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Synthesis of Binary Group-Complementary Sequences

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ABSTRACT: Algorithm of binary {-1, 1} group-complementary sequences direct construction for any sequences length N is suggested. On the base of these sequences the signals with very wide frequency bandwidth may be constructed (up to the ultra wideband (UWB) signals). Synthesized sequences may found their application in radar and communication.

1 INTRODUCTION

A pair of finite binary sequences of the same length with the sum of their autocorrelation functions equals zero was introduced by Golay with connection with study of infrared spectrometry in [1, 2, 3]. These sequences are called a pair of complementary sequences. Generalized complementary sequences were considered in [4]. These generalized complementary sequences, named group-complementary sequences, may contain more than two sequences and find their applications in radar [5-19] and communication [20, 21]. The properties of complementary sequences and their relation to other types of codes were investigated by several authors in [3, 4, 5, 6, 7, 22] and in survey article [22]. The problems for further resolving were pointed in [23]. A pair of finite binary complementary sequences doesn't exist for any lengths of sequences. The necessary condition for existence a pair of finite binary complementary sequences had been specified in [3]. Latter, necessary condition was extended to the cases of Group-complementary sequences [4]. Next extension to the complementary sequences with the mismatched filtering were done in [8, 11, 12, 13], where filter design problem was addressed for a given set of sequences with the considered signal-to-noise losses constraints and complementary properties. At the approach [8, 11] no other types of interference were supposed. The number of equations to be solved in [8] was equal to $PN + 2N - 1$, where P – number sequences in set, N – length of each sequence. In [12, 13] another approach for the filter design with the considered signal-to-noise losses constraints and complementary properties was suggested, where only NP equations are needed to be solved. Most importantly, this approach not only allows for signal-to-noise ratio maximization, but also provides a possibility for interfering reflection suppression with given range-velocity distribution. Also synthesis of sequence code-filter pairs under additional constraints with group complementary properties is suggested in [12]. The necessary condition for a sequence code-filter set to be complementary, under the mismatched filter conditions was obtained in [12]. It should be noted that the classical Golay complementary pair does not exist for odd N, but does exist for the mismatched case. In [13] demonstrated the efficiency of suggested approach to the filter synthesis under additional constraints with group-complementary properties on the base of numerical calculations. It was shown that signal-to-noise ratio loses is decrease with increasing memory of optimizing filters in set. Methods of sequences and corresponding mismatched (in common case) filters synthesis, which worked out in [12] and provides complementary properties, relates to the direct methods of sequences and filters construction. Those direct construction methods give the possibility to obtain sequences and corresponding filters with complementary properties for any given length. Besides in [12, 13] were suggested the recursive constructions of new sets of sequences and filters on the base of known sets of sequences and filters with complementary properties without any additional calculations. The counting of filters, which provides the tolerance for Doppler shifts together with proper choosing the order of sequences and corresponding filter dislocation was considered in [14]. Design of the binary complementary sequences under matched filtering more frequently is provided by recursive constructions. It is known only one method for direct construction in condition of match sequences filtering, suggested by Golay [3]. But it was worked out for the sequences with lengths $N = 2^m$. Thou essentially restricted the possibility of practical using of this approach. It not only couldn't be use for odd N, but also for most of even N either. For example it couldn't be used for N=6; 10; 12; 14; 18; 20; 22; 24; 26 and so on. In paper [20] was given a particularly compact description of this construction by using Algebraic Normal Forms.

In this paper we present the direct construction method for binary complementary sequences under matched filtering for any sequences lengths N. Choosing N big enough one can get signals with big product time length on frequency bandwidth, so can get UWB signals with complementary properties.

2 DIRECT CONSTRACTION

Set of P sequences (length N) $S_p^{(N)t} = [s_{0p}, s_{1p}, ..., s_{N-1p}]$ where s^t means transposed vector S, order number $p=1 \div P$, with elements $s_{np} \in \{1,-1\}$, ($n=0 \div N$-1), which for providing group-complementary properties may be derived by the next way:

$$s_{np} = e^{j\pi z_{np}} \tag{1}$$

$$z_{np} = \left\lfloor \left(p - 1 - \sum_{m=1}^{n-1} z_{mp} 2^{m-1}\right) / 2^{n-1} \right\rfloor_{mod\,2} \tag{2}$$

$\sum_{m=1}^{n-1} z_{mp} 2^{m-1} = 0$, for $n=0$, $n=1$; $p=1 \div P$, $P = 2^{N-1}$

In fact $\sum_{m=1}^{N-1} z_{mp} 2^{m-1} = p-1$, which is the 2-adic decomposition of $p-1$. Thus a code of sequence in set is determined only by its length N and order number p in set. For example, direct construction of the group-complementary binary sequences set with length N=3 will be considered. Number of sequences is equal $P = 2^{N-1}$ =4 From (2), (1) we get:

$z_{01}=0$; $z_{11}=0$; $z_{21}=0$; $s_{01}=1$; $s_{11}=1$; $s_{21}=1$;

$z_{02}=0$; $z_{12}=1$; $z_{22}=0$; $s_{02}=1$; $s_{12}=-1$; $s_{22}=1$;

$z_{03}=0$; $z_{13}=0$; $z_{23}=1$; $s_{03}=1$; $s_{13}=1$; $s_{23}=-1$;

$z_{04}=0$; $z_{14}=1$; $z_{24}=1$; $s_{04}=1$; $s_{14}=-1$; $s_{24}=-1$;

Thus the matrix of group-complementary sequences set:

$$[S]_P^{(N)} = \begin{bmatrix} S_1^{(N)t} \\ \vdots \\ S_P^{(N)t} \end{bmatrix} \tag{3}$$

for considered example $N=3$; $P=4$ have the form :

$$[S]_4^{(3)} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \tag{4}$$

The aperiodic auto-correlation function of $S_p^{(N)t}$

$R_k^{aS_p^{(N)}} = \sum_{n=0}^{N-k-1} s_{np} s_{(n+k)p}$, $0 \le k \le N$-1:

$$R_k^{S_p^{(N)}} = R_{-k}^{S_p^{(N)}} \tag{5}$$

For $[S]_4^{(3)}$ we have:

$$\sum_{p=1}^{4} \sum_{k=0}^{2} R_k^{S_p^{(3)}} = 2^{3-1} \cdot 3 \tag{6}$$

So, group- complementary property is fulfilled. The proving group-complementary property of sequences, derived on the base of expressions (1), (2), for any N can be gotten by means of mathematical induction method. Suggest that the property is true for N. Consider what follows from this fact for $N+1$. Matrix for N is $[S]_P^{(N)}$ (P=2^{N-1}). For $N+1$ this matrix transforms by the next:

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} \otimes [S]_P^{(N)} = \begin{matrix} [S]_P^{(N)} \\ [S]_P^{(N)} \end{matrix} \tag{7}$$

To matrix, getting by this way (7), with the size $N \times 2^N$ should be added at the right new last column with size:

$$1 \times 2^N; \begin{bmatrix} s_{N1} \\ s_{N2} \\ \vdots \\ s_{N2^N} \end{bmatrix} \tag{8}$$

The resulting matrix $[S]_{2^N}^{(N+1)}$ consists of matrix (7) with additional column (8) and has the size $(N+1) \times 2^N$. For row p of the matrix $[S]_{2^N}^{(N+1)}$ the sum of aperiodic auto-correlation function values (5) over all k ($k=1 \div N$) with considering (5), (7) and (8) we have:

$$\sum_{k=0}^{N} R_k^{S_p^{(N+1)}} = \sum_{k=0}^{N-1} R_k^{S_p^{(N)}} + s_{Np} \sum_{k=0}^{N} s_{(N-k)p} = R_0^{S_p^{(N)}} + \\ + (s_{Np})^2 + \sum_{k=1}^{N-1} R_k^{S_p^{(N)}} + s_{Np} \sum_{k=1}^{N} s_{(N-k)_p} \quad (9)$$

The sum of auto-correlation functions for all rows can be written as:

$$\sum_{p=1}^{2N} \sum_{k=0}^{N} R_k^{S_p^{(N+1)}} = \sum_{p=1}^{2^N} R_0^{S_p^{(N+1)}} + \sum_{p=1}^{2^N} \sum_{k=1}^{N-1} R_k^{S_p^{(N)}} + \\ + \sum_{p=1}^{2^N} \sum_{k=1}^{N-1} R_k^{S_p^{(N)}} + \sum_{p=1}^{2^N} s_{Np} \sum_{k=1}^{N} s_{(N-k)_p} \quad (10)$$

where $R_0^{S_p^{(N+1)}} = R_0^{S_p^{(N)}} + (s_{Np})^2$. Considering (7), expression (10) may be rewritten in the next form:

$$\sum_{p=1}^{2N} \sum_{k=0}^{N} R_k^{S_p^{(N+1)}} = \sum_{p=1}^{2^N} R_0^{S_p^{(N+1)}} + 2\sum_{p=1}^{2^{N-1}} \sum_{k=1}^{N-1} R_k^{S_p^{(N)}} + \\ + \sum_{k=1}^{N} s_{(N-k)_p} \left[\sum_{p=1}^{2^{N-1}} s_{Np} + \sum_{p=2^{N-1}+1}^{2^N} s_{Np} \right] \quad (11)$$

According to (2) and (1) expression in square brackets in (11) $\left[\sum_{p=1}^{2^{N-1}} s_{Np} + \sum_{p=2^{N-1}+1}^{2^N} s_{Np} \right] = 0$ is equal zero. Next term in (11) is also equal zero $2\sum_{p=1}^{2^{N-1}} \sum_{k=1}^{N-1} R_k^{S_p^{(N)}} = 0$ - according our supposing for N. Expression (11) can be written as

$$\sum_{p=1}^{2^N} \sum_{k=0}^{N} R_k^{S_p^{(N+1)}} = \sum_{p=1}^{2^N} R_0^{S_p^{(N+1)}} \quad (12)$$

Compare with (6). Thus, group-complementary property of binary sequences for *N+1* is proved.

So, expressions (1), (2) for direct construction of binary sequences with group-complementary property are true for any sequences length N .

3 REDUCTION OF THE NUMBER SEQUENCES IN SET

Above-cited direct method of construction gives the possibility to get binary sequences with group-complementary property for any length *N*. The number of sequences in set for that is equal $P = 2^{N-1}$. It does mean that the set of 2^{N-1} sequences with length *N* are group-complementary. But inside of this set exist a lot another sets with the same lengths and with less number sequences in them. For example if *N=4*, so *P=8*. Sequences with order numbers *p=3* and *p=5* create complementary pair, the same for *p=2* and *p=8*. Sequences with order numbers *p=1, p=4, p=6, p=7* create the group-complementary set of four signals, and with order numbers *p=1, p=3, p=4, p=5, p=6, p=7 –* of six signals. For *N=5*, the number of sequences is *P=16*. Sequences with order numbers *p=2, p=5, p=10, p=13* create the group-complementary set of four signals; the same we have for sequences with order numbers *p=3, p=8, p=11, p=16*. Sequences with order numbers *p=1, p=4, p=6, p=7, p=9, p=12, p=14, p=15* create group-complementary set of eight signals. And so on. For *N*=7 we have *P*=64 and will discuss only a part of sequences with reduction number signals in a set. Sequences with order numbers *p=5, p=27, p=50, p=59; p=17, p=23, p=45, p=47; p=5, p=12, p=50, p=53; p=2, p=14, p=47, p=58;* create group-complementary sets with four signals in each. Sequences with order numbers *p=8, p=9, p=19, p=30, p=34, p=47, p=53, p=60; p=1, p=12, p=23, p=30, p=40, p=45, p=50, p=59;* create group complementary sets which contain eight signals in each. Using given length and corresponding order numbers one can get the group-complementary sequences on the base of expressions (2) and (1). For example, in the case of sequences length *N=7* and order numbers sequences in set *p=17, p=23, p=45, p=47*; which mentioned above, on the base (2),(1) can be got z_{np} , s_{np} :

$z_{n17} = [0,0,0,0,0,1,0]$; $s_{n17} = [1,1,1,1,1,-1,1]$; $n=0 \div 6$; $p=17$;

$z_{n23} = [0,0,1,1,0,1,0]$; $s_{n23} = [1,1,-1,-1,1,-1,1]$; $n=0 \div 6$; $p=23$;

$z_{n45} = [0,0,0,1,1,0,1]$; $s_{n45} = [1,1,1,-1,-1,1,-1]$; $n=0 \div 6$; $p=45$;

$z_{n47} = [0,0,1,1,1,0,1]$; $s_{n47} = [1,1,-1,-1,-1,1.-1]$; $n=0 \div 6$; $p=47$.

For *N=10* we have *P=512.* Sequences with order numbers *p=201, p=390; p=236, p=320;* create complementary pairs. From (2), (1) can be got for these cases:

$z_{n201} = [0,0,0,0,1,0,0,1,1,0]$; $s_{n201} = [1,1,1,1,-1,1,1,-1,-1,1]$; $n=0 \div 9$; $p=201$;

$z_{n390} = [0,1,0,1,0,0,0,0,1,0]$; $s_{n390} = [1,-1,1,-1,1,1,1,1,-1,-1]$; $n=0 \div 9$; p=390;

$z_{n236} = [0,1,1,0,1,0,1,1,1,0]$; $s_{n236} = [1,-1,-1,1,-1,1,-1,-1,-1,1]$; $n=0 \div 9$; $p=236$;

$z_{n320} = [1,-1,-1,-1,-1,-1,-1,1,1-1]$; $s_{n320} = [1,-1,-1,-1,-1,-1,-1,1,1,-1]$; $n=0 \div 9$; $p=320$.

Obtained complementary pairs coincides with Golay complementary pairs for *N=10* [3]. Sequences with order numbers *p=32, p=107, p=133, p=218, p=232, p=280, p=298, p=373, p=387, p=496;* create the group-complementary set of ten signals with length *N=10.* For *N=11* sequences with order numbers *p=245, p=421, p=581,p=789;* create the group-complementary set of four signals and with order numbers *p=1, p=184, p=234, p=286, p=367, p=467, p=571, p=584, p=733, p=882, p=908, p=933;* create the group-complementary set of twelve signals. And so on. So, these examples show that for any length *N* inside general set with $P = 2^{N-1}$ exist a lot of sets with less number signals in it. As example, *N=18* contain the set of four group-complementary sequences:

[1-11-11111-1-111-11111-1]
[1-11-11111-1-1-1-11-1-1-1-11]
[1111-111-1-1111-11-1-1-11]
[1111-111-1-11-1-11-1111-1],

Many others sets of sequences with group-complementary properties and different number sequences in each group exist in *N=18* general set (the numbers of sequences in each group are always even numbers). For *N=24* general set of sequences also contain a lot of sets with group-complementary property with different number sequences. We can demonstrate one of those sets with four sequences in it:

[11-1111 1 -111-111-1-1-111-11111-1]
[11-11111-111-111-1-1-1-1-11-1-1-1-11]

[11-11111-1-1-11-1111-111-11-1-1-11]
[11-11111-1-1-11-1111-1-1-11-1111-1].

For *N=26* can be demonstrated a few set with group-complementary property which been contained in general set:

The set with two sequences in it

[111-1-1111-11-1-11-11-11-1-111-11111]
[111-1-1111-11-1-1-1-1-11-111-1-11-1-1-1-1].

This pair of sequences is coincide with pair that was obtained in [25] using a 'by hand' technique. It was shown that complementary sequences of length 26 have only one basic 'kernel'. The next set contains four sequences in it:

[1-1-11-11111111-11111-111-11-1-1-11]
[1-1-11-111111-1-11-1-1-1-11-1-11-1111-1]
[11-1-1111-11-111-11111-1-1-11-1111-1]
[11-1-1111-11-1-1-11-1-1-1-1111-11-1-1-11].

4 CONCLUSION

In the paper the direct method of group-complementary binary sequences construction for any length *N* is suggested. It was shown that inside of general group-complementary set of sequences exist a lot of group-complementary sets with less number sequences in it. That gives wide possibility for reduction number of sequences in each set.

REFERENCES

[1] M.J.E. Golay. Multislit spectroscopy. *J. Opt. Soc. Amer.*, vol. 39, pp. 437-444, 1949.
[2] M.J.E. Golay. Static multislit spectrometry and its application to the panoramic display of infrared spectra. . *J. Opt. Soc. Amer.*, vol. 41, pp. 468-472, 1951.
[3] M.J.E. Golay. Complementary series. *IRE Trans. Inform. Theory*, vol. T -7, pp. 82-87, 1961.
[4] C.C. Tseng, C.L. Liu. Complementary sets of sequences. *IEEE Trans. Inform. Theory,* vol. IT -18, pp.644-652, Sept. 1972.
[5] G. Welti. Quaternary codes for pulsed radar. *IRE Trans. Inform. Theory,* vol. IT-6, pp. 400-408, June 1960
[6] R.J. Turyn. Ambiguity functions of complementary sequences. *IEEE Trans. Inform. Theory,* vol. IT-9, pp. 46-47, Jan. 1963.
[7] R. Sivaswamy. Digital and analog sub-complementary sequences for pulse compression. *IEEE Trans. on Aerosp. and Electron. Syst.*, vol. AES-14, pp. 343-350, Mar. 1978.
[8] G. Weathers, E. M. Holiday. Group-complementary array coding for radar clutter rejection. *IEEE Trans. on Aerosp. and Electron. Syst.*, vol. AES-19, pp. 369-379, May 1983.
[9] N. Levanon, E. Mozeson. Radar signals. New York: Wiley, 2004.
[10] T.R. Quereshi, M.D. Zolowski, R. Calderbank. Target detection in MIMO radar in the presence of Doppler using complementary sequences. Proc. ICASSP-2010, pp.2766-2769.
[11] J. Bi, H. Rohling, Complementary binary code design based on mismatched filter. . *IEEE Trans. on Aerosp. and Electron. Syst.*, vol. AES-48, Jan. 2012.
[12] V.M. Koshevyy. Synthesis of waveform-filter pairs under additional constraints with group complementary properties. *IEEE Radar Conference 2015, Arlington, VA (USA),* pp. 0616-0621, May 2015.
[13] V.M. Koshevyy. Efficiency of filter synthesis under additional constraints with group-complementary properties. IEEE 2016 International Conference "Radioelectronics & InfoCommunications" (UkrMico), Kiev, Ukraine, pp. 978-982, September 11-16, 2016.
[14] V. Koshevyy, V. Popova. Complementary coded waveforms sets in marine radar application. *IEEE First Ukraine Conference on Electrical and Computer Engineering (UKRCON*), May 29-June 2, 2017, Kyiv, pp. 215-222.
[15] V. Koshevyy, V. Popova. Sets of Waveform and Mismatched Filter Pairs for Clutter Supression in Marine Radar Application. TransNav the International Journal on Marine Navigation and Safety of Sea Transportation, vol. 11, N 3, pp. 505-510, Sept. 2017.
[16] V.M. Koshevyy, O.L. Pashenko. Improved compound multiphase waveforms with additional amplitude modulation (periodic mode) for marine radars. *The second International Conference on Information and Telecommunication Technologies and Radio Electronics (UkrMiCo'2017), 11-15 September 2017, Odessa, Ukraine,* pp. 1-6.
[17] A. Pezeshki, A.R. Calderbank, W. Moran, S.D. Howard. Doppler resilient Golay complementaru waveforms. IEEE Tras. Inform. Theory, vol. 54, no. 9, pp. 4254-4266, Sep. 2008.
[18] N. Levanon, I. Cohen. Complementary Pair Radar Waveforms – Evaluating and Mitigating Some Drowbacks. IEEE A&E Systems Magazin, no.10, pp.40-50, March 2017.
[19] A. Jayakumar, A. Durafe. FPGA Based Ultra-Wideband Psevdo-Noise Radar. Proceedings of IEEE ICSCON 2011, pp. 46- 49. 2011.
[20] J.A. Davis, J. Jetwab. Peak-to-mean power control in OFDM, Golay complementary sequences and Reed-Muller codes. IEEE Trans. Inform. Theory, vol. IT-45, no. 7, pp. 2397-2417, Nov. 1999.
[21] S.Emami. UWB Communication Systems: Conventional and 60 GHz. Principles, Design and Standards. Springer, 2013
[22] M.G. Parker, K.G. Paterson, Ch. Tellambura. Golay Complementary Sequences. In *Wiley Encyclopedia of Telecommunications* (J.G. Proakis, ed.), Wiley, 2004.
[23] J.Jetwab, M.G. Parker. Golay complementary array pairs. Designs, Codes and Cryptography.pp. 1-10, May 2007.
[24] P.B. Borwein, R.A. Ferguson. A complete description of Golay pairs for lenghts up to 100. Mathmetics of Computation, vol.73, no.246, pp.967-985, Julay 2003.
[25] S. Jauregui, Jr. Complementary series of length 26, IRE Tras. Inform. Theory. Vol. IT-7,p. 323,1962