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Optimization of Operation Costs in Complex, Multi-State, Aging Technical Systems

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ABSTRACT: This paper presents a methodological framework for the joint optimization of operation costs and safety performance in complex, multi-state ageing technical systems. Two original models of system operation cost are proposed: one based on total operation time, and the other on operation within safety state subsets. The models are integrated with linear programming procedures and applied to a real-world maritime transport system. The approach enables a balanced consideration of operational efficiency and system reliability, offering a structured way to support decision making in the management of ageing infrastructure. The methodology relies on both statistical operational data and expert knowledge. Potential extensions of this work include incorporating external factors such as environmental conditions, as well as integrating repair, preventive maintenance, and evolutionary multi objective optimization algorithms to enhance resilience and sustainability in critical technical systems.

1 INTRODUCTION

This work focuses on the modeling and optimization of operation costs for complex, multi-state, aging technical systems, with particular emphasis on their changing operation and safety states over time. The main objective is to develop and apply cost models that not only reflect the technical and economic realities of system operation but also contribute to the improved performance of real-world maritime and port transportation systems.

A novel approach to cost modeling is presented, which considers two distinct categories of system states: operation states and safety states. Based on this dual-state concept, two original cost models have been proposed. The first, referred to as Model 1, represents the total operation cost for fixed operation time, incorporating both direct and indirect costs related to maintenance, repairs, consumables, training, energy usage, data protection, and system downtime. The second, Model 2, quantifies the total cost incurred while the system resides within the safety state subset, focusing exclusively on the period from entry to exit from the defined subset.

To support the safety-oriented classification of system behaviour, five discrete safety states have been introduced: full safety, high-level safety, medium-level safety, low-level safety, and hazard state. The transitions between these states depend on system-specific characteristics, risks, and external factors, and are managed by appropriate procedures or control algorithms. The proposed cost models allow for the assessment and optimization of expenditures under varying safety conditions, enabling targeted analysis of system behaviour within different safety regimes.

In industrial practice, optimizing the long-term operation costs of such systems is crucial. Therefore, linear programming techniques have been employed to minimize the average total operation cost for fixed time (Model 1) or while the system remains in safety states not worse than a critical safety state (Model 2). The optimization is based on the calculation of limit transient probabilities - fundamental to Markov processes - which approximate the long-run behaviour of the system.

Furthermore, the study explores joint optimization procedures that simultaneously address cost efficiency and system safety. Two strategies are proposed: one begins with cost minimization followed by safety assessment, while the other prioritizes safety maximization and then evaluates corresponding costs. In both cases, system modifications are guided by optimal limit transient probabilities, providing operators with actionable strategies to balance reliability, safety, and cost.

The proposed models and optimization procedures offer a comprehensive framework for the economic and safety analysis of complex systems, particularly in critical infrastructure sectors. Their practical application has the potential to enhance operation efficiency and strategic decision-making in real-world settings.

2 LITERATURE REVIEW

The literature on operation costs and optimization of technical systems is largely grounded in the concept of Life Cycle Costing (LCC) and optimization models in maintenance management. The LCC approach, widely introduced in the 1970s, involves analyzing costs incurred throughout the entire life cycle of a technical system. Early contributions to this field include [1], [2], which define LCC as the total of direct, indirect, recurring, and one-time costs associated with the design, research, production, operation, and maintenance of a system. The international standard further develops this definition by detailing the life cycle stages included in cost assessment frameworks [3].

Three main approaches to LCC cost estimation are commonly distinguished in the literature: parametric, analogous, and detailed models, as outlined in [4], [5]. The inclusion of system reliability in LCC analysis is emphasized in several studies [6]–[8], which highlight the need to categorize costs into preventive (e.g., inspections) and failure related categories - both internal and external. Such classifications are essential in decision making regarding investment and system operation.

In the domain of optimization models, the gap between academic research and practical applications is underscored in [9], [10], where the limited translation of theoretical models into industrial use is discussed. The need for more practice-oriented case studies is further emphasized in [11]. Excessive focus on abstract model development without regard to implementation feasibility is criticized in [12]. Additionally, many models optimize a single objective, which may not reflect the multidimensional nature of real world decision-making [13].

Research on multi criteria optimization - such as that presented in [14]–[16] - suggests that practical decision-making requires the simultaneous consideration of multiple criteria, including cost, reliability, and availability. A unifying optimization framework integrating these criteria is discussed in [17], highlighting the importance of not only constructing models but understanding and aligning them with organizational requirements.

In the context of maintenance strategies for multi component systems, the complexity of component interactions and the effects of imperfect maintenance are addressed in [18]. The significance of component interdependencies is examined in [19]–[21], while [22], [23] develop preventive maintenance models that incorporate continuous degradation and condition-based indicators.

Finally, studies such as [24] focus on the economic impact of imperfect repairs in critical infrastructure, emphasizing the importance of optimal maintenance planning to ensure reliability and minimize cost.

Existing research on the operation costs of complex, multi-state, aging technical systems [25]–[28] has primarily focused on the analysis and development of optimization strategies aimed at minimizing operation expenses while maintaining acceptable levels of performance, reliability, and safety. This field is particularly relevant in industries operating long-life systems, where aging and degradation significantly affect both maintenance and operational costs.

In summary, the literature review reveals that research on life cycle costs and maintenance policy optimization is essential for the effective management of technical systems. The adoption of multi-criteria approaches - taking into account cost, reliability, availability, and business-specific requirements - is indispensable for developing practical and efficient management strategies for such systems.

3 OPERATION COSTS OF A GENERIC SYSTEM

In the literature, the concept of a generic system is commonly used in systems theory to describe a general, abstract model that can be adapted to a variety of specific applications and domains. Such a system is not directly tied to any particular implementation but serves as a universal structure that can be modified according to context and requirements. This concept has been widely discussed in the scientific discourse, including the work of Professor Krzysztof Kolbusz, who emphasized its versatility in modeling various processes and phenomena. Its generality facilitates the application of the model across diverse operational environments. In the present study, the term generic system refers to the baseline subject of analysis - a complex, multi state, aging technical system. The chapter introduces a dedicated approach to modeling the operational costs of such a system, taking into account its inherent complexity and state-dependent behaviour. Because the system under consideration exhibits multiple degradation states over time, the modeling of operational costs must also incorporate transitions between different safety states. These safety states represent the system's varying levels of operational security and reliability throughout its life cycle. While the primary focus of this study is the optimization of operational costs, integrating safety-related aspects is essential to achieve a more comprehensive and realistic assessment. By jointly considering costs and safety dynamics, the model provides a more robust foundation for decision-making in the management of complex technical systems. The analysis considers two types of states: operation states and safety states. It also allows for the evaluation of different conditional costs, depending on whether a specific operational or safety state is fixed. Based on this framework, two original models of system operational costs have been proposed: Model 1 – System operation cost model for fixed operation time, Model 2 – System operation cost model in safety state subsets.

3.1 *System operation cost model for fixed operation time*

Model 1 is a mathematical representation of all costs associated with the maintenance and operation of a given system over a specified period. This model accounts for both direct and indirect costs incurred during system operation. Direct costs refer to expenses that are directly related to system maintenance and usage within the considered time interval, such as service and repair costs, the cost of consumable materials, or personnel training costs. In contrast, indirect costs are those not directly tied to system operation but result from its usage. These may include electricity consumption, expenses related to data protection and cybersecurity, and potential costs associated with operational delays.

The total operational cost of the system over the time interval θ

$$\hat{\boldsymbol{C}}(\theta) = \sum_{b=1}^{\nu} p_b \left[\hat{\boldsymbol{C}}(\theta)\right]^{(b)}, \theta > 0, \tag{1}$$

where p_b, $b = 1,2,...,\nu$, are limit transient probabilities at operation states, and

$$\left[\hat{\boldsymbol{C}}(\theta)\right]^{(b)} = \int_0^{\hat{M}_b} \left[\boldsymbol{C}(t)\right]^{(b)} dt, \theta > 0,\ \text{b} = 1,2,...,\nu, \tag{2}$$

are the total values of the system total conditional operation costs at the particular system operation states during the operation time θ, [27]–[28].

3.2 *System operation cost model in safety state subsets*

Model 2 is a mathematical representation of all costs associated with the maintenance and operation of the system while it resides within a defined subset of safety states from the moment the system enters the subset until it exits. This model includes both direct and indirect costs incurred specifically during the system's presence in the safety state subsets. Costs associated with other states that do not belong to the defined safety subset are not included in the analysis. The definitions of direct and indirect costs used in this model are analogous to those in Model 1.

The expected total operational cost of the system within safety states is defined by a vector:

$$\hat{\boldsymbol{C}}(t,\cdot) = \left[\hat{\boldsymbol{C}}(\geq 1), \hat{\boldsymbol{C}}(\geq 2), ..., \hat{\boldsymbol{C}}(z)\right], \tag{3}$$

with components that are expected values of the total system operating costs in safety states given by linear equations

$$\hat{\boldsymbol{C}}(\geq u) \cong \sum_{b=1}^{\nu} p_b [\hat{\boldsymbol{C}}(\geq u)]^{(b)}, ,\ \text{u} = 1,2,...,\text{z},\ \text{b} = 1,2,\ldots,\nu. \tag{4}$$

Where, analogously to the previous model, p_b, $b = 1,2,\ldots,\nu$, are limit transient probabilities at operation states.

The values

$$\left[\hat{\boldsymbol{C}}(\geq u)\right]^{(b)} = \int_0^{[\boldsymbol{\mu}(\geq u)]^{(b)}} \left[\boldsymbol{C}(t,u)\right]^{(b)} dt,\ \text{u} = 1,2,...,\text{z},$$

$$\text{b} = 1,2,...,\nu. \tag{5}$$

are the mean total operation costs of the system in safety states while it resides in individual operation states [27]–[30].

4 OPTIMIZATION APPROACH

For the purpose of cost optimization, linear programming has been proposed to minimize the total operational cost for fixed operation time, as well as to minimize the total operational cost in the safety state subsets not worse than the critical safety state based respectively on Cost Model 1 and Cost Model 2. Similar to the approach applied in safety optimization, the optimization procedures here make use of previously established results related to safety performance. This integration enables a consistent and unified framework for optimizing both economic and safety aspects of system operation. [28], [31]

The optimization of operation costs in a multi-state, aging technical system can be effectively addressed through linear programming, where the key decision variables are the limit values of transient probabilities of the system residing in particular operational states. [28], [31].

4.1 *Step 1: Definition of the Cost Function*

The total unconditional operation cost of the system over a fixed operation time is modeled as a weighted sum of conditional operation costs associated with each operational state. The weights are the values of limit transient probabilities p_b, $b = 1,2,...,\nu$, of the system operation process at the operation states, and the costs $\left[\hat{C}(\theta)\right]^{(b)}, \theta > 0,$ are considered to be fixed and known.

4.2 *Step 2: Formulation of the Linear Programming Problem*

The goal is to minimize the total unconditional operation costs by selecting an optimal probability distribution under the following constraints:

- The probabilities must sum to 1 (normalization condition),
- Each probability p_b must remain within a defined lower and upper bound $[\hat{p}_b, \check{p}_b]$,
- All costs $\left[\hat{C}(\theta)\right]^{(b)}$ must be non-negative.

This problem leads to a standard linear programming formulation where the objective function is linear in the probabilities.

4.3 *Step 3: Variable Transformation for Computational Efficiency*

To streamline the computation, the conditional cost values are first sorted in non-decreasing order, and the corresponding probabilities are relabeled accordingly. That is, new variables x_i are introduced to represent the reordered p_{b_i}, along with corresponding bounds $\breve{x}_i$ and $\hat{x}_i$.

4.4 *Step 4: Allocation Strategy*

To solve the problem, an efficient allocation strategy is used based on the structure of the linear objective function:

1. Calculate the remaining probability mass $\breve{x} = \sum_{i=1}^{v} \breve{x}_i,\ \hat{y} = 1 - \breve{x}$ and $\breve{x}^0 = 0, \hat{x}^0 = 0$ and $\breve{x}^I = \sum_{i=1}^{I} \breve{x}_i,\ \hat{x}^I = \sum_{i=1}^{I} \hat{x}_i$ for $I = 1,2,\ldots,v$.
2. Identify the largest index III for which the accumulated width $\hat{x}^I - \breve{x}^I < \hat{y}$.
3. Depending on the value of III, the optimal solution is constructed as:
   - Case if $I = 0$, the optimal solution is $\dot{x}_1 = \hat{y} + \breve{x}_1$ and $\dot{x}_i = \hat{x}_i$ for i = 2,3,…,ν,
   - Case if $0 < I < v$, the optimal solution is $\dot{x}_i = \hat{x}_i$ for $i=1,2,\ldots,I$, $\dot{x}_{I+1} = \hat{y} - \hat{x}^I + \breve{x}^I + \breve{x}_{I+1}$ and $\dot{x}_i = \breve{x}_i$ for $i=I+2,I+3,\ldots,v$;
   - Case $I = v$: assign all probabilities to their upper bounds.

4.5 *Step 5: Back-Transformation and Interpretation*

After determining the optimal values x_i, they are transformed back to the original probability variables p_b according to the earlier index mapping. This yields the optimal steady-state distribution that minimizes the expected total operation cost.

A similar procedure is applied when optimizing the operation cost within selected safety state subsets. In this case, the objective is to minimize the expected cost incurred only while the system operates within a subset of acceptable safety states, typically starting from a critical safety threshold r. The same steps, formulating the linear cost function, defining bounds, sorting, transforming variables, allocating probability mass, and performing inverse substitution are repeated, but now with the cost function and constraints restricted to the safety state subsets. [28], [30]-[31]

5 JOINT OPTIMIZATION OF OPERATION COST AND SAFETY IN MULTI-STATE AGING TECHNICAL SYSTEMS

The joint optimization of system operation cost and safety in complex, multi-state, aging technical systems involves an integrated approach that leverages the relationship between the system's probabilistic behaviour and its economic and safety performance metrics [28], [32]-[35]. This approach is centered on the determination of optimal limit transient probabilities of the system's operation process in its various states, which serve as the decision variables linking both cost and safety analyses. When the goal is to determine the operation cost corresponding to the system's maximum safety, the first step is to apply a general safety model along with a dedicated safety optimization procedure. This enables the identification of optimal limit transient probabilities that maximize the system's safety indicators such as extended residence time in safe states, minimal exposure to risk, or highest resilience. Once these optimal probabilities are obtained, they are substituted into the previously developed cost models. This substitution yields the expected total operational cost over a fixed operation period or within a predefined subset of safety states not worse than a critical level. The result is a quantification of the system's operational costs under the assumption of its best achievable safety configuration. This methodology allows for a consistent transition from safety optimization to cost evaluation, using a shared probabilistic foundation. Conversely, when the objective is to evaluate the safety characteristics corresponding to the system's minimal operation cost, the optimization process begins with the application of the operation cost model. Through linear programming techniques, the limit transient probabilities that minimize the system's total cost either over a specified time interval or within a critical safety region are determined. These cost optimal probabilities are then inserted into the safety function formulations, enabling the calculation of the conditional safety indicators associated with the lowest cost configuration. This process makes it possible to assess how minimizing costs affects the system's risk exposure, operational security, and long term reliability. In both approaches, the key lies in the use of limit transient probabilities as a common decision vector. Whether derived through safety or cost optimization, these probabilities can be propagated through the complementary domain's model to obtain the full profile of system behaviour. This bidirectional strategy creates a unified decision support framework that quantifies the trade-offs between safety and cost. It enables operators to answer critical questions, such as how much additional cost must be incurred to achieve a desired safety level, or what safety risks are introduced by stringent cost minimization. Ultimately, the joint optimization procedure contributes to the effective and rational management of aging, multi-state systems, particularly in safety-critical sectors. It offers a foundation for designing operational strategies that simultaneously satisfy economic constraints and maintain acceptable levels of safety, making it a valuable tool in infrastructure management, transport operations, and industrial system engineering.

6 CASE STUDY: OPERATIONAL COST AND SAFETY MODELING OF A MULTI-STATE FERRY SYSTEM

6.1 *System description*

In this case study, we examine the operation and cost structure of a passenger ferry operating daily on the Gdynia–Karlskrona route across the Baltic Sea. The ferry constitutes a complex technical system composed of several interdependent subsystems that significantly influence its operational safety and performance.

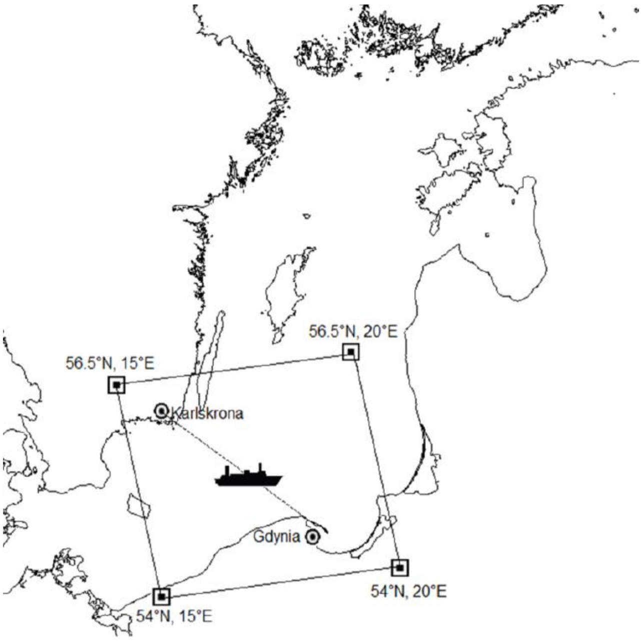

Figure 1. Ferry connection between Gdynia and Karlskrona

The ferry's technical infrastructure is modeled as a system of interconnected subsystems: navigation (S_1), propulsion and steering (S_2), loading and unloading (S_3), stability control (S_4), anchoring and mooring (S_5), as well as protection and rescue (S_6) and social services (S_7). For the purposes of this analysis, only the core technical subsystems S_1– S_5 are considered, collectively referred to as the technical system of the ferry. Each subsystem is hierarchically decomposed into lower level components (e.g., main engines, steering thrusters, gangways), and the overall safety structure is modeled as a series system in which the failure of any active subsystem implies a degradation of the system's operational safety. The ferry's operation is represented by a stochastic process consisting of 18 defined operational states which reflect the complete voyage cycle, from docking, loading, and departure in Gdynia, through navigation in various maritime zones, to arrival and unloading in Karlskrona, and vice versa. The process follows a sequential and cyclic structure and is characterized by limit transient probabilities, empirically derived from real voyage data owing to the high frequency and regularity of the ferry's operation [28], [30], [32], [34]-[35].

To assess the operational behavior of the ferry system, two primary sets of characteristics are used:

The limit transient probabilities of being in each operational state (e.g., p_5=0.363, p_{13}=0.351), which indicate that the ferry spends the majority of its time navigating in open waters under Polish and Swedish jurisdiction.

The approximate mean values of total sojourn times of the ferry in each state during a fixed time period (θ = 30 days = 720 hours), derived from M_b=$p_b \cdot \theta$, which allows for time based allocation of operational costs.

In the cost analysis, each subsystem is assigned a set of state-dependent hourly cost rates, differentiated by active usage versus standby conditions. For instance, the navigation subsystem S_1 incurs a constant operational cost of 20c when active and 10 *c* otherwise. Similarly, the propulsion system S_2 has differentiated costs depending on whether it operates in restricted or open waters (75*c* or 55*c*, respectively), while its standby cost is set at 25*c*. *c* is a coefficient, adopted arbitrarily, used to determine the costs of subsystems. Other subsystems follow similar differentiated costing schemes.

6.2 *Results for Model 1*

A comparison of the ferry's operation cost results based on Model 1 representing the total operation cost for fixed operation time is presented in Table 1., [28], [32]-[35].

Table 1. Comparison of the ferry system's total operation cost before and after optimization based on Model 1

Model 1 – cost for fixed operation time	Before optimization	After optimization	Joint optimization
	19 490.69c 100%	17 056.54c 87.5%	19 308.006c 99%

The optimization procedure led to a reduction in total operational costs of approximately 12.5%. Furthermore, the conditional total operation cost of the ferry, calculated under the assumption of maximum system safety, is found to be lower than the before optimization cost associated with the unoptimized safety profile, yet higher than the minimum cost obtained through direct cost optimization. This outcome implies that if the primary objective is to maintain a high level of operational safety rather than to minimize operational expenses, the system's operation process can be modified accordingly. In particular, the limit transient probabilities within the cost model can be replaced by their optimal values derived from the safety optimization procedure. This allows for the adjustment of the expected residence times of the system in particular states, supporting safety-oriented operational strategies while keeping costs within acceptable bounds.

A comparison of the ferry's safety performance results is presented in Table 2., [28], [32]-[35].

Table 2. Safety indicator values

Model 1 – safety indicator values		Before optimization	After optimization	Joint optimization
The expected values in the safety state subsets	{1, 2, 3, 4}	1.694	1.697	1.6923
	{2, 3, 4}	1.395	1.397	1.39364
	{3, 4}	1.244	1.247	1.243
	{ 4}	1.114	1.115	1.11318
The moment, when system risk function exceeds a permitted level		0.073	0.073	0.04634

The mean intensities of aging	{1, 2, 3, 4}	0.590363	0.589275	0.59091
	{2, 3, 4}	0.716869	0.715819	0.71755
	{3, 4}	0.803573	0.801925	0.8045
	{ 4}	0.897470	0.869565	0.89833
The coefficients of the operation process impact intensities of ageing	{1, 2, 3, 4}	1.044942	1.043	1.04591
	{2, 3, 4}	1.058098	1.057	1.0591
	{3, 4}	1.044645	1.043	1.04585
	{ 4}	1.044655	1.043	1.04566
The coefficient of resilience to the operation process impact	{1, 2, 3, 4}	95.70 %	95.88 %	95.61 %
	{2, 3, 4}	94.51 %	94.61 %	94.42 %
	{3, 4}	95.73 %	95.88 %	95.62 %
	{ 4}	95.73 %	95.88 %	95.63 %

The expected values (expressed in years) in the safety state subsets ({1,2,3,4}, {2,3,4}, {3,4}, {4}) show minor increases after safety optimization, indicating a slightly enhanced presence of the system in safer operational states. However, in the joint optimization scenario, these expected values are marginally reduced compared to post-safety optimization results. For example, in the critical safety state {4}, the expected value decreases from 1.115 (after safety optimization) to 1.11318 in joint optimization. This suggests that while joint optimization introduces minimal trade-offs in terms of safety positioning, the degradation is negligible and may be acceptable when cost reduction is also a priority. A significant improvement is observed in the moment when the system risk function exceeds a permitted level. While this threshold remains constant at 0.073 year before and after safety optimization, joint optimization leads to a noticeable delay in risk emergence, reducing the threshold to 0.04634. This outcome indicates that the combined optimization of cost and safety results in a more robust operational profile, potentially deferring critical risk exposure.

The mean intensities of aging generally decline after safety optimization, reflecting a reduction in the rate of system degradation. For instance, in safety state {4}, the aging intensity decreases from 0.897470 to 0.869565. However, under joint optimization, these intensities slightly increase again (e.g., to 0.89833 for {4}), reflecting the partial reallocation of optimization focus toward cost efficiency. This pattern illustrates the delicate balance between maintaining system longevity and achieving economic objectives. The coefficients of the operation process impact on aging intensities mirror this behavior. Post safety optimization results show a marginal reduction in values across all state subsets, while joint optimization slightly increases them, exceeding even the pre optimization baseline in some cases (e.g., {2,3,4}: from 1.058098 to 1.0591). Although the increases are modest, they suggest that the cost efficiency gains of joint optimization are accompanied by slightly intensified aging processes. Despite this, the coefficient of resilience to the operation process impact, a key safety metric, remains relatively stable and high across all conditions. While resilience improves after safety optimization (e.g., from 95.70% to 95.88% in {1,2,3,4}), it only marginally decreases under joint optimization (to 95.61%). This indicates that even with increased operational efficiency, the system retains a strong capacity to withstand the effects of aging under the influence of operational conditions.

6.3 *Results for Model 2*

A comparison of the ferry's operation cost results based on Model 2 representing the total operation cost in the safety state subsets is presented in Table 3., [28], [32]-[35].

Table 3. Comparison of the ferry system's total operation cost before and after optimization based on Model 2

Model 2 – cost in safety state subsets		Before optimization	After optimization	Joint optimization
	{1, 2, 3, 4}	175.15054c	173.034c	173.034c
	{2, 3, 4}	144.13643c	142.433c	142.433c
	{3, 4}	128.71551c	127.060c	127.060c
	{ 4}	115.24016c	113.795c	113.795c

In each of the analyzed safety state subsets ({1,2,3,4}, {2,3,4}, {3,4}, and {4}), a reduction in the total operation cost is observed after the optimization process. This confirms the effectiveness of the proposed optimization approach in reducing system operation costs while maintaining a predefined safety level. The operation costs after joint optimization, i.e., the integrated approach to simultaneously optimizing safety and cost, are equal to the costs obtained through safety-focused optimization alone. This suggests that in this case, the safety constraints were dominant in the optimization process, meaning the minimization of costs did not compromise the safety levels within the considered state subsets. A compromise was achieved without deterioration in safety performance. The largest absolute cost reduction in the safety state subset ({1,2,3,4}), where the cost decreased from 175.15054c to 173.034c. Although the relative change is modest (approximately 1.2%), it demonstrates that even within restricted operational modifications, meaningful cost savings can be realized.

A comparison of the ferry's safety performance results is presented in Table 4., [28], [32]-[35].

Table 4. Safety indicator values

Model 2 – safety indicator values		Before optimization	After optimization	Joint optimization
The expected values in the safety state subsets	{1, 2, 3, 4}	1.694	1.697	1.69629
	{2, 3, 4}	1.395	1.397	1.39654
	{3, 4}	1.244	1.247	1.24527
	{ 4}	1.114	1.115	1.11535
The moment, when system risk function exceeds a permitted level		0.073	0.073	0.06467
The mean intensities of aging	{1, 2, 3, 4}	0.590363	0.589275	0.58952
	{2, 3, 4}	0.716869	0.715819	0.71605
	{3, 4}	0.803573	0.801925	0.80304
	{ 4}	0.897470	0.869565	0.89658
The coefficients of the operation process impact intensities of ageing	{1, 2, 3, 4}	1.044942	1.043	1.04345
	{2, 3, 4}	1.058098	1.057	1.05689
	{3, 4}	1.044645	1.043	1.04358
	{ 4}	1.044655	1.043	1.04362

The coefficient of resilience to the operation process impact	{1, 2, 3, 4}	95.70 %	95.88 %	95.83 %
	{2, 3, 4}	94.51 %	94.61 %	94.61 %
	{3, 4}	95.73 %	95.88 %	95.82 %
	{ 4}	95.73 %	95.88 %	95.82 %

The expected values in safety state subsets show marginal improvements after both safety targeted and joint optimization. For each considered subset ({1,2,3,4}, {2,3,4}, {3,4}, and {4}), a slight increase in the expected value is observed, indicating a higher likelihood of the system remaining in safer states for longer periods. The joint optimization results are nearly identical to those achieved after safety specific optimization, suggesting a negligible compromise in safety due to the inclusion of cost factors. The moment when the system risk function exceeds the permitted level remains constant at 0.073 before and after safety only optimization but decreases to 0.06467 under joint optimization. This indicates that joint optimization not only preserves safety performance but slightly improves system resilience by delaying the onset of critical risk thresholds. The mean intensities of aging decrease slightly after safety optimization and are maintained at nearly the same levels under joint optimization. For example, in the critical state subset {4}, the intensity drops from 0.897470 before optimization to 0.869565, and then marginally increases to 0.89658 in joint optimization. These values reflect reduced degradation rates of the system components during optimized operation, which implies improved long term reliability. The coefficients of the operation process impact on intensities of aging are consistently reduced after optimization across all state subsets, again with negligible differences between safety only and joint optimization results. This suggests that the optimization process effectively minimizes the operational load contributing to system wear. The coefficients of resilience to the operation process impact improve slightly after safety optimization and are preserved under joint optimization. The values range from 94.51% to 95.88%, indicating a high level of system robustness. For instance, in the {1,2,3,4} subset, resilience increases from 95.70% to 95.88%, and is maintained at 95.83% in joint optimization.

7 CONCLUSIONS

The article presents original models and optimization procedures for assessing and improving both the operation costs and safety levels of complex, multi-state, ageing technical systems. These models were applied to the technical system of a Baltic Sea ferry operating between Gdynia and Karlskrona. Cost and safety evaluations were based on detailed operational data and expert derived estimations of parameters related to safety and subsystem operation costs. The only significant limitation in practical application of the models is the availability of sufficiently accurate statistical data from system users. The study shows that optimized ferry operation reduces costs by over 12%, while maintaining satisfactory safety levels.

Future research could explore the influence of non-operational factors, such as climatic or weather conditions [36], on system reliability and cost. Additionally, expanding the optimization framework to include repair strategies [37], preventive actions [22], and system corrections could enable more holistic infrastructure management. The use of multi criteria optimization methods and evolutionary algorithms is also recommended [38], especially for managing ageing systems with irreversible physical and chemical degradation [39].

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