e_u} = \mathbf{W_u u} \tag{2}$$

$$\mathbf{v} = \mathbf{W_z \tilde{r}} - \mathbf{W_p \tilde{p}} - \mathbf{W_n \tilde{n}} - \mathbf{W_s G u} \tag{3}$$

$$\mathbf{u} = \mathbf{K v} \tag{4}$$

Above equations can be rewritten in more compact form:

$$\begin{bmatrix} \mathbf{e_y} \\ \mathbf{e_u} \\ \mathbf{v} \end{bmatrix} = \mathbf{P} \times \begin{bmatrix} \mathbf{\tilde{r}} \\ \mathbf{\tilde{p}} \\ \mathbf{\tilde{n}} \\ \mathbf{u} \end{bmatrix} \tag{5}$$

where matrix $\mathbf{P}$ has the form:

$$\mathbf{P} = \begin{bmatrix} \mathbf{W_s W_z} & -\mathbf{W_s W_p} & \mathbf{0} & -\mathbf{W_s G} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{W_u} \\ \mathbf{W_z} & -\mathbf{W_p} & -\mathbf{W_n} & -\mathbf{G} \end{bmatrix} \tag{6}$$

Matrix $\mathbf{P}$ is called the augmented plant (model plant) due to weighting functions vectors included in it. Introducing the input vector $d = [\tilde{r} \quad \tilde{p} \quad \tilde{n}]^T$ and the weighting error vector $e = [e_y \quad e_u]^T$ one can write:

$$\begin{bmatrix} \mathbf{e} \\ \mathbf{v} \end{bmatrix} = \mathbf{P} \times \begin{bmatrix} \mathbf{d} \\ \mathbf{u} \end{bmatrix} \tag{7}$$

$$\mathbf{u} = \mathbf{K} \times \mathbf{v} \tag{8}$$

The last equations enable to build the generalized configuration exposed in Fig.4.

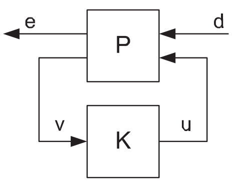

Figure 4: The generalized closed-loop system configuration.

Now the weighting error vector can be expressed in the form:

$$\mathbf{e} = \mathbf{T_{ed}}(\mathbf{P}, \mathbf{K}) \times \mathbf{d} \tag{9}$$

where matrix $\mathbf{T_{ed}}$ can be obtained by means of the Lower Linear Fractional Transformation (Redheffer 1960).

The control system design can be treated as a process of calculating a controller $\mathbf{K}$ such which maintain small certain weighted signals (e.g. control errors). One of the possible way to define the 'smallness' of signals (or transfer matrices) are matrix norms H_{inf} and H_2 (Skogestad & Postlethwaite 2003) expressed by the following equations:

$$\begin{aligned} \|\mathbf{T_{ed}}(\mathbf{s})\|_2 &= \sqrt{\frac{1}{2\pi} \int_{-\infty}^{+\infty} tr[\mathbf{T_{ed}}(\mathbf{j}\omega) \times \mathbf{T_{ed}}(\mathbf{j}\omega)] d\omega} \\ \|\mathbf{T_{ed}}(\mathbf{s})\|_\infty &= \max_{\omega \in \langle 0, \infty \rangle} \bar{\sigma}[\mathbf{T_{ed}}(\mathbf{j}\omega)] \end{aligned} \tag{10}$$

2.2 *The H_2 regulator*

The H_2 optimal control problem is to find a controller $\mathbf{K}$ which stabilizes the closed-loop system (presented in Fig. 4) and minimizes the H_2 norm of this system. The minimization of the H_2 norm is performable only for strictly proper systems. When the plant $\mathbf{P}$ is written in state model form:

$$\begin{bmatrix} \dot{x} \\ e \\ v \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix} \times \begin{bmatrix} x \\ d \\ u \end{bmatrix} \tag{11}$$

the part D11 and D22 must be a matrices of zeros for such a system.

The well-known LQG controller can be treated as a special case of the H_2 regulator, when a weighting factor in LQG performance criterion is included into weighting function $\mathbf{W_u}$ (Zhou 1998).

The regulator which minimizes the H_2 norm of the system ensures the proper steering quality represented by the matrix weighting functions $\mathbf{W_s}$ and/or $\mathbf{W_u}$ (see Fig.3), but under assumption that the plant model is adequate and accurate.

2.3 *The H_{inf} regulator*

The goal of H_{inf} regulator is similar to that of the H_2 one, but now one wants to minimize the H_{inf} norm with the condition:

$$\|\mathbf{T_{ed}}(\mathbf{P}, \mathbf{K})\|_\infty < \gamma, \qquad \gamma > 0, \qquad \gamma \in \Re$$

The value γ has the sense of the energy ratio between error vector $\mathbf{e}$ and exogenous input vector $\mathbf{d}$. When the γ tends to its minimal value the above formulation is often named the optimal H_∞ control problem (Skogestad & Postlethwaite 2003).

The regulator which minimizes the H_{inf} norm of the system ensures similar quality of the steering **for**

any combinations of exogenous input signals formed by matrix weighting functions $\mathbf{W_p}$, $\mathbf{W_n}$ and $\mathbf{W_z}$ (note that this is not warranted by H_2 regulator).

2.4 *The robust regulator*

However this steering quality is only achieved under the same assumption that the plant model is accurate. If the real plant differs (e.g. due to operating conditions) from the model used during controller synthesis this quality can be significantly poor. The differences between the object and the model are usually named the system uncertainties (Doyle 1982).

There are several sources of uncertainties which can be introduced into the ship model:

- changes the physical parameters of the vessel due to different work conditions (e.g. load, trim, depth of water, etc.),
- errors in estimation process for model coefficients values,
- neglected nonlinearities inside the object (e.g. related to hydrodynamics phenomena),
- measurement and filtration process errors (e.g. biases),
- unmodelled dynamics, especially in the high frequency range,

accepted (chosen) limitation of the model order. All uncertainties can be divided into two classes: parametric ones, related to the particular model coefficients and others - nonparametric ones. Introduction of the concept of uncertainties into the modelling process means that one considers not only the one nominal model of the object $\mathbf{G_n(j\omega)}$, but a family of models $\mathbf{G_D}$ spread around this nominal model.

The uncertainties can be introduced into the system model in different ways, depending on their types and locations, but all of them are represented by means of two components:

- the first one is the "pure" uncertainty Δ, bounded in the H_{inf} norm sense i.e. $\|\Delta\|_\infty \le 1$
- the second one it is the weighting function modeling the magnitude and shape of the uncertainty in the frequency domain.

Consequently, any closed-loop system with uncertainties contains three basic components: the generalized (augmented) plant **P**, the controller **K** that has to be obtained and the set of "pure" uncertainties Δ, collected in the matrix form (see Fig.5).

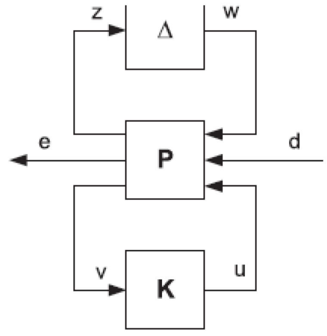

Figure 5: The generalized closed-loop system configuration with uncertainties.

The augmented plant **P** consists of the nominal object model $\mathbf{G_n}$ and of all matrices of weighting functions (modeling the performance requirements, forming input signals and describing the uncertainties). Note that the augmented plant **P** for H_{inf} controller synthesis slightly different from this plant for H_2 one.

2.5 *The ship subsystems as a control object*

The control object denoted Gn (see Fig.3) in the considered system consists of four elements: the allocation unit, thrusters set, the ship and the filters system (Gierusz 2006). It has three inputs: two demanded forces τ_x and τ_y for longitudinal and lateral directions of movement and one moment τ_p for turning (in the ship-fixed frame) and three outputs: estimated values of velocities surge $\hat{u}$, sway $\hat{v}$ and yaw $\hat{r}$ (see Fig.6).

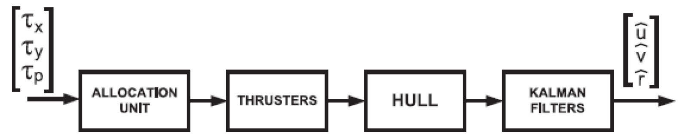

Figure 6: The block diagram of control object.

3 CASE STUDY

3.1 *The training ship*

The H_2 and H_{inf} robust controllers was applied to steer a floating training ship. The vessel named 'Blue Lady' is used by the Foundation for Safety of Navigation and Environment Protection at the Silm lake near Ilawa in Poland for training of navigators. It is one of the series of 7 various training ships exploited on the lake.

The ship 'Blue Lady' is an isomorphous model of a VLCC tanker, built of the epoxide resin laminate in 1:24 scale. It is equipped with battery-fed electric drives and the two persons control steering post at the stern T˙ he silhouette of the ship is presented in Fig.7.

The main parameters of the ship are as follows:

Length over all	LOA	=	13,78[m]
Beam	B	=	2,38[m]
Draft (average) - load condition	Tl	=	0,86[m]
Displacement - load condition	Δl	=	22,83[t]
Speed	V	=	3,10[kn]

The high-fidelity, fully coupled, nonlinear simulation model of this ship was built for controllers synthesis. Special attention was paid to the proper modeling of the ship's behaviour during movement with any drift angle (e.g. astern or askew). The block diagram of the model is presented in Fig. 8 (see (Gierusz 2001) for detailed description of this model).

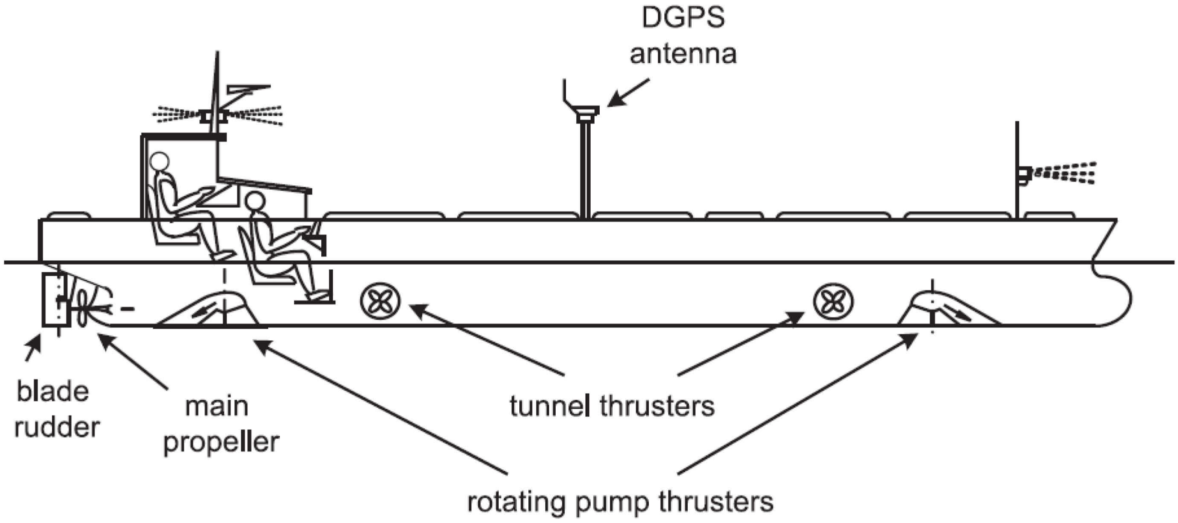

Figure 7: The outline of the training ship "Blue Lady"

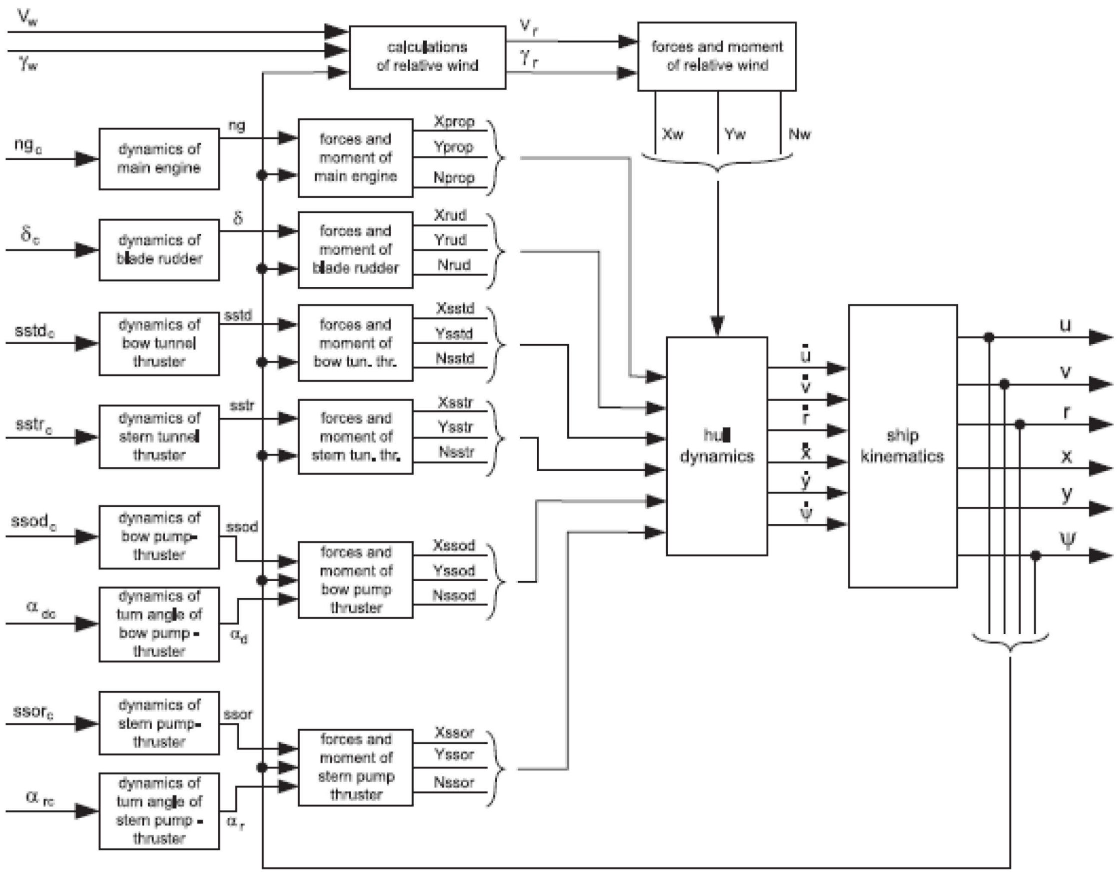

Figure 8: The block diagram of the 'Blue Lady' simulation model. Input signals for the model are as follows (from top to bottom): mean wind velocity - Vw , mean wind direction - γw , revolutions of the main propeller - ngc, blade rudder angle - δc, relative thrust of the bow (stern) tunnel thruster - sstdc (sstrc), relative thrust of the bow pump thruster - ssodc, turn angle of the bow pump thruster - αdc, relative thrust of the stern pump thruster - ssorc, turn angle of the stern pump thruster - αrc. The output signals of the model are: surge - u, sway - v, yaw - r, position coordinates - x,y and the heading - Ψ.

3.2 *The linear model identification*

The synthesis processes of both controllers described in this paper need a linear model of the object. There are two ways to create it: a linearization of a nonlinear (e.g. simulation) model of the vessel dynamics or identification way. The second approach was used in presented work.

Every identification experiment was performed as a simulation run in Simulink environment. More than one hundred of experiments were performed for this purpose (Gierusz 2006).

During identification process, it turned out, that three subsystems demonstrated weak correlation between output and input signals $\tau_x \to \upsilon, \tau_y \to u, \tau_p \to u$, therefore these subsystems were canceled from the whole model (see Fig. 9).

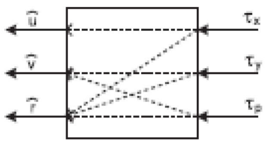

Figure 9: Control object paths to be identified.

Finally, the third order state model was obtained. The average values of coefficients, obtained in all identification experiments were chosen as the values of parameters of the nominal model Gn. Note values of coefficients equal 0 in the channels cancelled during identification process (see Fig. 9).

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{uu} & 0 & 0 \\ 0 & a_{\upsilon\upsilon} & a_{\upsilon r} \\ a_{ru} & a_{r\upsilon} & a_{rr} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_{uu} & 0 & 0 \\ 0 & b_{\upsilon\upsilon} & b_{\upsilon r} \\ b_{ru} & b_{r\upsilon} & b_{rr} \end{bmatrix} \times \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} u \\ \upsilon \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{14}$$

The resultant model is state controllable and observable - see (Gierusz & Tomera 2006) for details. This model was used for H_2 controller synthesis.

For synthesis of the robust regulator five parametric uncertainties (denoted δ_i, i = 1, ..., 5) were introduced into the state model due to the wide range of variations of parameter values acquired in various experiments. This model had the form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} a_{uu}+\delta_{11} & 0 & 0 \\ 0 & a_{\upsilon\upsilon}+\delta_{12} & a_{\upsilon r} \\ a_{ru} & a_{r\upsilon} & a_{rr}+\delta_{13} \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_{uu}+\delta_{14} & 0 & 0 \\ 0 & b_{\upsilon\upsilon} & b_{\upsilon r}+\delta_{15} \\ b_{ru} & b_{r\upsilon} & b_{rr} \end{bmatrix} \times \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} \tag{15}$$

$$\begin{bmatrix} u \\ \upsilon \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} x \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{16}$$

The coefficients values of the state model of the ship dynamics with values of uncertainties are collected in the table 1 below.

Table 1: The values of model coefficients.

Wsp.	Nominal value	Real uncertainty value	Relative uncertainty value [%]
a_{uu}	$-3.36*10^{-3}$	$2.64*10^{-3}$	78
a_{vv}	$-9.00*10^{-3}$	$5.00*10^{-3}$	64
a_{vr}	$-2.00*10^{-4}$		
a_{ru}	$-3.00*10^{-3}$		
a_{rv}	$-1.00*10^{-3}$		
a_{rr}	$-7.75*10^{-3}$	$4.05*10^{-3}$	52
b_{uu}	$+3.62*10^{-3}$	$1.51*10^{-3}$	42
b_{vv}	$+2.06*10^{-3}$		
b_{vr}	$+1.61*10^{-5}$	$2.89*10^{-5}$	179
b_{ru}	$+3.00*10^{-5}$		
b_{rv}	$+1.15*10^{-5}$		
b_{rr}	$+8.00*10^{-3}$		

3.3 *The controllers synthesis*

3.3.1 *H_2 regulator*

The state model, presented via equations (13) and (14), could be arranged into 'augmented state model of the open-loop process' (Balas, Doyle, Glover, Packard & Smith 2001), which was necessary to compute the multivariable controller which minimized H_2 norm.

The three tracking velocity errors e_u, e_v and e_r were chosen as a performance criterion. It was assumed that these expected errors would depend on frequency of the reference signals. These requirements were transferred into the matrix of the weighting functions $\mathbf{W_s}$ for each velocity. The matrix of the weighting function $\mathbf{W_{zad}}$ was introduced instead, to moderate the reference signals rate and consequently to constrain the possibly large amplitude of the steering signals.

The block diagram of model for this process is presented in Fig. 10.

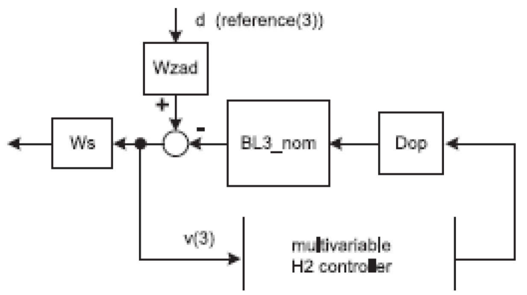

Figure 10: The block diagram of the augmented open- loop process for H2 controller synthesis. Symbols denote: BL3 nom - state model of the control object; Dop - adaptation matrix; Wzad - filters for reference signals; Ws - weighting functions for control perfor- mance. Numbers in parentheses denote sizes of the signal vectors.

The synthesis of the regulator was made by means of the algorithm named 'h2syn' from 'μ Analysis and Synthesis Toolbox'(see (Balas et al. 2001) for more details).

The computed regulator is of order 15:

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{A}_{\mathbf{r}}^{\mathbf{15x15}} \times \mathbf{x}(\mathbf{t}) + \mathbf{B}_{\mathbf{r}}^{\mathbf{15x3}} \times \mathbf{v}(\mathbf{t}) \tag{17a}$$

$$\tau_c = C_{\mathbf{r}}^{3\mathbf{x15}} \times \mathbf{x}(\mathbf{t}) + D_{\mathbf{r}}^{3\mathbf{x3}} \times \mathbf{v}(\mathbf{t}) \tag{17b}$$

The value of the closed-loop system H_2 norm was 12.14 and the value of the H_∞ norm was between 23.9365 and 23.9604. This last value means that the H_2 controller is not a robust one for the described sys- tem.

3.3.2 *Hinf regulator*

Apart from uncertainties related to changing proper- ties of the plant, (see equations (15) and (16)) two multiplicative, nonparametric uncertainties were introduced to the presented ship control sys- tem. The first one modelled inaccuracy in input sig- nals (related to transmission errors) with the matrix of weighting function $\mathbf{W_{wyk}}$, and the second one modelled measuring and filtering errors in the output plant with the matrix of weighting function $\mathbf{W_{pom}}$. The state model of the control object with all weighting functions was rebuilt into 'augmented state model of the open-loop process' much more complicated then one presented in Fig.10:

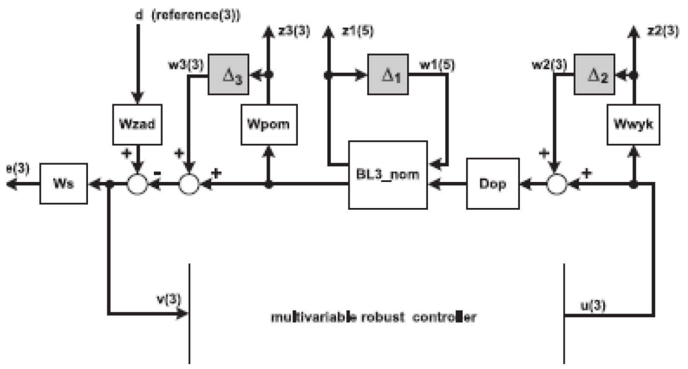

Figure 11: The block diagram of the augmented open- loop process. Symbols denote: Δ_1 - structured uncertainties block; Δ_2 - input uncertainty with weighting functions **Wwyk**; Δ_3 - measuring and filtering uncertainty with weighting functions **Wpom**; **BL3_nom**- state model of the control object; **Dop** - adaptation matrix; **Wzad** - filters for reference signals; **Ws** - weighting functions for robust performance. Numbers in paren- theses denote sizes of the signal vectors.

The algorithm named 'D-K iteration' from men- tioned Matlab toolbox was used to compute the ro- bust Hinf controller for the system presented in Fig. 11. The obtained regulator in state model form was of high order equal to 41 - the same as the open- loop system (with the scaling matrices D - see (Balas et al. 2001) for the meaning of such matrices).

The value of H_{inf} norm was 0.56 < 1 which en- sures the robust property of the controller.

Therefore the order reduction procedures were performed. Finally the controller of the order 21 was obtained.

The regulator order seems to be quite high, but it is worth to remember what the introduction of para- metric uncertainties to the plant model is. It means that the obtained controller should steer properly (in weighing functions sense) the object which can change its characteristic in a very wide range. There- fore, the controller for such object should not be so simple.

4 RESULTS ANALYSIS AND FINAL REMARKS

The examination of both control systems was per- formed during simulation runs with the ship's non- linear simulation model.

Every Figure is divided into two parts. The left- hand side presents the results of the steering with the H_2 controller and the right-hand side presents the same trials performed with the robust regulator.

This example is illustrated by means of 3 Figures:

– the trajectory, drawn by ship's silhouettes eve- ry 60[s],

- ship's velocities (reference signals and real values), supplemented by wind velocity runs (presented in Beaufort scale)
- command signals from the regulators.

The results were recalculated to start both trajectories from point (0,0) and the initial heading was chosen as 0 [deg].

One can compare the tracking errors for all velocities in all presented examples . The following formula was used for this purpose:

$$J_q = \frac{1}{T}\sum_{i=1}^{T}\left(q_c(i) - \hat{q}(i)\right)^2, \qquad q = \{u, \upsilon, r\} \qquad (18)$$

where

q_c reference signal for particular velocity,

$\hat{q}$ estimated value from Kalman filter,

T = 1000, 1400, 2800 successively for first, second and third example.

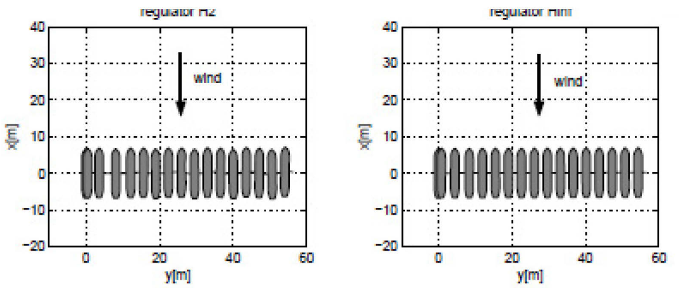

Figure 12: The trajectory of the ship in the first exam- ple drawn by silhouettes every 60[s]. Initial heading ψ0 = 0[deg], the trial period t = 1000[s]. An arrow indicate the average wind direction.

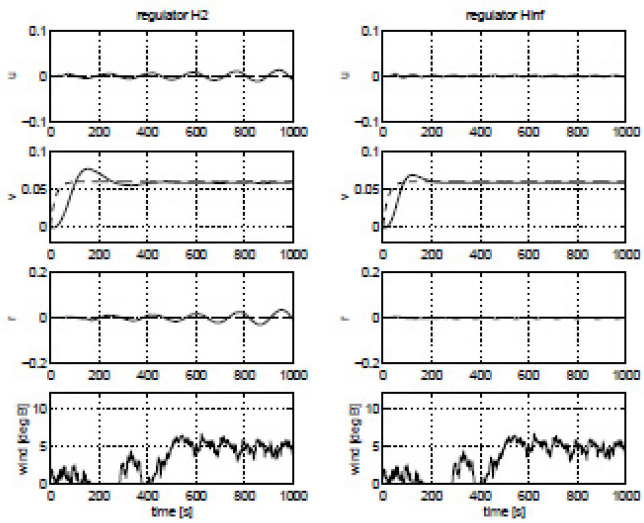

Figure 13: The velocities of the ship in the first example - from the top: surge, sway and yaw. The bottom figures present the wind speed in Beaufort scale (recalculated in the ship model scale 1:24). Solid lines denote real values, dashed lines - commands.

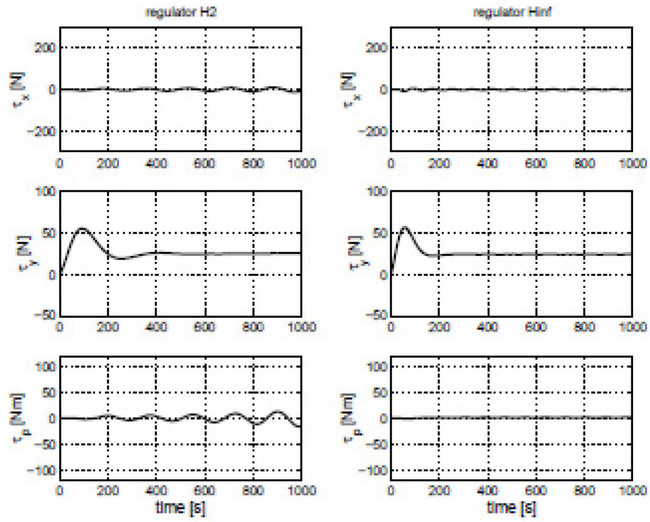

Figure 14: The commands from controllers - from the top: for surge - τ_x, for sway - τ_y and for yaw - τ_p.

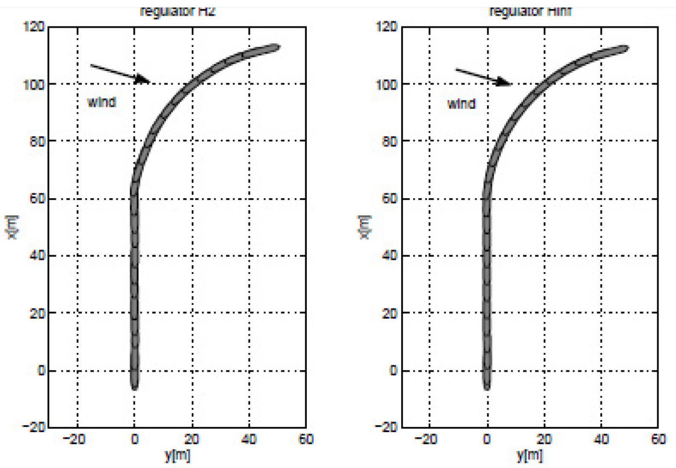

Figure 15: The trajectory of the ship in the second example drawn by silhouettes every 60[s]. Initial heading ψ_0 = 0[deg], the trial period t = 1400[s]. An arrow indicate the average wind direction.

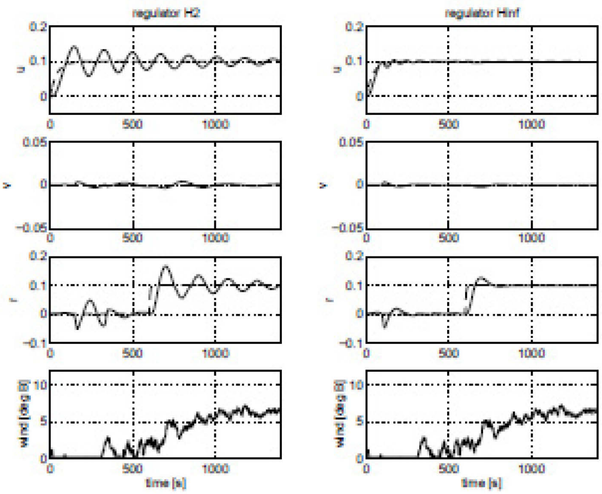

Figure 16: The velocities of the ship in the second example - from the top: surge, sway and yaw. The bottom figures present the wind speed in Beaufort scale (recalculated in the ship model scale 1:24). Solid lines denote real values, dashed lines - commands.

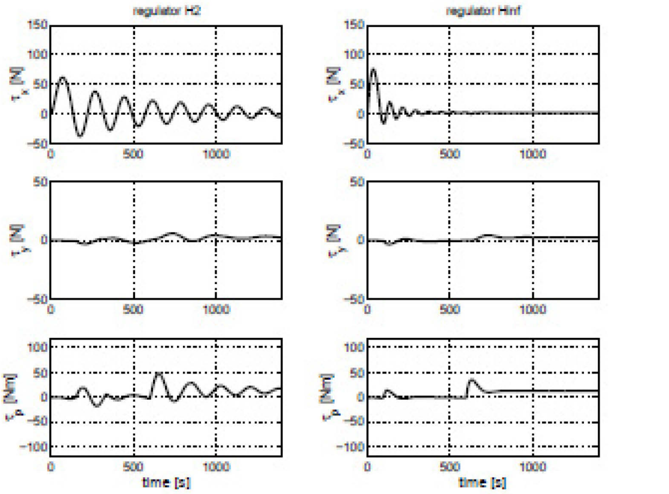

Figure 17: The commands from controllers - from the top: for surge - τ_x, for sway - τ_y and for yaw - τ_p.

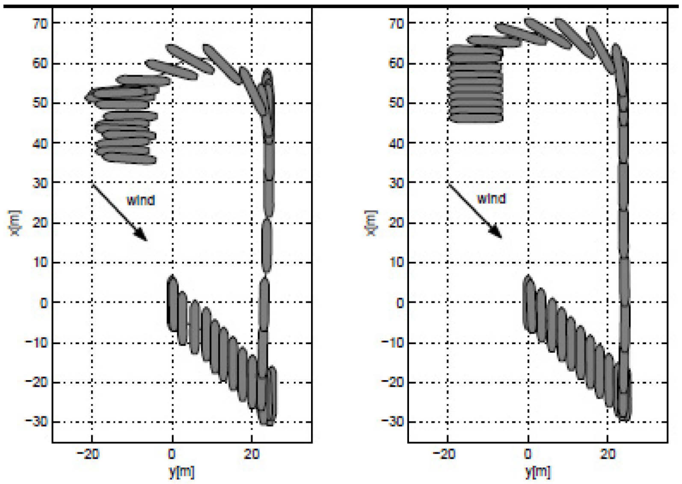

Figure 18: The trajectory of the ship in the third example drawn by silhouettes every 60[s]. Initial heading ψ_0 = 0[deg], the trial period t = 2800[s]. An arrow indicate the average wind direction.

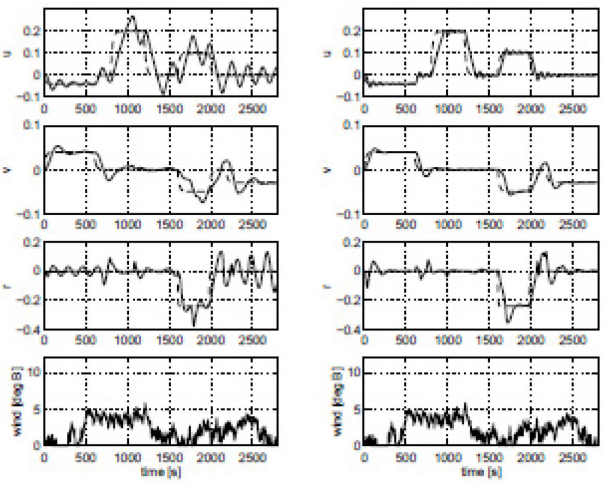

Figure 19: The velocities of the ship in the third example - from the top: surge, sway and yaw. The bottom figures present the wind speed in Beaufort scale (recalculated in the ship model scale 1:24). Solid lines denote real values, dashed lines - commands.

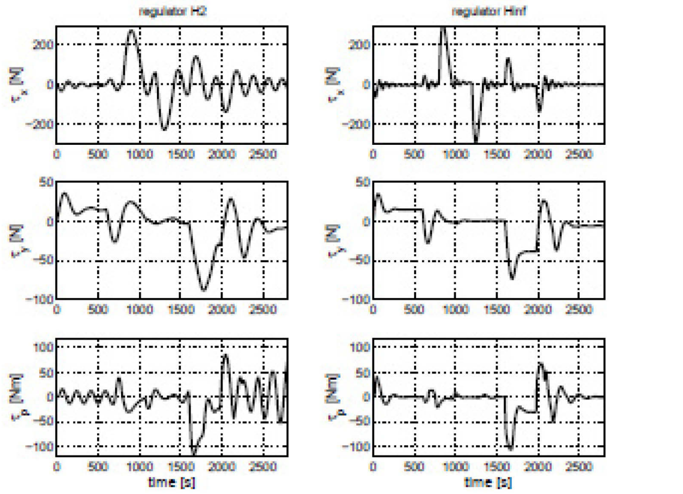

Figure 20: The commands from controllers - from the top: for surge - τ_x, for sway - τ_y and for yaw - τ_p.

The comparisons are presented in the tables (values x 10^6):

Example 1

Controller	J_u	J_v	J_r
H_2	35	127	191
H_{inf}	1	73	29

Example 2

Controller	J_u	J_v	J_r
H_2	369	3	660
H_{inf}	37	1	230

Example 3

Controller	J_u	J_v	J_r
H_2	2650	205	3280
H_{inf}	920	109	2270

The similar calculations one can perform for control effort for both regulators using the formula:

$$J_{\tau s} = \frac{1}{T}\sum_{i=1}^{T}\left(\tau_s(i)\right)^2, \qquad s = \{x, y, p\}$$

where

τ_s control signal from regulator in the particular channel,

T = 1000, 1400, 2800 successively for first, second and third example.

The results are presented in the tables:

Example 1

Controller	$J_{\tau x}$	$J_{\tau y}$	$J_{\tau p}$
H_2	30	805	34
H_{inf}	1	728	1

Example 2

Controller	$J_{\tau x}$	$J_{\tau y}$	$J_{\tau\psi}$
H_2	393	7	225
H_{inf}	180	5	132

Example 3

Controller	$J_{\tau x}$	$J_{\tau y}$	$J_{\tau\psi}$
H_2	7720	723	1104
H_{inf}	5120	444	633

5 REMARKS

- The fully coupled, simulation model of the ship with acceptable accuracy gives possibilities to perform the identification trials instead of costs and time consuming full-scale experiments. One can the build the multidimensional linear model and estimate the system uncertainties: their ranges and sources, based on the results from simulation runs.
- The introduction of parametric uncertainties into the plant model enables to cover the changes of object characteristics (even nonlinear) in the all range of assumed work conditions. On the other hand it causes the increasing difficulty in the controller synthesis.
- Very important advantage (or attribute) of both regulators is its fixed structure and constant values of coefficients. It means that navigators do not need to adjust any coefficients of these controllers.
- The H_2 controller works worse than the robust one. One can compare tables with results for control quality and steering effort. One of the main reasons for such a steering can be the lack of the robust properties of the regulator (see the H_{inf} norm of this regulator).
- Both systems were tested in the presence of a medium level of wind, in spite of fact that external disturbances were not taken into account during controllers synthesis processes. The robust regulator still seems to be a better one in such work conditions. The external disturbances one can try to introduce into the controller synthesis process but often no enough adequate regulator is obtained (eg. without robust properties).
- As one can see in Fig. 12 - Fig.19, the steering is almost de-coupling despite the full matrices B, C and D in the controllers.
- The both closed-loop systems are stable under all tested work conditions.
- The most important problems are related to yaw steering (especially for H_2 controller). One of the possible sources was the gyrocompass (with its accuracy 0.2[deg]) and one was the fact that the training ship is high weatherly.
- In general regulator calculated for one ship can not be transferable to another one due to linear object model specified for particular ship. It is a similar situation like with PID controllers in many industrial processes. But the possibility of using a simulation model of the ship's dynamics instead a real ship for experiments for H_2 or H_{inf} robust controller synthesis seems to be a great advantage of described approach.

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