

# Residual Ultimate Strength of Box Girders with Variable Cracks

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**ABSTRACT:** The aim of the present study is to investigate the residual ultimate strength characteristics of box girders with variable cracks under torsional loading. A series of finite element models are established by changing the crack length and crack angle using a commercial FEA program, ABAQUS. The cracks are located at the center and torques are applied on both ends of the box beam. Different aspect ratios are considered to evaluate the effects of cracks on box beams for various widths and lengths of panels in the middle yielding region. The accuracy of the nonlinear FEA results is verified by a comparison with previous predicted formulas. Based on the FEA results, the relationship between the residual ultimate strength and crack parameters can be indicated in a function with period of in the form of Fourier series.

## 1 INTRODUCTION

The ultimate strength of an intact ship has providing an important approach to ensure the structure will not collapse under maximum expected bending moment, torsional loading or combined action of loads. For an ageing thin walled structure, it is vulnerable to various types of defects and damages induced by different phenomena such as corrosion and fatigue cracking. In addition to ship's intact strength, it is necessary to have an assessment of the residual ultimate strength of ship structures in damaged conditions. Cracks of any size may be produced at various locations and orientations throughout plates in the process of ship operation due to corrosion and fatigue damage which will lead to the reduction of ultimate load carrying capacity of the hull structure. Giving exact prediction of the residual ultimate strength has great significance to avoid catastrophic failures of damaged structures over the lifetime and to avoid uneconomical over design.

In recent years, the residual ultimate strength of stiffened or unstiffened panels with cracks under compression, tension, edge shear loading or torsion have been widely studied using numerical and experimental methods and related prediction formulas have been proposed (Hu 2004, Alinia 2007a,b, Alinia 2008, Paik 2008, Paik 2009, Rahman et al. 2011, Rahman et al. 2013). The required mesh refinements around the crack and the influence of relative geometrical and mechanical characteristics of cracked panels on the residual ultimate strength have been investigated and related prediction formulas have been proposed.

Invaluable data can be obtained from physical experiments for validating theoretical modeling approaches and demonstrating how structures behave under closely damaged loading conditions. However destructive testing of large scale structures, such as ships and bridges, are normally limited by size, complexity of structure and cost constraints. These factors have placed a great emphasis on

developing more effective simplified models and robust theoretical techniques to examine structural characteristics. For ships, box girders are extensively used on hull structure research as the hull is mainly consisted of thin plates, beams and other aggregates. At present, the study of box beam ultimate strength is concentrated in the consideration of the influence of initial imperfection while researches concerning about cracking damage are still less.

Paik(2001) and Sun(2003) investigated the ultimate strength characteristics of ship hulls with large hatch openings under torsion respectively in numerical and experimental ways and the ultimate strength of cracked open box girders subjected to variant loads was researched by Shi(2012) and a simple prediction model of the residual ultimate strength was proposed.

Kim(2008) investigated the ultimate strength interaction between bending and torsion of rectangular steel box beams considering the effects of residual stresses and initial imperfections applying different aspect ratios, width-to-thickness ratios and yield stresses by a nonlinear numerical approach. Simple forms of prediction equations for ultimate strengths were proposed by a means of regression analysis on the numerical results.

The ultimate strength of cracked box girders subjected to pure torque was investigated by Shi(2012) and related simple model for predicting the ultimate strength reduction of the box girders due to cracking damage in various crack sizes and crack locations was proposed.

Considering the arbitrarily shaped path of crack propagation for cracked marine structures(SUMI 1998, Okawa 2006), it necessary to involve the influence of crack inclination on ship structures which has not been investigated before. Groups of box beam models are established through changing crack parameters such as various crack sizes and inclined angles and the strength variation regularity of box beams under different cracking damage forms is investigated. FEA results show that crack length and angle have large effect on the variation of the residual ultimate strength and the relationship between the strength of the box girder and the crack parameters is indicated by simplified prediction formulas based on the finite element numerical results.

## 2 FINITE ELEMENT MODELS

The whole calculation in this paper is conducted using commercial software ABAQUS 6.11. The nonlinear shell finite element S4R is used for modeling thin plates as a general four nodes shell element including both reduction of integral method and hourglass control mode which can be used for large deformation analysis of thin plates. The RIKS algorithm is applied as a method of incremental-solution to trace the proper collapse and post-buckling process in structural nonlinear analysis which is a widely used method in structural nonlinear analysis because of advantage of overcoming the difficulties of traditional Newton method across critical points during structural nonlinear buckling

equilibrium path and automatically adjusting incremental steps during iterative processes.

To analyze the influence of different forms of cracks on residual ultimate strength of box girders, A model with 1/2 +1+1/2 transverse frame spacing as shown in Figure 1 was used for all non-linear FE analyses in the present study for box girders so that the middle part can be the yielding region to ensure the damage happens in the middle part first and lateral parts were loading regions. One crack was considered to be distributed in the center of each plate of box beams (four cracks in total with the same size) and the cracks were presumed to be through thickness, having no friction between their edges and no propagation was allowed. The plate initial deflection was not considered for simplicity. Without special stated, the box length and width were set as  $a=b=1000\text{mm}$ , the box thickness was fixed at  $t=10\text{mm}$ . Both material and geometrical nonlinearities were included in the study. Materials were considered to behave in an elasto-plastic manner having bi-linear stress-strain relationship. Unless otherwise specified, the default material, for most parts of the work, was assumed to be mild steel with Young's modulus, yield stress and Poisson's ratio given by:

$$E = 205800\text{MPa}, \sigma_y = 345\text{MPa} \text{ and } \nu = 0.3$$

In order to facilitate the related analysis, the effects of stiffeners or surrounding members on the ultimate strength of box beams were not considered and the Von Mises yield criterion, known to be the most suitable one for mild steel, was used throughout this research.

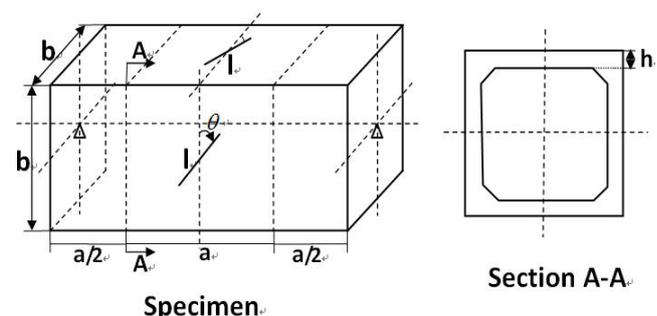


Figure 1. Geometry model of box beams under torque.

The box model was meshed with uniform gird generally and local mesh refinement was taken around the cracks. Figures 2-3 show the variation of the torsional strength behavior for different mesh sizes and crack widths considering the possible influence of mesh size and crack width on computational results. It is observed a mesh size  $\delta b$  no higher than 20mm is good enough for accurate results. Different crack width values within certain limits have little effect on the ultimate strength of the girder which can be neglected. In this case, the default gape size between crack faces  $\delta l$  is taken as  $\delta l = 4\text{mm}$  for most part of the work considering the convenience of meshing around the cracks and the total gird size is taken as  $\delta b = 20\text{mm}$  and  $\delta b/b = 1/50$  which can give satisfied results.

The dimension of the transverse frame can have effects on the ultimate strength of cracked girders

possibly because the frame plays a supporting role for the longitudinal components. Figure 4 shows the variation of box's ultimate strength as the frame size ranges from  $h=0\text{mm}$  to  $h=160\text{mm}$ . It can be seen that changing the frame size has little influence on the ultimate strength when increasing to  $h=80\text{mm}$ . A critical coefficient of frame size can be defined as  $\delta h_c = h_c / b$  and the critical value of box frame can be given as  $h_c = \delta h_c \cdot b$  for different cross sections.

### 3 THE ULTIMATE STRENGTH OF BOX GIRDERS UNDER TORSION BENDING

The residual ultimate strength of a series of cracked box beams in large torsion deflection was investigated by varying the crack length and angle through nonlinear finite element analysis. As to relate the length of cracks to the dimensions of box section width  $b$ , the crack length was varied as the ratio of the width  $b$  given by ( $l=$ )  $0.2b, 0.3b, 0.4b, 0.5b$  while the crack inclination changed as ( $\theta=$ )  $0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ$  and  $90^\circ$ . The crack was considered to be perpendicular to the axis of the box when angle taken as  $\theta=0^\circ$ . The boundary condition was assumed as simply supported on both ends of the box girder. In the torsional center of the ends of the box beam, one side was constrained as  $u_x = u_y = u_z = 0$ , the other was constrained as  $u_x = u_y = 0$ , while direction was along the axis of the box. The torque was imposed by giving a torsional displacement on both ends of the box beam as  $\delta\phi_\tau = 0.015\text{rad}$ , the conversion formula between bending moment and shear stress is as followed,

$$\tau = \frac{T}{2At} \quad (1)$$

where  $T$  is the Torque applied at every end of the box girder,  $A$  is the enclosed area of the box cross section,  $t$  is the box wall thickness.

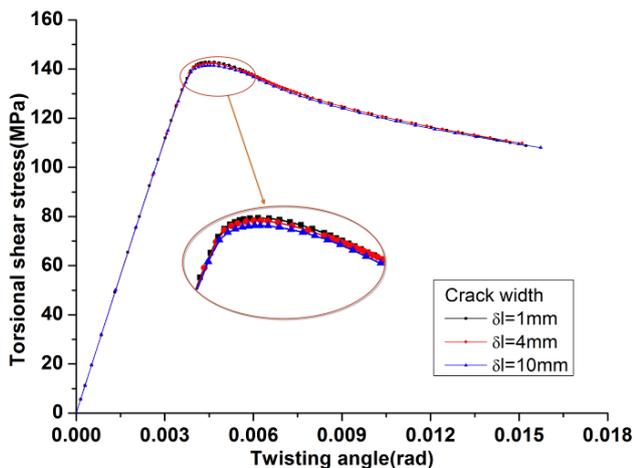


Figure 2. Effect of crack width on the ultimate torsional strength of cracked box ( $\theta=45^\circ$ ).

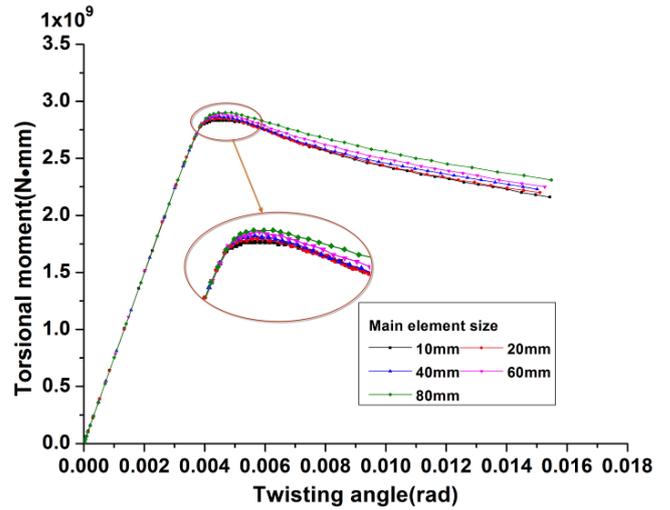


Figure 3. Effect of mesh size on the ultimate torsional strength of cracked box ( $\theta=45^\circ$ ).

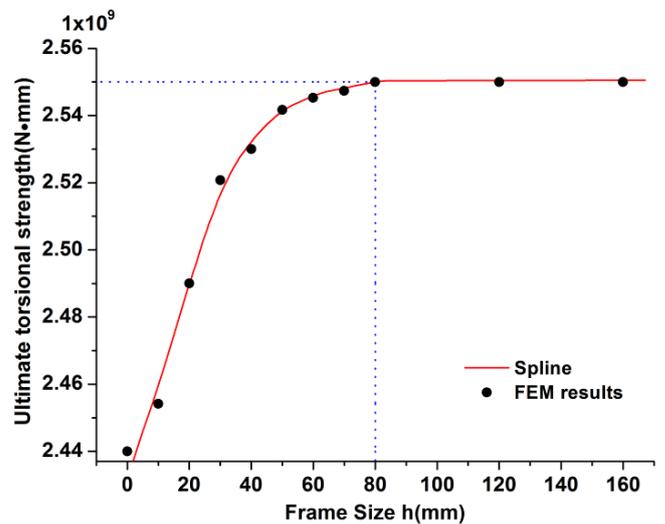


Figure 4. Effect of frame size on the ultimate torsional strength of cracked box.

Figure 5 shows the average torsional shear stress-twisting angle curve when varying the angle of crack in different related crack length  $l/b$ . It is observed from figure 5 the crack inclination has much more impact on the box's strength as the crack length increases. The increasement of angle enhances the box strength within certain limits for a certain length. Changing the angle  $\theta$  has little influence on the strength of box beams while having the least length as  $l/b=0.2$ , where the reduction ratio is only  $\Delta\tau/\tau_{\min}=3.87\%$  between the min angle and max  $\theta=90^\circ$ . On the contrary, the angle has great impact on the box ultimate strength as the min-max changing ratio reaches to  $\Delta\tau/\tau_{\min}=35.5\%$  while large crack size is given at  $l/b=0.5$ . It is shown from the diagram that the ultimate torsional shear stress has no linear proportional relationship with the inclination at certain crack lengths as the stress changes little along with the angle  $\theta$  changing around  $0^\circ$  and  $90^\circ$ , however it ranges a lot when  $\theta$  is around.

Figure 6 shows the distribution of membrane stress when the crack angle varies as the crack length fixed at  $l/b=0.3$  (Figs 6a, b, c are corresponding to  $\theta=0, 30^\circ, 60^\circ$ ). The left figure shows the membrane stress component in  $x$  direction

S11 (namely  $\sigma_x$ ) and the right shows the membrane stress component in y direction S22 (namely  $\sigma_y$ ). It is observed that the compressive stress mainly focuses on the crack tips and then develops along the crack while tensile stress develops from the tip towards outside. Certain angle is fixed between the box longitudinal axis and tensile stress in spite of the variation of crack angles which may have little influence on the stress distribution.

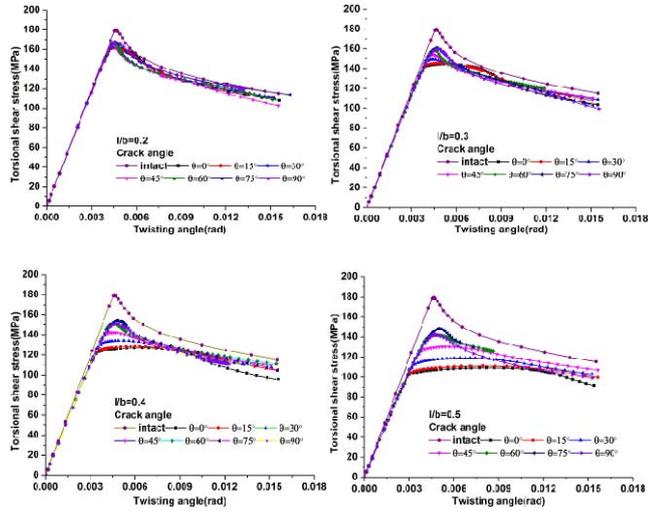


Figure 5. The torsional shear stress-twisting angle curve by changing the crack inclination in varying related crack lengths.

Due to the practical purposes, the length of beam panels may vary. Figure 7 shows the influence of different aspect ratios on the box ultimate strength to evaluate the effects of cracks on box beams for various widths ( $a$ ) and lengths ( $b$ ) of panels in the middle yielding region. However, the value of aspect ratio was taken equal to 1 when other parameters were concerned. It can be seen that the relation between  $\sigma_u / \sigma_y$  and  $a/b$  will be expressed by a formula of  $\sigma_u / \sigma_y \cdot (a/b)^{-0.06}$ .  $\sigma_{u0}$  is the ultimate shear stress when the aspect ratio is taken to 1.

#### 4 SIMPLIFIED PREDICTION FORMULAS FOR ULTIMATE STRENGTH OF BOX BEAM

The following formula gives the reduction factor of torsional ultimate strength for box girders with transverse cracks:

$$R_f = \frac{\tau_u}{\tau_{u0}} = \frac{A_c}{A_0} = \frac{4(b-2c)t}{4bt} = 1 - \frac{2c}{b} \quad (2)$$

where  $R_f$  is the reduction factor of torsional ultimate strength,  $\tau_u$  is the ultimate torsional shear stress for box beam with cracks,  $\tau_{u0}$  is the ultimate torsional shear stress for perfect girder,  $t$  is the panel thickness, is the section width of the box girder,  $2c$  is the crack length.

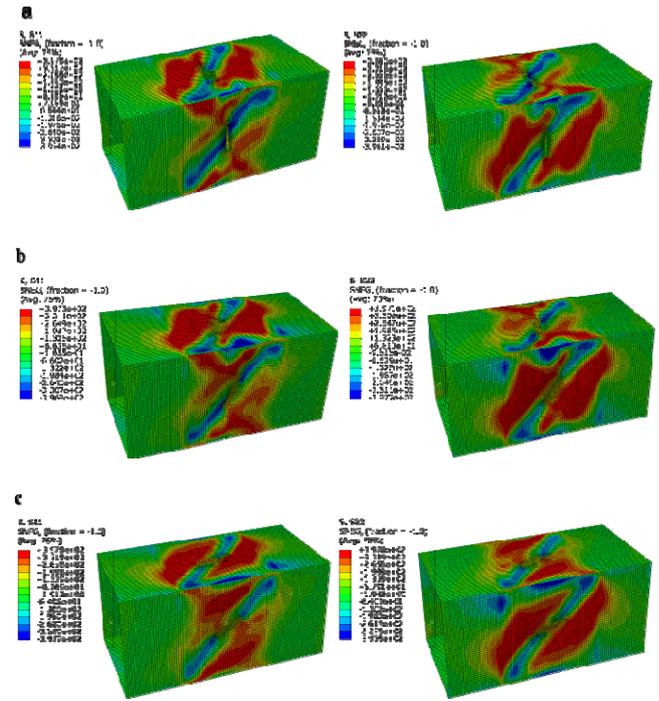


Figure 6. Membrane stress distribution corresponding for different crack angles at crack size of  $l/b=0.3$ : (a)  $\theta = 0^\circ$ , (b)  $\theta = 30^\circ$ , (c)  $\theta = 60^\circ$ .

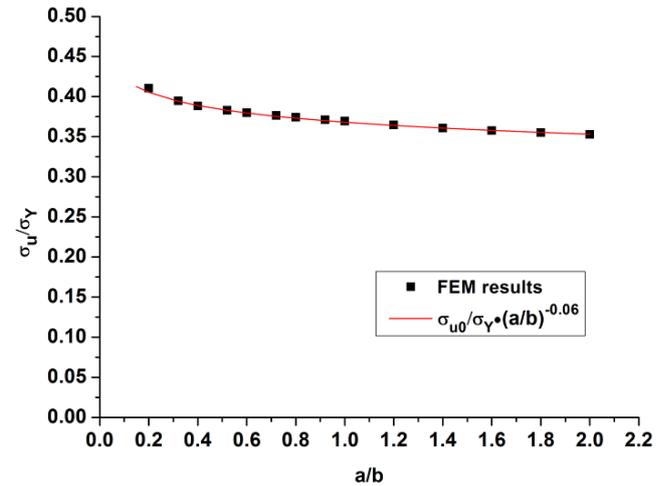


Figure 7. Effect of aspect ratio on the ultimate torsional strength of cracked box ( $\theta=45^\circ$ ).

It has been recognized that the proposed formula in Eq.(2) yields conservative values for the ultimate torsional strength of box girders with transverse cracks ( $\theta=0^\circ$ ) (Shi 2012). Figure 8 compares the torsional ultimate strength of cracked box girders as obtained from the FEA and from Eq.(2) by varying crack lengths and angles where the solid line represents for the predicted results of Eq.(2) and the dotted lines for results from the FEM. It shows conservative results still can be obtained by Eq.(2) even though the length and inclination angle of crack happen to change and results from formula in Eq.(2) are parallel to that from numerical calculation by ABAQUS in general as  $\theta=0^\circ$ . However, the predicted results have large difference compared with the FEA results along with the increase of crack angle which can't be reflected by equation (2). Results from Eq.(2) are smaller than numerical results because the prime formula is derived based on an

isolated thin plate while the four panels are not independent plates but fixed together for box beams. It is clear that the ultimate strength and crack length have a certain linear proportional relationship in spite of the variation of angle.

Considering the geometric symmetry of cracks with respect to the box beam, it can be treated the angle  $\theta$  is larger than  $90^\circ$  for the torque direction reverse. For example,  $\theta = 30^\circ$  with the torque direction reverse can be replaced by  $\theta = 150^\circ$  where the direction did not change. Taking into account the triangular characteristic reflected in the numerical results, the ultimate strength reduction characteristics of cracked box beam can be indicated as a function with period of  $\pi$  in the form of Fourier series as following:

$$R_f = \left(1.07 - 0.77 \frac{l}{b}\right) + \left(0.29 \frac{l}{b} - 0.042\right) \sin(2\theta - \alpha) + \left(0.08 \frac{l}{b} - 0.016\right) \cos(4\theta + \beta) \quad (3)$$

where  $\sin(\alpha) = 0.884$ ,  $\cos(\beta) = 0.386$ ,  $l/b$  is relative crack length,  $\theta$  is the crack inclination angle.

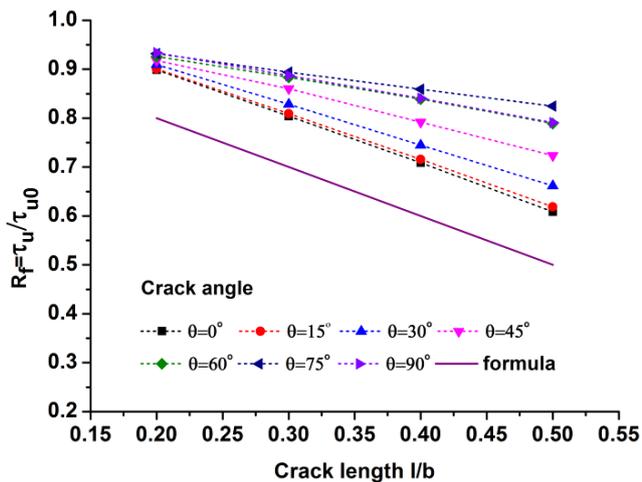


Figure 8. Comparison of ultimate torsional strength reduction between results from Eq.(2) and that from FEA for various crack parameters.

Figure 9 shows the relationship between the torsional reduction factor and crack parameters in a whole cycle. It is found that the largest ultimate shear stress  $\tau_{u(max)}$  is corresponding to crack angle  $\theta = 75^\circ$  while the minimum corresponding to  $\theta = 165^\circ$  with the crack size of  $l/b \geq 0.3$ . The dotted lines in figure 9 are calculated results obtained from Eq.(3) which agrees well with the numerical results from FEA. However, it should be noticed that both Eq.(2) and Eq.(3) are based on the beam model with cracks of the same size on each of all four sides. The slenderness ratio of girder plates  $\beta = (b/t)\sqrt{\sigma_y/E}$  may have effects on the beam ultimate strength corresponding to different beam widths( $b$ ) which need to be confirmed for further investigation and the formula validity need to be verified for beams with cracks only for deck or sides which are not presented in this paper.

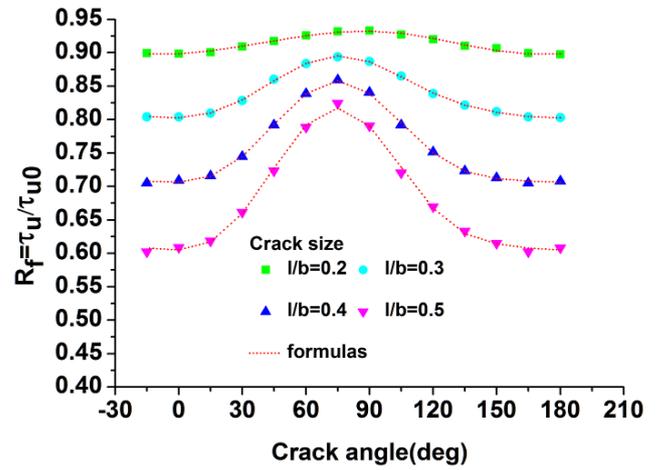


Figure 9. Ultimate torsional strength reduction characteristics in a whole cycle of for different crack sizes.

## 5 CONCLUSION

In this paper, groups of box beam models under torque force were established by changing the crack length  $l$  and angle  $\theta$ . The influence of crack parameters and torsional direction on box ultimate strength was investigated using nonlinear finite element method and following conclusions can be drawn:

- 1 It is clear that when the crack size is less than  $l/b = 0.2$ , changing crack angle has little influence on the ultimate strength of a box girder where the reduction ratio is only  $\Delta\tau/\tau_{min} = 3.87\%$  between the min angle  $\theta = 0^\circ$  and max  $\theta = 90^\circ$ . However, the angle has great impact on the box ultimate strength when crack length increases to  $l/b = 0.5$  as the min-max changing ratio reaches to  $\Delta\tau/\tau_{min} = 35.5\%$ .
- 2 It is observed that the ultimate strength and crack size have a certain linear proportional relationship in spite of the variation of angle. However, no linear relationship but a trigonometric relationship happens between the ultimate strength and crack angle especially when relative crack length  $l/b > 0.2$ .
- 3 Considering the geometric symmetry of cracks with respect to the box beam and the triangular characteristics with changing of crack angle, the ultimate strength reduction characteristics of box girders due to cracking damage can be indicated in a function with period of  $\pi$  in the form of Fourier series. It is found that the largest ultimate shear stress  $\tau_{u(max)}$  is corresponding to crack angle  $\theta = 75^\circ$  while the minimum corresponding to  $\theta = 165^\circ$  with crack size of  $l/b \geq 0.3$ .
- 4 In order to facilitate the analysis and design, the mid body region or other local parts of a ship can be treated as a thin box girder structure. Given exact parameters of cracks, the simple formula proposed in Eq. (3) can be used to predict the residual ultimate strength of simplified hull girder structures under torsional loading.

## ACKNOWLEDGMENTS

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