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The Dynamic Game Models of Safe Navigation

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ABSTRACT: The paper introduce the application of selected methods of a game theory for automation of the processes of moving object steering, the game control processes in marine navigation and mathematical models of the safe ship control. The control goal has been defined first and then description of the base model, the approximated models of multi-stage positional game and multi-step matrix game of the safe ship steering in a collision situation has been presented.

1 INTEGRATED NAVIGATION

1.1 *Multilevel system*

The control of the ship's movement may be treated as a multilevel problem shown on Figure 1, which results from the division of the entire control system of ship - within the frame of the performance of the cargo carriage by the ship's operator - into clearly determined subsystems which are ascribed appropriate layers of control (Lisowski 2004b).

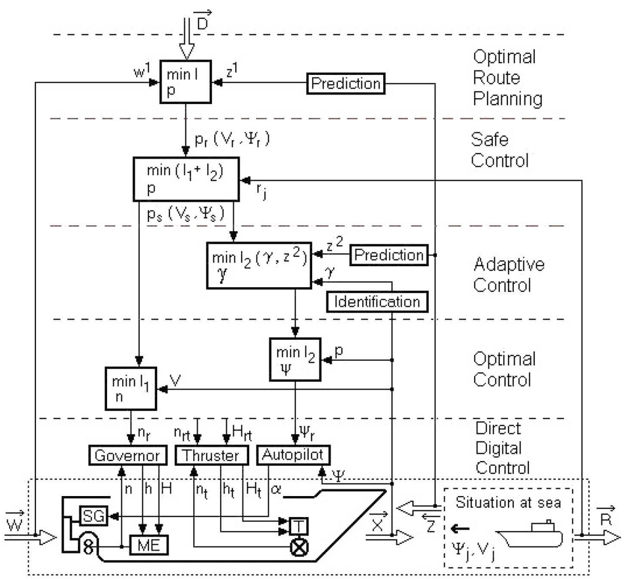

Fig. 1. Multilevel ship movement steering system

This is connected both with a large number of dimensions of the steering vector and of the status of the process, its random, fuzzy and decision making characteristics - which are affected by strong interference generated by the current, wind and the sea wave motion on the one hand, and a complex nature of the equations describing the ship's dynamics with non-linear and non-stationary characteristics. The determination of the global control of the steering systems has in practice become too costly and ineffective (Lisowski 2005e).

1.2 *Control processes*

The integral part of the entire system is the process of the ship's movement control, which may be described with appropriate differential equations of the kinematics and dynamics of a ship being an object of the control under a variety of the ship's operational conditions such as:

- stabilisation of the course or trajectory,
- adjustment of the ship's speed,
- precise steering at small speeds in port with thrusters or adjustable-pitch propeller,
- stabilisation of the ship's rolling,
- commanding the towing group,
- dynamic stabilisation of the drilling ship's or the tanker's position.

The functional draft of the system corresponds to a certain actual arrangement of the equipment. The increasing demands with regard to the safety of navigation are forcing the ship's operators to install the systems of integrated navigation on board their ships. By improving the ship's control these systems increase the safety of navigation of a ship - which is a very expensive object of the value, including the cargo, and the effectiveness of the carriage goods by sea (Lisowski 2000a, 2005b, 2007).

2 SAFE SHIP CONTROL

2.1 *ARPA acquisition and tracking*

The challenge in research for effective methods to prevent ship collisions has become important with the increasing size, speed and number of ships participating in sea carriage. An obvious contribution in increasing safety of shipping has been firstly the application of radars and then the development of ARPA (Automatic Radar Plotting Aids) anti-collision system (Cahill 2002).

The ARPA system enables to track automatically at least 20 encountered j objects as is shown on Figure 2, determination of their movement parameters (speed V_j, course ψ_j) and elements of approach to the own ship ($D^j_{\min} = DCPA_j$ - Distance of the Closest Point of Approach, $T^j_{\min} = TCPA_j$ - Time to the Closest Point of Approach) and also the assessment of the collision risk r_j (Lisowski 2001a).

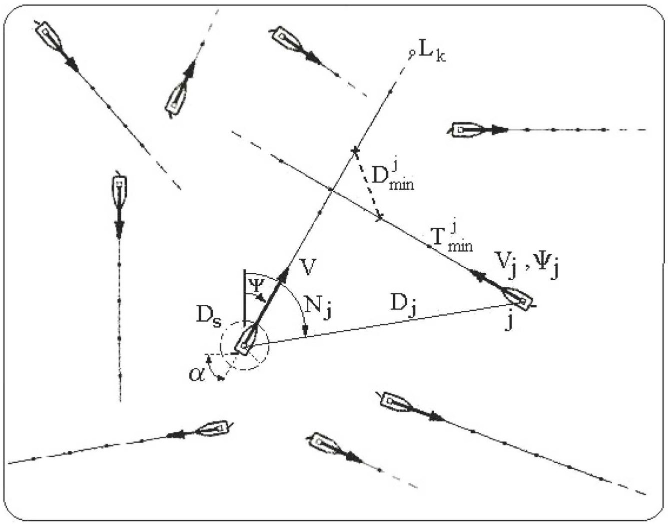

Fig. 2. Navigational situation representing the passing of the own ship with the *j-th* object

The risk value is possible to define by referring the current situation of approach, described by parameters $D^j_{\min}$ and $T^j_{\min}$, to the assumed evaluation of the situation as safe, determined by a safe distance of approach D_s and a safe time T_s – which are necessary to execute a collision avoiding manoeuvre with consideration of distance D_j to *j-th* met object - shown on Figure 3 (Lisowski 2001b, 2004a, 2006):

$$r_j = \left[k_1 \left(\frac{D^j_{min}}{D_s} \right)^2 + k_2 \left(\frac{T^j_{min}}{T_s} \right)^2 + \left(\frac{D_j}{D_s} \right)^2 \right]^{-\frac{1}{2}} \quad (1)$$

The weight coefficients k_1 and k_2 are depended on the state visibility at sea, dynamic length L_d and dynamic beam B_d of the ship, kind of water region and in practice are equal:

$$0 \le [k_1(L_d, B_d),\ k_2(L_d, B_d)] \le 1 \quad (2)$$

$$L_d = 1.1\,(1 + 0.345\ V^{1.6}) \quad (3)$$

$$B_d = 1.1\,(B + 0.767\ LV^{0.4}) \quad (4)$$

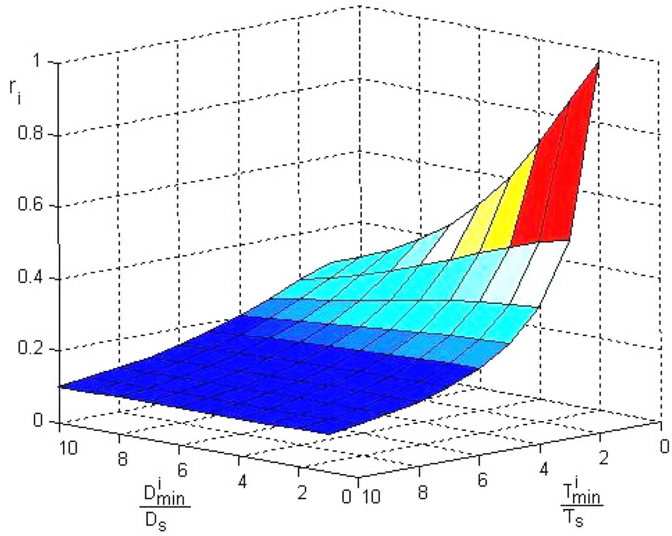

Fig. 3. The ship's collision risk space in a function of relative distance and time of approaching the *j-th* object

2.2 *ARPA manoeuvre simulation*

The functional scope of a standard ARPA system ends with the simulation of the manoeuvre altering the course $\pm\Delta\psi$ or the ship's speed $\pm\Delta V$ selected by the navigator as is shown on Figure 4.

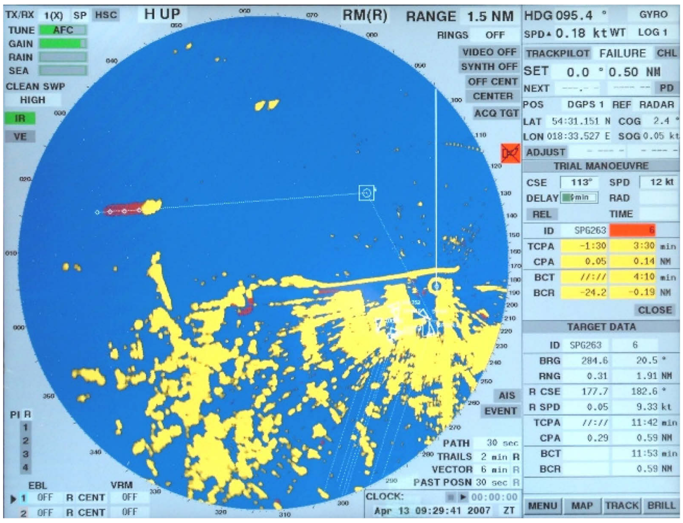

Fig. 4. The screen of SAM Electronics ARPA on the sailing vessel s/v DAR MLODZIEZY

2.3 *Computer support of navigator*

The problem of selecting such a manoeuvre is very difficult as the process of control is very complex since it is dynamic, non-linear, multi-dimensional, non-stationary and game making in its nature.

In practice, methods of selecting a manoeuvre assume a form of appropriate steering algorithms supporting navigator decision in a collision situation. Algorithms are programmed into the memory of a Programmable Logic Controller PLC. This generates an option within the ARPA anti-collision system or a training simulator (Lisowski 2005a, c, d).

3 GAME CONTROL IN MARINE NAVIGATION

3.1 *Dynamic game of ship control*

The classical issues of the theory of the decision process in marine navigation include the safe steering of a ship. The problem of non-collision strategies in the steering at sea appeared in the Isaacs' works (Isaacs 1965) called "the father of the differential games" and was developed by many authors both within the context of the game theory (Baba & Jain 2001, Segal & Miloh 1998), and also in the steering under uncertainty conditions (Engwerda 2005).

The definition of the problem of avoiding a collision seems to be quite obvious, however, apart from the issue of the uncertainty of information which may be a result of external factors (weather conditions, sea state), incomplete knowledge about other objects and imprecise nature of the recommendations concerning the right of way contained in International Regulations for Preventing Collision at Sea COLREG.

The problem of determining safe strategies is still an urgent issue as a result of an ever increasing traffic of vessels on particular water areas. It is also important due to the increasing requirements as to the safety of shipping and environmental protection, from one side, and to the improving opportunities to use computer supporting the navigator's duties. In order to ensure safe navigation the ships are obliged to observe legal requirements contained in the COLREG Rules.

However, these Rules refer exclusively to two ships under good visibility conditions, in case of restricted visibility the Rules provide only recommendations of general nature and they are unable to consider all necessary conditions of the real process. Therefore the real process of the ships passing exercises occurs under the conditions of indefiniteness and conflict accompanied by an imprecise co-operation among the ships in the light of the legal regulations.

Consequently, it is reasonable - for ship operational purposes - to present this process and to develop and examine methods for a safe steering of the ship by applying the rules of the game theory.

A necessity to consider simultaneously the strategies of the encountered objects and the dynamic properties of the ships as the steering objects is a good reason for the application of the differential game model - often called the dynamic game - for the description of the processes (Osborne 2004, Straffin 2001).

3.2 *Processes of game ship control*

Assuming that the dynamic movement of the ships in time occurs under the influence of the appropriate sets of steering:

$$[U_0^{(\mu_0)}, U_j^{(\mu_j)}] \tag{5}$$

where:

$U_0^{(\mu_0)}$ – a set of the own ship's strategies,

$U_j^{(\mu_j)}$ – a set of the *j-th* ship's strategies,

$\mu(\mu_0, \mu_j) = 0$ – denotes course and trajectory stabilisation,

$\mu(\mu_0, \mu_j) = 1$ – denotes the execution of the anti-collision manoeuvre in order to minimize the risk of collision, which in practice is achieved by satisfying the following inequality:

$$D_{\min}^{j} = \min D_j(t) \geq D_s \tag{6}$$

$D_{\min}^j$ – the smallest distance of approach of the own ship and the *j-th* encountered object,

D_s – safe approach distance in the prevailing conditions depends on the visibility conditions at sea, the COLREG Rules and the ship's dynamics.

D_j – current distance to the *j-th* object taken from the ARPA anti-collision system.

$\mu(\mu_0, \mu_j) = -1$ – refers to the manoeuvring of the ship in order to achieve the closest point of approach, for example during the approach of a rescue vessel, transfer of cargo from ship to ship, destruction the enemy's ship, etc.).

In the adopted describing symbols we can discriminate the following type of steering ship in order to achieve a determined goal:

- basic type of steering – stabilization of the course or trajectory: $[U_0^{(0)}\ U_j^{(0)}]$
- avoidance of a collision by executing:
  - own ship's manoeuvres: $[U_0^{(1)} U_j^{(0)}]$
  - manoeuvres of the *j-th* ship: $[U_0^{(0)} U_j^{(1)}]$
  - co-operative manoeuvres: $[U_0^{(1)} U_j^{(1)}]$
- encounter of the ships: $[U_0^{(-1)} U_j^{(-1)}]$
- situations of a unilateral dynamic game: $[U_0^{(-1)} U_j^{(0)}]$ *and* $[U_0^{(0)} U_j^{(-1)}]$
  Dangerous situations resulting from a faulty assessment of the approaching process by one of the party with the other party's failure to conduct observation - one ship is equipped with a radar or an anti-collision system, the other with a damaged radar or without this device (Lisowski 2002).
- chasing situations which refer to a typical conflicting dynamic game: $[U_0^{(-1)} U_j^{(1)}]$ *and* $[U_0^{(1)} U_j^{(-1)}]$.

The first case usually represents regular optimal control, the second and third are unilateral games while the fourth and fifth cases represent the conflicting games.

4 MATHEMATICAL MODELS OF SAFE SHIP CONTROL

4.1 *Base model*

As the process of steering the ship in collision situations, when a greater number of objects is encountered, often occurs under the conditions of indefiniteness and conflict, accompanied by an inaccurate co-operation of the objects within the context of COLREG Regulations then the most adequate model of the process which has been adopted is a model of a dynamic game, in general of *j* tracked ships as objects of steering.

The diversity of selection of possible models directly affects the synthesis of the ship's handling algorithms which are afterwards effected by the ship's handling device directly linked to the ARPA system and, consequently, determines the effects of the safe and optimal control.

4.1.1 *State equation*

The most general description of the own control object passing the *j* number of other encountered moving objects is the model of a differential game of a *j* number of objects - shown on Figure 5.

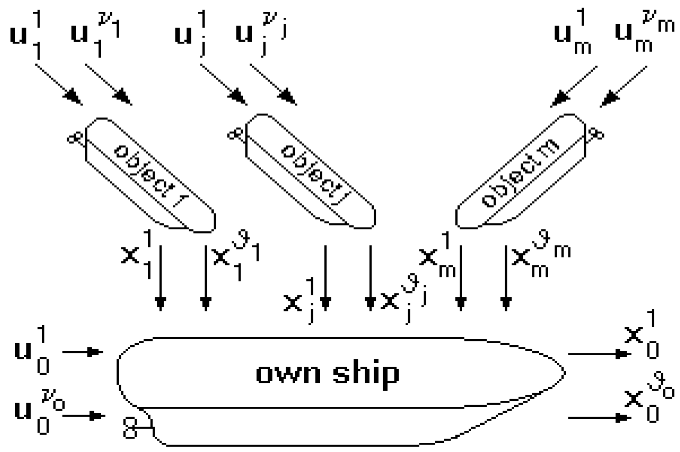

Fig. 5. Block diagram of a base dynamic game model

The properties of the process are described by the state equation:

$$\dot{x}_i = f_i[(x_0^{\vartheta_0}, x_1^{\vartheta_1}, ..., x_j^{\vartheta_j}, ..., x_m^{\vartheta_m}), (u_0^{\nu_0}, u_1^{\nu_1}, ..., u_j^{\nu_j}, ..., u_m^{\nu_m}), t$$
$$i = 1, 2, ..., (j \cdot \vartheta_j + \vartheta_0),\ j = 1, 2, ..., m \qquad (7)$$

where:

$\vec{x}_0^{\vartheta_0}(t)$ – ϑ_0 dimensional vector of the process state of the own control object determined in a time span $t \in [t_0, t_k]$,

$\vec{x}_j^{\vartheta_j}(t)$ – ϑ_j dimensional vector of the process state for the *j-th* object,

$\vec{u}_o^{\nu_0}(t)$ – ν_0 dimensional control vector of the own control object,

$\vec{u}_j^{\nu_j}(t)$ – ν_j dimensional control vector of the *j-th* object.

Taking into consideration the equations reflecting the own ship's hydromechanics and equations of the own ship's movement relative to the *j-th* encountered object, the equations of the general state of the process (7) take the following specific form (8).

$$
\begin{aligned}
\dot{x}_0^1 &= x_0^2 \\
\dot{x}_0^2 &= a_1 x_0^2 x_0^3 + a_2 x_0^3 \left| x_0^3 \right| + b_1 x_0^3 \left| x_0^3 \right| u_0^1 \\
\dot{x}_0^3 &= a_4 x_0^3 \left| x_0^3 \right| \left| x_0^4 \right| x_0^4 \left(1 + x_0^4 \right) + a_5 x_0^2 x_0^3 x_0^4 \left| x_0^4 \right| + \\
&\quad a_6 x_0^2 x_0^3 x_0^4 + a_7 x_0^3 \left| x_0^3 \right| + a_8 x_0^5 \left| x_0^5 \right| x_0^6 + b_2 x_0^3 x_0^4 \left| x_0^3 u_0^1 \right| \\
\dot{x}_0^4 &= a_3 x_0^3 x_0^4 + a_4 x_0^3 x_0^4 \left| x_0^4 \right| + a_5 x_0^2 x_0^4 + a_9 x_0^2 + b_2 x_0^3 u_0^1 \\
\dot{x}_0^5 &= a_{10} x_0^5 + b_3 u_0^2 \qquad\qquad (8) \\
\dot{x}_0^6 &= a_{11} x_0^6 + b_4 u_0^3 \\
\dot{x}_j^1 &= -x_0^3 + x_j^2 x_0^2 + x_j^3 \cos x_j^3 \\
\dot{x}_j^2 &= -x_0^2 x_j^1 + x_j^3 \sin x_j^3 \\
\dot{x}_j^3 &= -x_0^2 + b_{4+j} x_j^3 u_j^1 \\
\dot{x}_j^4 &= a_{11+j} x_j^4 \left| x_j^4 \right| + b_{5+j} u_j^2
\end{aligned}
$$

The state variables are represented by the following values:

$x_0^1 = \psi$ – course of the own ship,

$x_0^2 = \dot{\psi}$ – angular turning speed of the own ship,

$x_0^3 = V$ – speed of the own ship,

$x_0^4 = \beta$ – drift angle of the own ship,

$x_0^5 = n$ – rotational speed of the screw propeller of the own ship,

$x_0^6 = H$ – pitch of the adjustable propeller of the own ship,

$x_j^1 = D_j$ – distance to *j-th* object, or x_j – its coordinate,

$x_j^2 = N_j$ – bearing of the *j-th* object, or y_j – its coordinate,

$x_j^3 = \psi_j$ – course of the *j-th* object, or β_j – relative meeting angle,

$x_j^4 = V_j$ – speed of the *j-th* object,

where: $\vartheta_o = 6, \vartheta_j = 4$.

While the control values are represented by:

$u_0^1 = \alpha_r$ – reference rudder angle of the own ship, or $\dot{\psi}$ - angular turning speed of the own ship, or ψ - course of the own ship, depending of a kind approximated model of process,

$u_0^2 = n_r$ – reference rotational speed of the own ship's screw propeller, or force of the propeller thrust of the own ship, or speed of the own ship,

$u_0^3 = H_r$ – reference pitch of the adjustable propeller of the own ship,

$u_j^1 = \psi_j$ – course of the *j-th* object, or $\dot{\psi}_j$ - angular turning speed of the *j-th* object,

$u_j^2 = V_j$ – speed of the *j-th* object, or force of the propeller thrust of the *j-th* object,

where: $\nu_o = 3, \nu_j = 2$.

Values of coefficients of the process state equations (8) for the 12 000 DWT container ship are given in Table 1.

Table 1. Coefficients of basic game model equations

Coefficient	Measure	Value
a_1	m^{-1}	$-4.143 \cdot 10^{-2}$
a_2	m^{-2}	$1.858 \cdot 10^{-4}$
a_3	m^{-1}	$-6.934 \cdot 10^{-3}$
a_4	m^{-1}	$-3.177 \cdot 10^{-2}$
a_5	-	- 4.435
a_6	-	- 0.895
a_7	m^{-1}	$-9.284 \cdot 10^{-4}$
a_8	-	$1.357 \cdot 10^{-3}$
a_9	-	0.624
a_{10}	s^{-1}	- 0.200
a_{11}	s^{-1}	- 0.100
a_{11+j}	$s \cdot m^{-1}$	$-7.979 \cdot 10^{-4}$
b_1	m^{-2}	$1.134 \cdot 10^{-2}$
b_2	m^{-1}	$-1.554 \cdot 10^{-3}$
b_3	s^{-1}	0.200
b_4	s^{-1}	0.100
b_{4+j}	m^{-1}	$-3.333 \cdot 10^{-3}$
b_{5+j}	$m \cdot s^{-1}$	$9.536 \cdot 10^{-2}$

In example for *j=20* objects the base game model is represented by *i=86* state variables of process control.

4.1.2 *Constraints*

The constraints of the control and the state of the process are connected with the basic condition for the safe passing of the objects at a safe distance D_s in compliance with COLREG Rules, generally in the following form:

$$
g_j(x_j^{\vartheta_j}, u_j^{\nu_j}) \leq 0 \qquad (9)
$$

The constraints referred to as *the ships domains* in the marine navigation, may assume a shape of a circle, ellipse, hexagon, or parabola and may be generated for example by an artificial neural network as is shown on Figure 6 (Lisowski et al. 2000b).

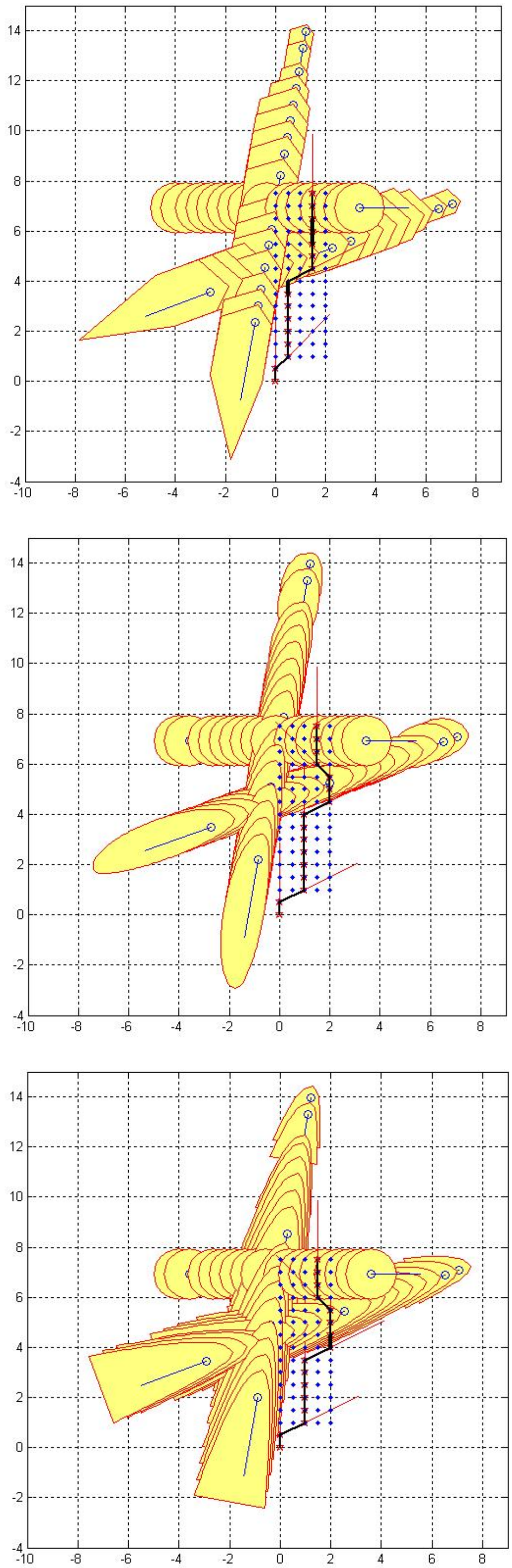

Fig. 6. The shapes of the neural ship's domains in the situation of three encountered objects on Gdanska Bay

4.1.3 *Goal function*

The synthesis of the decision making pattern of the object control leads to the determination of the optimal strategies of the players who determine the most favourable, under given conditions, conduct of the process. For the class of non-coalition games, often used in the control techniques, the most beneficial conduct of the own control object as a player with *j-th* object is the minimization of her goal function in the form of the payments – the integral payment and the final one:

$$I_0^j = \int_{t_0}^{t_k} [x_0^{\vartheta_0}(t)]^2\, dt + r_j(t_k) + d(t_k) \;\to \min \tag{10}$$

The integral payment represents loss of way by the ship while passing the encountered objects and the final payment determines the final risk of collision $r_j(t_k)$ relative to the *j-th* object and the final deflection of the ship $d(t_k)$ from the reference trajectory.

Generally two types of the steering goals are taken into consideration - programmed steering $u_0(t)$ and positional steering $u_0[x_0(t)]$. The basis for the decision making steering are the decision making patterns of the positional steering processes, the patterns with the feedback arrangement representing the dynamic games.

The application of reductions in the description of the own ship's dynamics and the dynamic of the *j-th* encountered object and their movement kinematics lead to approximated models.

4.2 *Approximate models*

4.2.1 *Multi-stage positional game*

The general model of dynamic game is simplified to the multi-stage positional game of *j* participants not co-operating among them.

State variables and control values are represented by:

$$\left.\begin{array}{l} x_0^{(1)} = X_0,\; x_0^{(2)} = Y_0,\; x_j^{(1)} = X_j,\; x_j^{(2)} = Y_j \\ u_0^{(1)} = \psi,\; u_0^{(2)} = V,\; u_j^{(1)} = \psi_j,\; u_j^{(2)} = V_j \\ \qquad\qquad j = 1, 2, \ldots, m \end{array}\right\} \tag{11}$$

The essence of the positional game is to subordinate the strategies of the own ship to the current positions $p(t_k)$ of the encountered objects at the current step *k*. In this way the process model takes into consideration any possible alterations of the course and speed of the encountered objects while steering is in progress. The current state of the process is determined by the co-ordinates of the own ship's position and the positions of the encountered objects:

$$\left.\begin{array}{l} x_0 = (X_0, Y_0),\ x_j = (X_j, Y_j) \\ j = 1, 2, \ldots, m \end{array}\right\} \tag{12}$$

The system generates its steering at the moment t_k on the basis of data received from the ARPA anti-collision system pertaining to the positions of the encountered objects:

$$p(t_k) = \begin{bmatrix} x_0(t_k) \\ x_j(t_k) \end{bmatrix} \quad j = 1, 2, \ldots, m \qquad k = 1, 2, \ldots, K \tag{13}$$

It is assumed, according to the general concept of a multi-stage positional game, that at each discrete moment of time t_k the own ship knows the positions of the objects.

The constraints for the state co-ordinates:

$$\{\, x_0(t), x_j(t) \,\} \in P \tag{14}$$

are navigational constraints, while steering constraints:

$$u_0 \in U_0,\ u_j \in U_j \quad j = 1, 2, \ldots, m \tag{15}$$

take into consideration: the ships' movement kinematics, recommendations of the COLREG Rules and the condition to maintain a safe passing distance as per relationship (6).

The closed sets U_0^j and U_j^0, defined as the sets of acceptable strategies of the participants to the game towards one another:

$$\{U_0^j[p(t)], U_j^0[p(t)]\} \tag{16}$$

are dependent, which means that the choice of steering u_j by the *j-th* object changes the sets of acceptable strategies of other objects.

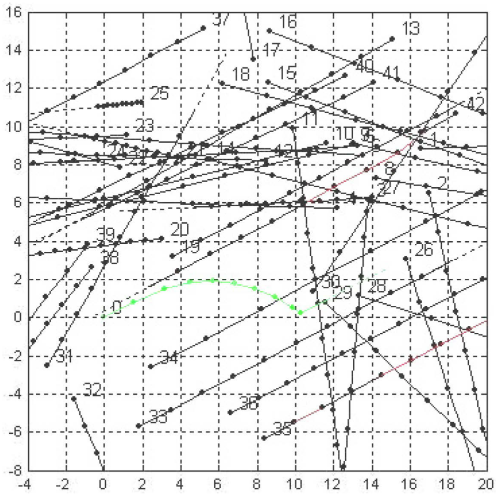

Fig. 7. Positional game trajectories in good visibility, D_s=0.6 nm, $r(t_k)$=0, $d(t_k)$=4.67 nm in a situation of passing 42 encountered objects

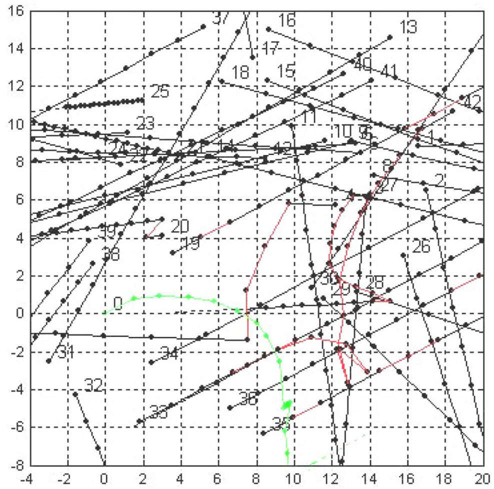

Fig. 8. Positional game trajectories in restricted visibility, D_s=3.0 nm, $r(t_K)$=0, $d(t_K)$=12.11 nm in a situation of passing 42 encountered objects

Examples of safe positional game trajectories are shown on Figures 7 and 8.

4.2.2 *Multi-step matrix game*

When leaving aside the ship's dynamics equations the general model of a dynamic game for the process of preventing collisions is reduced to the matrix game of *j* participants non-co-operating among them (Lisowski 2004b).

The state and steering variables are represented by the following values:

$$\begin{array}{l} x_j^{(1)} = D_j,\ x_j^{(2)} = N_j,\ u_0^{(1)} = \psi,\ u_0^{(2)} = V,\ u_j^{(1)} = \psi_j,\ u_j^{(2)} = V_j \\ j = 1, 2, \ldots, m \end{array} \tag{17}$$

The game matrix $\boldsymbol{R}[r_j(v_j, v_o)]$ includes the values of the collision risk r_j determined from relation (1) on the basis of data obtained from the ARPA anti-collision system for the acceptable strategies v_0 of the own ship and acceptable strategies v_j of any particular number of *j* encountered objects.

In a matrix game player I -own ship has a possibility to use v_0 pure various strategies, and player II -encountered objects have v_j various pure strategies:

$$R = [r_j(v_j, v_0)] = \begin{vmatrix} r_{11} & r_{12} & \ldots & r_{1,v_0-1} & r_{1v_n} \\ r_{21} & r_{22} & \ldots & r_{2,v_0-1} & r_{2v_n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ r_{v_1 1} & r_{v_1 2} & \ldots & r_{v_1,v_0-1} & r_{v_1 v_n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ r_{v_j 1} & r_{v_j 2} & \ldots & r_{v_j,v_0-1} & r_{v_j v_n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ r_{v_m 1} & r_{v_m 2} & \ldots & r_{v_m,v_0-1} & r_{v_m v_n} \end{vmatrix} \tag{18}$$

The constraints for the choice of a strategy (ν_0, ν_j) result from the recommendations of the way priority at sea.

Examples of safe risk game trajectories are shown on Figures 9 and 10.

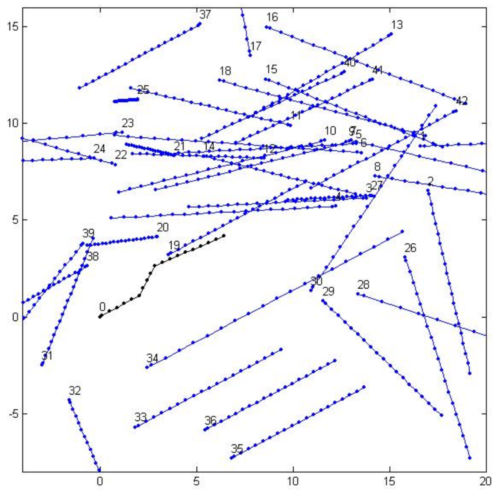

Fig. 9. Risk game trajectories in good visibility, D_s=0.6 nm, $r(t_k)$=0, $d(t_k)$=3.81 nm in a situation of passing 42 encountered objects

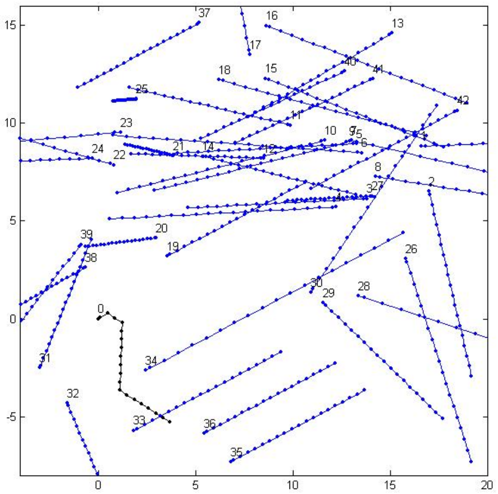

Fig. 10**.** Risk game trajectories in restricted visibility, D_s=3.0 nm, $r(t_K)$=0, $d(t_K)$=8.43 nm in a situation of passing 42 encountered objects

5 CONCLUSION

The application of the models of a game theory for the synthesis of an optimal manoeuvring makes it possible to determine the safe game trajectory of the own ship in situations when she passes a greater number of the encountered objects.

To sum up it may be stated that the control methods considered in this study are, in a certain sense, formal models for the thinking processes of a navigating officer steering of own ship and making decisions on manoeuvres.

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