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# Routing Planning As An Application Of Graph Theory with Fuzzy Logic

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ABSTRACT: The routing planning one of the classic problems in graph theory. Its application have various practical uses ranging from the transportation, civil engineering and other applications. The resolution of this paper is to find a solution for route planning in a transport networks, where the description of tracks, factor of safety and travel time are ambiguous. In the study the ranking system based on the theory of possibility is proposed.

## 1 INTRODUCTION

In many applications such as transportation, routing, communications, economical, and so on, graphs emerge naturally as a mathematical model of the observed real world system. Fuzzy logic is a form of many-valued logic or probabilistic logic; it deals with reasoning that is approximate rather than fixed and exact. Compared to traditional binary sets (where variables may take on true or false values) fuzzy logic variables may have a truth value that ranges in degree between 0 and 1. Fuzzy logic has been extended to handle the concept of partial truth, where the truth value may range between completely true and completely false. Djikstra's algorithm (named after its discover, E.W. Dijkstra) solves the problem of finding the shortest path from a point in a graph (the source) to a destination. It turns out that one can find the shortest paths from a given source to all points in a graph in the same time, hence this problem is sometimes called the single-source shortest paths problem. Dijkstra's algorithm keeps two sets of vertices:

- S: The set of vertices whose shortest paths from the source have already been determined and
- V:-S the remaining vertices.

In finding the shortest path under uncertain environment, an appropriate modelling approach is to make use of fuzzy numbers. One of the most used methods to solve the shortest path problem is the Dijkstra algorithm. In the case of crisp number to model arc lengths, the Dijkstra algorithm can be easily to implemented. However, due to the reason that many optimization methods for crisp numbers cannot be applied directly to fuzzy numbers, some modifications are needed before using the classical methods.(Boominathan and Kanchan, 2014)

Routing problems in networks are the problem in the context of sequencing and in recent times, they have to receive progressive note. Congruous issues usually take places in the zones of transportation and communications. A schedule problem engages identifying a route from the one point to the other because there are many of optional tracks in miscellaneous halting place of the passage. The cost, time, safety or cost of travel are different for each routes. Theoretically, the method comprises determining the cost of all prospective tracks and the find with minimal expense. In fact, however, the amount of such options are too large to be tested one after another. A traveling salesman problem is a routing problem associated with preferably strong restrictions. Different routing problem emerges when it can to go from one point to another point or a few points, and choose the best track with the at the lowest estimate length, period or cost of many options to reach the desired point. Such acyclic route network problem easily can be solved by job sequencing. A network is defined as a series of points or nodes that are interconnected by links. One way to go from one node to another is called a path. The problem of sequencing may have put some restrictions on it, such as time for each job on each machine, the availability of resources (people, equipment, materials and space), etc. in sequencing problem, the efficiency with respect to a minimum be measured costs, maximize profits, and the elapsed time is minimized. The graph image and the example of costs of borders are given in the figure 1. In this hypothetical idea the tract network is illustrated by a graph. Presented graph is given with an ordered pair G: = (V, E) comprising a set V of vertices or nodes together with a set E of edges (paths), which connect two nodes. The task is to reach the N1 node from N3 node in the graph at smallest cost.(Neumann, 2016)

## 2 PATH FINDING ALGORITHMS

A path finding algorithm for transit network is proposed to handle the special characteristics of transit networks such as city emergency handling and drive guiding system, in where the optimal paths have to be found. As the traffic condition among a city changes from time to time and there are usually a huge amounts of requests occur at any moment, it needs to quickly find the best path. Therefore, the efficiency of the algorithm is very important . The algorithm takes into account the overall level of services and service schedule on a route to determine the shortest path and transfer points. There are several methods for pathfinding: In Dijkstra's algorithm the input of the algorithm consists of a weighted directed graph G and a source vertexes in Graph. Let's denote the set of all vertices in the graph G as V. Each edge of the graph is an ordered pair of vertices (u, v) representing a connection from vertex u to vertex v. The set of all edges is denoted E. Weights of edges are given by a weight function w: E → [0, ∞]; therefore w (u, v) is the non-negative cost of moving from vertex u to vertex v. The cost of an edge can be thought of as the distance between those two vertices. The cost of a path between two vertices is the sum of costs of the edges in that path. For a given pair of vertices s and t in V, the algorithm finds the path from s to t with lowest cost (i.e. the shortest path). It can also be used for finding costs of shortest paths from a single vertex s to all other vertices in the graph (Boominathan and Kanchan, 2014).

An ordered pair of sets G = (V, E) where V is a nonempty finite set and E consisting of 2-element subsets of elements of V is called a graph. It is denoted by G = (V, E). V is called vertex and edge set respectively. The elements in V and E are called vertices and edges respectively. If elements of E are ordered pairs, then G is called a directed graph or digraph. The vertices between which an edge exists are called endpoints of the edge. An edge whose endpoints are the same is called a loop. A graph without loops is called a simple graph.

### 2.1 *Dijkstra's algorithm*

For a given source vertex (node) in the graph, the algorithm finds the path with lowest cost (ie the shortest path) between that vertex and every other vertex. It can also be used for finding the shortest cost path from one vertex to a destination vertex by stopping the algorithm is determined by the shortest path to the destination node. For example, if the vertices of the graph represent the city and are the costs of running paths edge distances between pairs of cities connected directly to the road, Dijkstra's algorithm can be used to find the shortest route between one city and all other cities. As a result, the shortest path algorithm is widely used routing protocols in a network, in particular the IS - IS and Open Shortest Path First. (Neumann, 2014)

Short characteristic of Dijsktra algorithm [2].

- The input of the algorithm consists of a weighted directed graph G and a source vertex s in G
- Denote V as the set of all vertices in the graph G.
- Each edge of the graph is an ordered pair of vertices (u,v)
- This representing a connection from vertex u to vertex v
- The set of all edges is denoted E
- Weights of edges are given by a weight function w: E → [0, ∞)
- Therefore w(u,v) is the cost of moving directly from vertex u to vertex v
- The cost of an edge can be thought of as (a generalization of) the distance between those two vertices
- The cost of a path between two vertices is the sum of costs of the edges in that path
- For a given pair of vertices s and t in V, the algorithm finds the path from s to t with lowest cost (i.e. the shortest path)
- It can also be used for finding costs of shortest paths from a single vertex s to all other vertices in the graph.

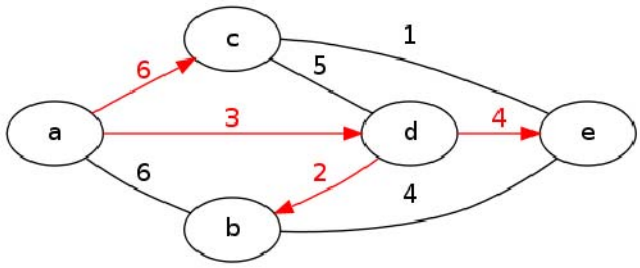


Figure 1. Dijkstras algorithm on tree graph

## 3 FUZZY NUMBERS AND POSSIBILITY THEORY

In case when there is need to model uncertainty that originates in indistinguishability, vagues etc. it is not suitable to use statistical approaches and alternative approaches is necessary (Ghatee and Hashemi, 2009).

Alternative framework for all essential operations can be found in Fuzzy set theory and Possibility theory. (Caha and Dvorsky, 2015)

### 3.1 *Fuzzy Numbers*

Fuzzy numbers are special cases of convex, normal fuzzy sets defined on $\mathbb{R}$ with at least piecewise continuous membership function, that represent vague, imprecise or ill-known value (Ghatee and Hashemi, 2009). There are several types of fuzzy numbers, commonly used are triangular and trapezoidal ones, however other shapes are possible as well (Ghatee and Hashemi, 2009). Triangular and trapezoidal fuzzy numbers are often used because calculations with them and their comparison can be done relatively easily, but it is much better if calculations and comparisons can be done for any shape of fuzzy numbers.

The most general type of fuzzy numbers that can be utilized for calculations are so called piecewise linear fuzzy numbers, these fuzzy numbers are defined as a set of $\alpha$-cuts (Ghatee and Hashemi, 2009). They can approximate any given shape and in their most simple representation are equal to triangular or trapezoidal fuzzy numbers.

If there is need to combine fuzzy numbers with classic crisp values then crisp numbers are treated as special case of fuzzy number, where all $\alpha$-cuts are the same degenerative interval (Ghatee and Hashemi, 2009).

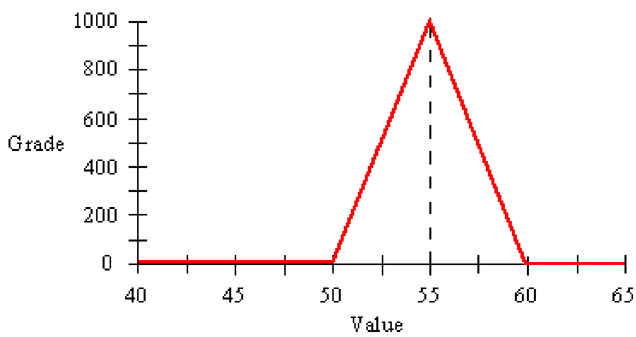


Figure 2. Example of triangular fuzzy number

### 3.2 *Fuzzy Arithmetic*

In order to perform basic arithmetic operations with fuzzy numbers there is need for apparatus that allows and specifies such operations. The most general form of such rule is specified by so called extension principle (Zadeh, 1975), however this particular definition is complicated in terms of implementation, so alternative approaches that utilize decomposition theorem and interval arithmetic are used (Ghatee and Hashemi, 2009). The decomposition theorem states that every fuzzy number (or generally any fuzzy set) $\tilde{A}$ can be described by associated sequence of $\alpha$-cuts. An $\alpha$-cut is an interval where all the objects have membership at least equal $\alpha$. Formally it can is written as: $cut_\alpha(\tilde{A}) = \tilde{A}_\alpha = \{x \in X \mid \mu_{\tilde{A}}(x) \geq \alpha\}$ (Ghatee and Hashemi, 2009). Such $\alpha$-cut of a fuzzy number is always closed interval $A_\alpha = [\underline{a}_\alpha + \underline{b}_\alpha]$ . The only necessary arithmetic operation for determination of shortest path is addition, using decomposition theorem and interval arithmetic the addition of fuzzy number $\tilde{A}, \tilde{B}$ is (Moore et al., 2009):

$$\tilde{A}_\alpha + \tilde{B}_\alpha = \left[\underline{a}_\alpha + \underline{b}_\alpha, \bar{a}_\alpha + \bar{b}_\alpha\right] \quad (1)$$

for each $\alpha \in [0,1]$. Using this approach the addition of any two fuzzy numbers is possible (Caha and Dvorsky, 2015).

### 3.3 *Possibility theory*

To allow decision making based on fuzzy numbers there is a need for a system that will allow ranking of fuzzy numbers. There are several such systems however most of them consider only one point of view on the problem (Dubois and Prade, 1983). The complete set of ranking indices in the framework of possibility theory was proposed in (Dubois and Prade, 1983). This ranking system uses possibility and necessity measures to determine relation of two fuzzy numbers (Caha and Dvorsky, 2015).

Utilization of possibility theory allows also semantically describe fuzzy numbers as possibility distributions (Zadeh, 1975). This semantic than help us explaining what such fuzzy numbers mean. The values with membership value 1 are believed to be absolutely possible or unsurprising, thus they should cove the most likely result. With decreasing degree of membership the possibility of obtaining given result decreases and the surprise rises. When membership value reaches 0 then such result is impossible (or almost impossible at some cases) and the surprise that such result would present is maximal. Such semantics helps with practical explanation what the results truly mean.

To asses position of fuzzy number $\tilde{X}$ to the fuzzy number $\tilde{Y}$ four indices are needed (Dubois and Prade, 1983). Two of them define possibility and necessity that $\tilde{X}$ is at least equal or greater than $\tilde{Y}$ :

$$\Pi_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right) = \sup_x \min\left(\mu_{\tilde{X}}(x), \sup_{y \leq x} \mu_{\tilde{Y}}(y)\right) \quad (2)$$

$$N_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right) = \inf_x \max\left(1 - \mu_{\tilde{X}}(x), \sup_{y \leq x} \mu_{\tilde{Y}}(y)\right) \quad (3)$$

The other two determine if $\tilde{X}$ is strictly greater than $\tilde{Y}$ :

$$\Pi_{\tilde{X}}\left(\left]\tilde{Y},\infty\right)\right) = \sup_x \min\left(\mu_{\tilde{X}}(x), \inf_{y \geq x} 1 - \mu_{\tilde{Y}}(y)\right) \quad (4)$$

$$N_{\tilde{X}}\left(\left]\tilde{Y},\infty\right)\right) = \inf_x \max\left(1 - \mu_{\tilde{X}}(x), \inf_{y \geq x} 1 - \mu_{\tilde{Y}}(y)\right) \quad (5)$$

Together these indices allow comparison of any two fuzzy numbers, based on pairwise comparison any set of fuzzy numbers can be sorted.

For both set of indices there are four situations of the combinations of possibility and necessity that can be outcome of the calculation. In this paragraph both relations – at least equal or greater, and strictly greater – are referred as relation, because the descriptions are valid for both pairs of indices. The first situation is when $\Pi_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right)=N_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right)=0$, which means that $\tilde{X}$ is definitely does not fulfil the given relation to $\tilde{Y}$. Then there is opposite situation in which $\tilde{X}$ completely satisfy the relation. The other two relations contains some uncertainty, because they indicate certain results but they cannot provide them absolutely. The first of those is situation when $\Pi_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right)>0$ and $N_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right)=0$. This means that there is possibility that $\tilde{X}$ might satisfy the relation, but it is not necessary. Obviously that means that the indicators are not strong. The last possible combination of values is $\Pi_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right)=1$ and $N_{\tilde{X}}\left(\left[\tilde{Y},\infty\right)\right)>0$. In such case again it the relation is not satisfied absolutely but the indicators are much stronger than in previous case (Caha and Dvorsky, 2015).

## 4 FUZZY GRAPH THEORY

It is quite well known that graphs are simply models of relations. A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. When there is vagueness in the description of the objects or in its relationships or in both, it is natural that we need to design a 'Fuzzy Graph Model' (Sunitha and Sunil, 2013).

That is well known that a graph is a symmetric binary relation on a nonempty set V. Similarly, a fuzzy graph is a symmetric binary fuzzy relation on a fuzzy subset. The concept of fuzzy sets and fuzzy relations was introduced by L.A.Zadeh (Zadeh, 1975). It was (Rosenfeld, 1975) who considered fuzzy relations on fuzzy sets and developed the theory of fuzzy graphs in 1975.

A graph is a convenient way of representing information involving relationship between objects. The objects are represented by vertices and relations by edges. In many real world problems, we get partial information about that problem. So there is vagueness in the description of the objects or in its relationships or in both (Myna, 2015).

## 5 CONCLUSIONS

The modification of the Dijkstra algorithm aims to provide support for better decision-making in situations of uncertainty. It should be helpful, but all solutions cannot be distinguished by providing. This is used of theory possibility to manage the uncertainty and the ambiguity.

The use of fuzzy numbers as weights in the graph allows for better modelling of real situations where the time travel from one point to another cannot be specified exactly or other similar cases. The time as a sharp number can indicate a much idealization and simplification of the problem because the algorithms for the search then optimal way idealized to produce solutions that are not in the calculation of either uncertainty or the amount of dissimilarity of the solutions.

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