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Ship Hull Construction Analysis to the Ultimate Strength Considering Damages

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ABSTRACT: Damage to ship construction causes its structure to lose its ultimate strength. The damage could result from a collision or grounding. The ship's structural integrity following a collision or grounding must be evaluated. This is done to satisfy the ship's structural design requirements. The objective of the present study is to analyze the effect of damage implemented on single and double hull construction to the ultimate strength of ship structure. The ultimate strength of a ship's hull girder following damage from a collision or grounding was ascertained a numerical approach is used in this study. The cross-section sample of the ship is taken in this study namely single and double hull of bulk carrier. The modeling of damage to the ship's bottom and side shell sections from collisions and groundings are taken into account to know the influence of damage when it is applied to single and double hull.

1 INTRODUCTION

The ship's construction is formed from elements consisting of plates and stiffened plates connected. These elements support local and global forces to withstand the loads on the ship. If the load works on different types of construction, then the response of the two types is also different. The two types of ship construction are single and double hull construction. Both types of hulls have advantages and disadvantages. A single hull allows for greater cargo capacity, but it is also more susceptible to structural damage from grounding or collisions. While the double hull has additional strength due to the inner hull, which functions to prevent liquid from directly entering the cargo space, the load carried is reduced due to the additional construction element, namely the inner hull. Damage that may occur to ship hulls is generally asymmetrical. This damage caused a reduction in the ship's strength. Thus, evaluating the impact of damage on both single and double-hull ship structures during a grounding or collision is crucial.

Numerical studies related to structural analysis focused on structures in general and their application to ships have also been carried out. Rakowski and Guminiak [1] studied the free vibrations of geometrically nonlinear elastic Timoshenko beams with fixed supports and used numerical method implemented in finite elements. Li and Chen [2] concentrated on creating empirical formulas for designing hull structures and estimating safety, employing nonlinear finite element analysis to evaluate plates under biaxial compression. The methodology can be applied to bio-composites using calculation methods detailed in current regulations, following an approach similar to that used in the study by Velasco-Parra et al [3] and their research focused on assessing the feasibility of using jute fiber and bioepoxy resin for constructing a boat hull. A finite element study of a special channel beam with thin walls and reinforced webs was carried out by Grenda and Paczos [4]. Zhong et al [5] used computational methods to examine the overall strength characteristics of a hull girder with a sandwich plate upper deck and a laser-welded web core. Additionally, they examined how the girder behaved under different load combinations, torsion, and vertical and horizontal bending. Quispe et al [6] tested a reduced-scale hull box girder for four-point bending using both experimental and computational modeling techniques.

The dynamic maximum load-bearing capability of ultra-large container ships under actual loading conditions was examined by Jagite et al [7] and their study concentrated on assessing the hull girder under lateral and localized loads brought on by various cargo loading circumstances, in addition to a composite bending moment obtained from hydro-elastic research over an extended period of time. The maximum strength of a ship hull girder model with apertures was investigated by Zhao et al [8]. The effects of coupled bending moments and lateral pressure were the main focus of Ma et al [9] investigation into the load-bearing properties and failure behaviors of hull girders at various scaling factors. Cui et al [10] extended their study by taking into account the impacts of elastic shakedown in order to evaluate the maximum strength of hull constructions. Deng et al [11] Using both experimental techniques and finite element analysis examined the ultimate strength and buckling failure behavior of single-hull and double-hull girders with broad deck apertures under cyclic ultimate bending moments. Babazadeh and Khedmati [12] expanded on earlier studies by examining the effects of fractures on a ship's hull girder's ultimate longitudinal strength. To ascertain the ultimate strength of an ISSC2000 bulk carrier, they conducted a progressive collapse analysis, taking into account crack damage at several points on the hull girder, such as the deck, sides, bottom, and double bottom.

Shi and Gao [13] used a steel model with superstructures to conduct a collapse experiment. The experiment's outcomes confirmed important factors including starting flaws and welding residual stress that were part of the modeling of nonlinear finite elements. To determine the crashworthiness of double-hull constructions, a conceptual design method for evaluating grounding and collisions was put forth by Liu et al [14]. Using the section modulus as a strength indicator, Zhang et al [15], introduced a unique technique for assessing strength decrease using stiffness loss analysis. According to their findings, a hull girder's decreased rigidity after damage does not always indicate a decline in strength. They confirmed the idea that strength and stiffness loss are not always equal by analyzing 13 standard cross-sections (UNSS). Using both experimental and computational techniques, [16] Wang and Wang (2020) examine the torsional failure response of a single-compartment hull girder that is intended to serve as a scaled model of the mid-ship section of a container vessel. The simplified progressive collapse approach was refined by Li et al [17] to better forecast how ship hull girders would react to cyclic loading. The cyclic progressive failure method is the name given to this improved strategy. In one study, Wang and Wang [18] examined changes in the thickness and length of the plate geometry at various scales using several genuine girders from the hull of a 10,000 TEU cargo ship. By analyzing the scaling properties of ultimate strength and collapse behavior in hull girders, this work sought to create a more precise scaling criterion for comparing the ultimate strength of scale models with full-sized ships. In contrast to situations when the bending force grows gradually, research shows that cyclic loading can lower a ship hull girder's ultimate strength. This serves as the foundation for previous research conducted by Liu and Guedes Soares [19]. Examining the collapse process of a hull girder with large deck apertures under torsional stress and determining the critical element causing the change from warping failure to shear failure were the objectives of the study.

According A double-hull tanker under biaxial bending was examined by Kuznecovs et al [20] in four different situations: an intact hull, a hull with collision damage, a newly built hull, and a hull impacted by corrosion. The purpose of the study was to evaluate the accuracy and processing requirements of two distinct approaches. Vu Van et al [21] study aimed to evaluate how different corrosion levels and beginning defects affected the maximum bending moment in two different bulk carrier sizes and kinds. Zhang et al [22] used both finite element modeling and experimental testing to expose scaled doublehull side structures to quasi-static impacts at the mid-span using conical and knife-edge indenters. Investigating fracture behavior and related energy dissipation mechanisms was the study's goal. Using a numerical approach, this study investigates the ultimate strength of ship hull girders in single and double hull configurations. The analysis considers the damage caused by longitudinal bending during both the hogging and sagging phases. The impact of damage on the ultimate strength of ship constructions with one or two hulls is another novel topic covered in the study.

2 SHIP PARTICULAR

This research examines how single and double-hull girder designs affect ultimate strength through analytical techniques. The study examines the cross-section of bulk carriers with single and double hull constructions, analyzing them under conditions of hogging and sagging. The grounding damage is located in the lower part and is assumed to be evenly distributed in both single and double hull configurations. The collision damage is found on the external surface of both single and double hull designs. The collision damage is believed to be situated on the shear strake at the corner of the deck. The ship measures 32.2 meters in width and 19.062 meters in depth. Along the longitudinal direction, the frame spacing is fixed at 5.1 meters. The ship's single and double hulls are the same size. Both single and double hull designs have the same material characteristics, such as density, yield strength, Poisson's ratio, and Young's modulus (see Table 1). The analysis excludes considerations of initial deflection, corrosion, or cracks. It is assumed that the cross-section remains planar during assessing progressive collapse. The final strength assessment for both single and double hull bulk carriers is performed under conditions of hogging and sagging.

Table 1. Material properties

Item	Value	Unit
Text	Text	Text
Density	7.89 × 10-9	N/mm^3
Young Modulus	206000	N/mm^2
Poisson Ratio	0.3	-
Yield Strength	315/355	N/mm^2

3 DAMAGED MODELLING ON SHIP CROSS SECTION

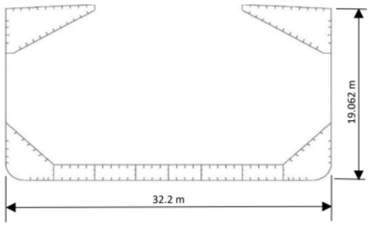


Figure 1. Section view of a single-hull bulk carrier

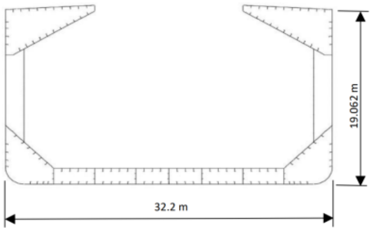


Figure 2. Section view of a double-hull bulk carrier

In this study, the cross sections of single and double hull constructions for intact vessels are shown in Figures 1 and 2, respectively, along with their dimensions. Figures 3 and 4 illustrate grounding and collision damages for both single and double hull constructions of bulk carriers. It is crucial to emphasize that, as seen in Figure 3, the symmetrical grounding damage is located in the cross-section's lower outer region in both single and double bulk carriers. In the event of collision damage, it is assumed to be positioned at the corner of the deck, as illustrated in Figure 4. The applied rotational force is given at one side of the cross section, while the other side is set up to be constrained.

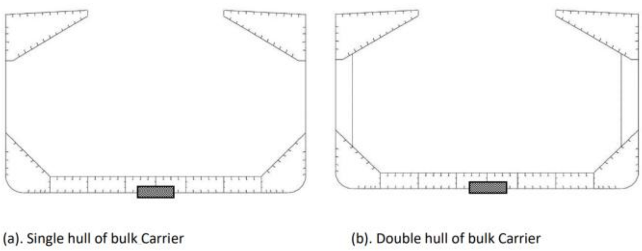


Figure 3. Grounding damages on single hull and double hull bulk carriers

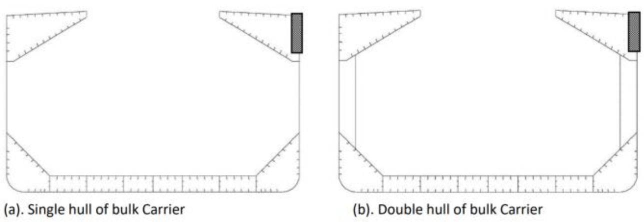


Figure 4. Collision damages on single hull and double hull bulk carriers

4 MODEL, LOAD AND BOUNDARY CONDITION

The finite element models for the single-hull and double-hull bulk carrier designs are shown in Figure 5. In the whole concept, a shell element is used for both kinds of bulk carriers. The loading and boundary conditions for single-hull and double-hull bulk carriers are shown in Figure 6. While the other side of the cross-section has an applied load in the form of a rotating force, the other side has a rigid body connection. In this instance, the Multiple Point Constrained (MPC) method is applied with the rotation force attached.

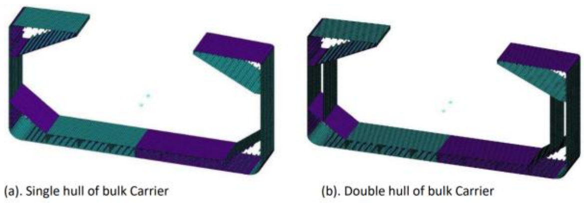


Figure 5. Finite Element Model of Bulk Carrier

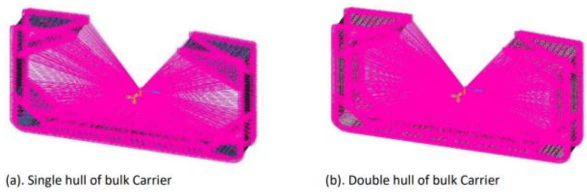
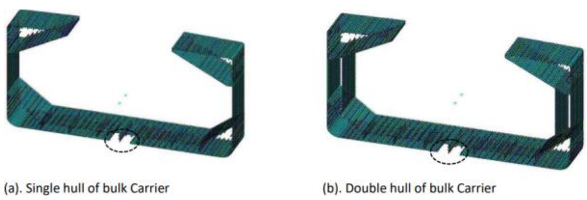


Figure 6. Load and Boundary Conditions

5 DAMAGES MODEL

The 3D Finite Element Model used to build single and double hulls in bulk carriers while taking collision and grounding damage into account is seen in Figures 7 and 8. The shell elements are implemented into the whole cross-section of bulk carriers. Assuming that the elements are removed from the 3D models of bulk carriers. The elements at the outer bottom plate, including stiffeners, are eliminated, and the length of the damage is 2 meters. The assumption of grounding damage is measured from the center line to the left and the right, as shown in Figure 7, marked by the circle line. The elements at the deck side corner, represented as collision damages, have also been removed, including their stiffeners, as expressed in Figure 8. The material is assumed to be homogenous and isotropic. As the fundamental case, the analysis does not consider the initial deflection, strain-hardening effects, and cracks.



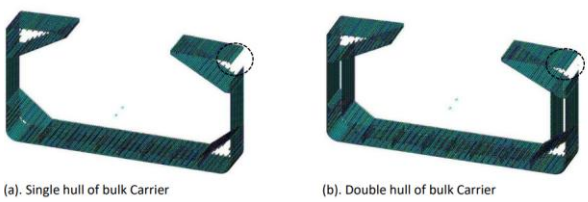


Figure 8. Collision Damages of Bulk Carriers

6 SIMPLIFIED ANALYTICAL METHOD

The procedure for the analytical method is derived irrespective of the influence of neutral axes and damages.

1. The cross-section is divided into elements, which consist of unstiffened and stiffened plates.
2. The average stress-average strain relationships are derived for individual elements, including the effects of buckling and yielding, as shown in Eq. 1.

$$\sigma = f_i\left(\varepsilon\right) \quad (1)$$

The axial stress corresponds to the axial strain, determined by the average stress-average strain relationship calculated in advance for the individual elements. Generally, the average stress-average strain relationship in accordance with buckling and yielding is a nonlinear function of strain. It is assumed that the cross-section remains plane, therefore, the axial strain at the i-th structural element due to the horizontal and vertical curvatures are stated as follows Eq.2:

$$\varepsilon_i\left(y_i, z_i\right) = \varepsilon_0 + y_i\phi_H + z_i\phi_V \quad (2)$$

P, M_V and M_H are the axial force, vertical, and horizontal bending moments, respectively, and they are obtained by integrating axial stresses over the intact cross-section as stated in Eqs. 3, 4, and 5.

$$P = \sum_{i=1}^{N} \sigma_i A_i = 0 \quad (3)$$

$$M_H = \sum_{i=1}^{N} \sigma_i y_i A_i \quad (4)$$

$$M_V = \sum_{i=1}^{N} \sigma_i z_i A_i \quad (5)$$

where N is the number of intact elements and A_i is a cross-section of the individual elements. However, to obtain the stress and moment at the deck and bottom, the longitudinal bending of the hull girder needs to fulfill the condition of the zero-axial force stated in Eq. 3. Eqs. 1 and 2 are substituted into Eqs. 3~5, thereby forming a set of nonlinear simultaneous equations concerning the axial strain ε_0 and curvatures ϕ_V and ϕ_H. This is used to determine the relationship between crosssectional forces and deformations. The equation states the location of the neutral axis in the *y-z* plane on a straight line Eq. 6:

$$\varepsilon_0 + y_i\phi_H + z_i\phi_V = 0 \quad (6)$$

3. Derive the tangential axial stiffness of individual elements Di, in Eq. 1, from the recent average stress-average strain curve Eq. 7.

$$\Delta\sigma = D_i \Delta\varepsilon \quad (7)$$

where $D_i = \dfrac{df_i}{d\varepsilon}$

4. Calculate the position of the neutral axis y_G and z_G, Eqs. 8 and 9

$$y_G = \frac{\left(\sum_{i=1}^{N} y_i D_i A_i\right)}{\left(\sum_{i=1}^{N} D_i A_i\right)} \quad (8)$$

$$z_G = \frac{\left(\sum_{i=1}^{N} z_i D_i A_i\right)}{\left(\sum_{i=1}^{N} D_i A_i\right)} \quad (9)$$

y_G and z_G the coordinates at the neutral axis are measured from the origin at the bottom keel. Here,

y_i and z_i are the coordinates of individual elements.

5. Evaluate the flexural stiffness of the cross-section regarding the neutral axis, Eq. 10

$$\begin{Bmatrix} \Delta M_H \\ \Delta M_V \end{Bmatrix} = \begin{bmatrix} D_{HH} & D_{HV} \\ D_{VH} & D_{VV} \end{bmatrix} \begin{Bmatrix} \Delta\phi_H \\ \Delta\phi_V \end{Bmatrix} \quad (10)$$

where, the stiffness is calculated as follows:

$$D_{AA} = \sum_{i=1}^{N} D_i A_i$$

$$D_{HH} = \sum_{i=1}^{N} D_i \left(y_i - y_G\right)^2 A_i$$

$$D_{VV} = \sum_{i=1}^{N} D_i \left(z_i - z_G\right)^2 A_i$$

$$D_{HV} = D_{VH} = \sum_{i=1}^{N} D_i \left(y_i - y_G\right)\left(z_i - z_G\right) A_i$$

where ΔM_H, ΔM_V, $\Delta\phi_H$, $\Delta\phi_V$ are the incremental of the horizontal and vertical bending moment, including horizontal and vertical curvatures, respectively.

6. Calculate the individual elements' strain, curvature, and stress increments using the slope of the average stress-average strain curve.
7. Generate a curve of bending moments versus rotations and convert it to curvature by dividing the rotation by length.
8. Plot a curve of the bending moments against the curvatures.
9. Proceed to the next step.

7 RESULT AND DISCUSSION

Single-hull construction has just one watertight outer layer extending across the entire structure. Because there is only one layer, single-hull ships present a higher risk to the marine environment in the event of an accident. In contrast, double hull construction incorporates an additional layer, with the space between the two layers serving as ballast tanks. These ballast tank spaces run the full length of the cargo area, providing a significant safety advantage. Single-hull designs lack these ballast spaces. However, double-hull construction requires more steel, making the building process longer. The ballast compartments in double-hull ships are more susceptible to hull fractures and small failures than singlehull designs. Operators of double-hull ships often report cargo leakage into ballast tanks due to stress, fatigue, or construction flaws. This study examines the damage to the outer bottom part of both singlehull and double-hull bulk carriers.

By showing the moment-curvature relationship for both hogging and sagging circumstances, which are pertinent to single and double-hull designs of bulk carriers, this paper illustrates the maximum strength achieved by the numerical technique. According to Babazadeh and Khedmati [12], Kuznecovs et al [20], and Liu et al [14], the moment-curvature curve shows the hull girder's bending moment capacity under both tension and compression. An alternative method was taken by Yao and Nikolov [24], who demonstrated the load-carrying capacity based on the connection between bending moment and curvature by integrating Smit's technique into the software "HULLST." For simple computations, the midship section's distance of one frame represents the ship's length Yao and Nikolov [24], Cui et al [10], Kuznecovs et al [20]. In bulk carriers exposed to vertical bending moments during hogging and sagging circumstances, the ultimate strength of single and double hulls is divided into two categories intact and damaged. Figures 9 and 10 use FEM and HULLST, respectively, to compare the ultimate strength of single and double-hull bulk carriers in their undamaged states.

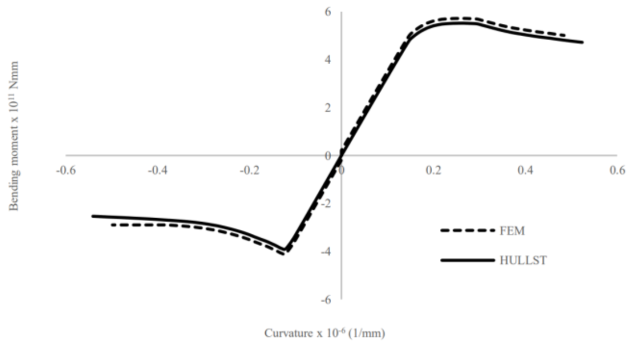


Figure 9. Comparison of moment-curvature of single hull Bulk Carrier (Intact)

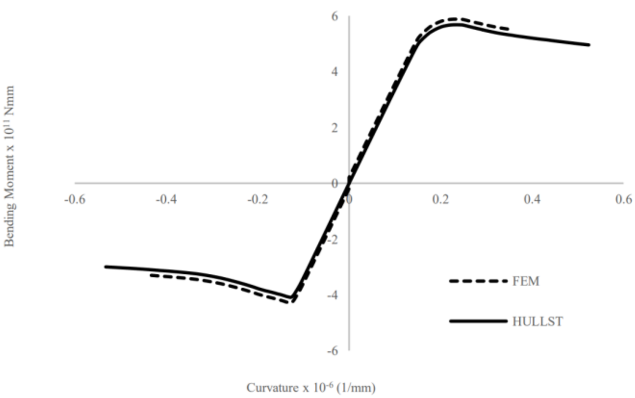


Figure 10. Comparison of moment-curvature of double hull Bulk Carrier (Intact)

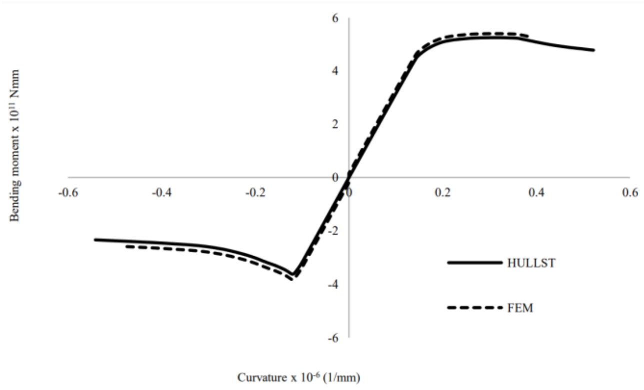


Figure 11. Comparison of moment-curvature of single hull Bulk Carrier (Collision)

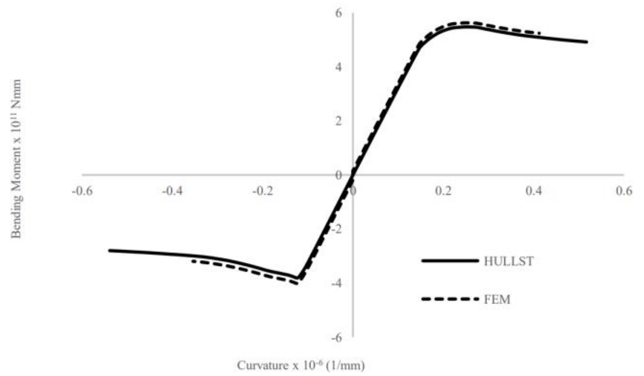


Figure 12. Comparison of moment-curvature of double hull Bulk Carrier (Collision)

Figures 11 and 12 illustrate the comparison of maximum strength between FEM and HULLST when exposed to collision damage in single and double hull bulk carriers. The moment-curvature curves produced by FEM are represented with solid lines, whereas those generated by HULLST are indicated with dashed lines. It has been noted that the final strength determined using the numerical method through FEM is slightly higher than that obtained from HULLST. This is also seen in the previous condition namely the intact condition where this phenomenon is found. The single and double hull construction gives influence to the ultimate strength. By the additional of construction element in term of an inner hull, it affects the inertia and section modulus. This undoubtedly affects the changes in the neutral axis's location.

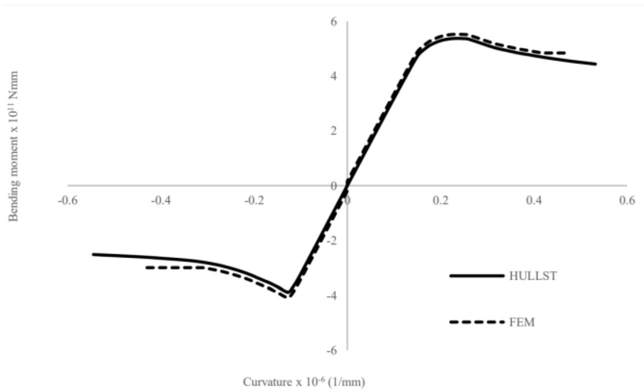


Figure 13. Comparison of moment-curvature of single hull Bulk Carrier (Grounding)

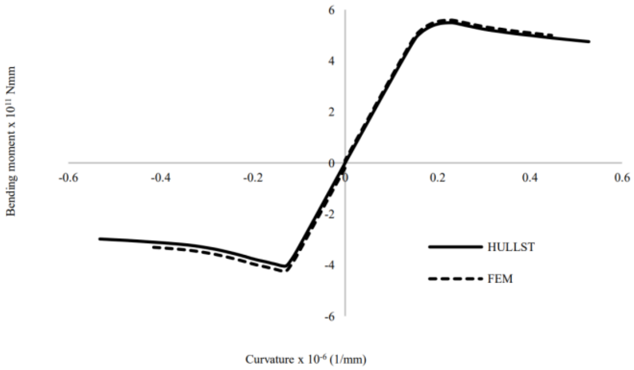


Figure 14. Comparison of moment-curvature of double hull Bulk Carrier (Grounding)

The ultimate strength analysis using another sample like box girder was also investigated by Ao and Wang [23], Deng et al [11] and Quiespe et al [6] using numerical analysis. In the present study, by using numerical analysis and simplified method/analytical method, the influence of damages to the single and double hull constructions of bulk carriers are investigated. Figures 13 and 14 compare the ultimate strength of bulk carriers with single and double hulls while accounting for the impact of grounding damage. The comparison of the ultimate strength single and double hull under hogging and sagging conditions subjected to grounding damages are 4.3% and 3.8%, respectively. While for collision damages, the comparison of the ultimate strength between single and double under hogging and sagging are 2.2% and 4.5%. It has been noted that adding an inner hull in double hull construction affects the ultimate strength, even though the damage from grounding happens at the outer bottom plate. In this instance, double-hulled ships exhibit greater bending moment strength than single-hulled ships.

8 CONCLUSIONS

This study used numerical analysis to evaluate the ultimate strength of bulk carriers with single and double-hull configurations. The maximum strength of both ships was assessed through an analytical approach, and these results were subsequently compared with numerical analysis. From the perspective of ultimate strength, a bulk carrier designed with a double hull and an inner hull is stronger than one with a single hull construction. As calculated using numerical methods, the maximum strength of the two ships exceeds that obtained through analytical methods. This is probably due to removing certain elements and how stress is distributed. The total strength under hogging and sagging situations is largely determined by the design of the single and double hulls. It is also found that the ultimate strength obtained by numerical method is in good agreement with analytical method. This study contributes to the guidelines for designing and constructing ship hulls.

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