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On Ship Systems Multi-state Safety Analysis

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ABSTRACT A multi-state approach to defining basic notions of the system safety analysis is proposed. A system safety function and a system risk function are defined. A basic safety structure of a multi-state series system of components with degrading safety states is defined. For this system the multi-state safety function is determined. The proposed approach is applied to the evaluation of a safety function, a risk function and other safety characteristics of a ship system composed of a number of subsystems having an essential influence on the ship safety. Further, a semi-markov process for the considered system operation modelling is applied. The paper also offers an approach to the solution of a practically important problem of linking the multi-state system safety model and its operation process model. Finally, the proposed approach is applied to the preliminary evaluation of safety characteristics of a ship system in varying operation conditions.

1 INTRODUCTION

Taking into account the importance of the safety and operating process effectiveness of technical systems it seems reasonable to expand the two-state approach to multi-state approach in their safety analysis (Dziula, Jurdzinski, Kolowrocki & Soszynska 2007). The assumption that the systems are composed of multi-state components with safety states degrading in time gives the possibility for more precise analysis and diagnosis of their safety and operational processes' effectiveness. This assumption allows us to distinguish a system safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system safety characteristic is the time to the moment of exceeding the system safety critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state safety function that is a basic characteristic of the multi-state system. Determining the multi-state safety function and the risk function of systems on the base of their components' safety functions is then the main research problem. Modelling of complicated systems operations' processes is difficult mainly because of large number of operations states and impossibility of precise describing of changes between these states. One of the useful approaches in modelling of these complicated processes is applying the semi-markov model (Grabski 2002). Modelling of multi-state systems' safety and linking it with semi-markov model of these systems' operation processes is the main and practically important research problem of this paper. The paper is devoted to this research problem with reference to basic safety structures of technical systems (Soszynska 2005, 2006) and particularly to safety analysis of a ship series system (Jurdzinski, Kolowrocki & Dziula 2006) in variable operation conditions. This new approach to system safety investigation is based on the multi-state system reliability analysis considered for instance in (Aven 1985, Hudson & Kapur 2985, Kolowrocki 2004, Lisnianski & Levitin 2003, Meng 1993, Xue & Yang 1995) and on semi-markov processes modelling discussed for instance in (Grabski 2002).

2 BASIC NOTIONS

In the multi-state safety analysis to define systems with degrading components we assume that:

- n is the number of system's components,
- E_i, $i = 1,2,...,n$, are components of a system,
- all components and a system under consideration have the safety state set $\{0,1,...,z\}$, $z \geq 1$,
- the safety state indexes are ordered, the state 0 is the worst and the state z is the best,
- $T_i(u)$, $i = 1,2,...,n$, are independent random variables representing the lifetimes of components E_i in the safety state subset $\{u,u+1,...,z\}$, while they were in the state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the safety state subset $\{u,u+1,...,z\}$ while it was in the state z at the moment $t = 0$,
- the system and its components safety states degrade with time t,
- $E_i(t)$ is a component E_i safety state at the moment t, $t \in <0,\infty)$.
- $S(t)$ is a system safety state at the moment t, $t \in <0,\infty)$.

The above assumptions mean that the safety states of the system with degrading components may be changed in time only from better to worse. The way in which the components and the system safety states change is illustrated in Figure 1.

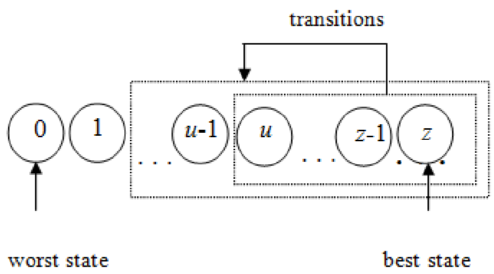

Fig. 1. Illustration of a system and components safety states changing

The basis of our further considerations is a system component safety function defined as follows.

Definition 1. A vector

$$s_i(t,\cdot) = [s_i(t,0),\ s_i(t,1),...,\ s_i(t,z)],\ t \in <0,\infty),$$

$$i = 1,2,...,n,$$

where

$$s_i(t,u) = P(E_i(t) \geq u \mid E_i(0) = z) = P(T_i(u) > t)$$

for $t \in <0,\infty)$, $u = 0,1,...,z$, $i = 1,2,...,n$, is the probability that the component E_i is in the state subset $\{u,u+1,...,z\}$ at the moment t, $t \in <0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state safety function of a component E_i.

Similarly, we can define a system multi-state safety function.

Definition 2. A vector

$$\boldsymbol{s}_n(t,\cdot) = [\boldsymbol{s}_n(t,0),\ \boldsymbol{s}_n(t,1),...,\ \boldsymbol{s}_n(t,z)],\ t \in <0,\infty),$$

where

$$\boldsymbol{s}_n(t,u) = P(S(t) \geq u \mid S(0) = z) = P(T(u) > t) \tag{1}$$

for $t \in <0,\infty)$, $u = 0,1,...,z$, is the probability that the system is in the state subset $\{u,u+1,...,z\}$ at the moment t, $t \in <0,\infty)$, while it was in the state z at the moment $t = 0$, is called the multi-state safety function of a system.

Under this definition we have

$$\boldsymbol{s}_n(t,0) \geq \boldsymbol{s}_n(t,1) \geq \ldots \geq \boldsymbol{s}_n(t,z),\ t \in <0,\infty).$$

Further, if we introduce the vector of probabilities

$$p(t,\cdot) = [p(t,0),\ p(t,1),...,\ p(t,z)],\ t \in <0,\infty),$$

where

$$p(t,u) = P(S(t) = u \mid S(0) = z)$$

for $t \in <0,\infty), u = 0,1,...,z$, is the probability that the system is in the state u at the moment t, $t \in <0,\infty)$, while it was in the state z at the moment $t = 0$, then

$$\boldsymbol{s}_n(t,0) = 1,\ \boldsymbol{s}_n(t,z) = p(t,z),\ t \in <0,\infty), \tag{2}$$

and

$$p(t,u) = \boldsymbol{s}_n(t,u) - \boldsymbol{s}_n(t,u+1),\ \ u = 0,1,...,z-1, \tag{3}$$

$$t \in <0,\infty).$$

Moreover, if

$$\boldsymbol{s}_n(t,u) = 1 \text{ for } t \leq 0,\ u = 1,2,...,z,$$

then

$$m(u) = \int_0^\infty \boldsymbol{s}_n(t,u)dt,\ u = 1,2,...,z, \tag{4}$$

is the mean value of the system lifetime in the safety state subset $\{u,u+1,...,z\}$, while

$$\sigma(u) = \sqrt{n(u) - [m(u)]^2},\ u = 1,2,...,z, \tag{5}$$

where

$$n(u) = 2\int_0^{\infty} t\boldsymbol{s}_n(t,u)dt,\ u = 1,2,...,z, \quad (6)$$

is the standard deviation of the system lifetime in the state subset $\{u, u+1,..., z\}$ and moreover

$$\overline{m}(u) = \int_0^{\infty} p(t,u)dt,\ u = 1,2,...,z, \quad (7)$$

is the mean value of the system lifetime in the state u upon that the integrals (4)-(5) and (6) are convergent.

Additionally, according to (2)-(4) and (7), we get the following relationship

$$\overline{m}(u) = m(u) - m(u+1),\ u = 0,1,...,z-1,$$

$$\overline{m}(z) = m(z).$$

Close to the multi-state system safety function, its basic characteristic is the system risk function defined as follows.

Definition 3. A probability

$$\boldsymbol{r}(t) = P(S(t) < r \mid S(0) = z) = P(T(r) \leq t),\ t \in <0,\infty),$$

that the system is in the subset of states worse than the critical state r, $r \in \{1,...,z\}$ while it was in the state z at the moment $t = 0$ is called a risk function of the multi-state system.

Under this definition, from (1), we have

$$\boldsymbol{r}(t) = 1 - P(S(t) \geq r \mid S(0) = z) = 1 - \boldsymbol{s}_n(t,r), \quad (8)$$

$$t \in <0,\infty),$$

and, if τ is the moment when the risk exceeds a permitted level δ, $\delta \in <0,1>$, then

$$\tau = \boldsymbol{r}^{-1}(\delta),$$

where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}(t)$ given by (8).

3 BASIC SYSTEM SAFETY STRUCTURES

The proposition of a multi-state approach to definition of basic notions, analysis and diagnosing of systems' safety allows us to define the system safety function and the system risk function. It also allows us to define basic structures of the multi-state systems of components with degrading safety states. For these basic systems it is possible to determine their safety functions. Further, as an example, we will consider a series system. Other safety structures can be defined and analysed similarly.

Definition 4. A multi-state system is called a series system if it is in the safety state subset $\{u, u+1,..., z\}$ if and only if all its components are in this subset of safety states.

Corollary 1. The lifetime $T(u)$ of a multi-state series system in the state subset $\{u, u+1,..., z\}$ is given by

$$T(u) = \min_{1 \leq i \leq n}\{T_i(u)\},\ u = 1,2,...,z.$$

The scheme of a series system is given in Figure 2.

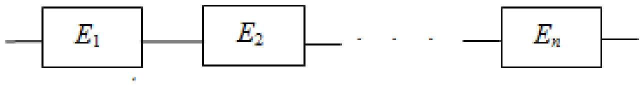

Fig. 2. The scheme of a series system

It is easy to work out the following result.

Corollary 2. The safety function of the multi-state series system is given by

$$\overline{\boldsymbol{s}}_n(t,\cdot) = [1,\ \overline{\boldsymbol{s}}_n(t,1),...,\ \overline{\boldsymbol{s}}_n(t,z)],\ t \in <0,\infty),$$

where

$$\overline{\boldsymbol{s}}_n(t,u) = \prod_{i=1}^{n} s_i(t,u),\ t \in <0,\infty),\ u = 1,2,...,z.$$

From Corollary 2, we immediately get the following result.

Corollary 3. If components of the multi-state series system have exponential safety functions, i.e., if

$$s_i(t,\cdot) = [1, s_i(t,1),..., s_i(t,z)],\ t \in <0,\infty),$$

where

$$s_i(t,u) = \exp[-\lambda_i(u)t] \text{ for } t \in <0,\infty),\ \lambda_i(u) > 0,$$

$$u = 1,2,...,z,\ i = 1,2,...,n,$$

then its safety function is given by

$$\overline{\boldsymbol{s}}_n(t,\cdot) = [1,\ \overline{\boldsymbol{s}}_n(t,1),...,\ \overline{\boldsymbol{s}}_n(t,z)],$$

where

$$\overline{\boldsymbol{s}}_n(t,u) = \exp[-\sum_{i=1}^{n}\lambda_i(u)t] \text{ for } t \in <0,\infty),$$

$$u = 1,2,...,z.$$

4 SHIP SAFETY IN CONSTANT OPERATION CONDITIONS

We preliminarily assume that the ship is composed of a number of main subsystems having an essential influence on its safety. These subsystems are illustrated in Figure 3.

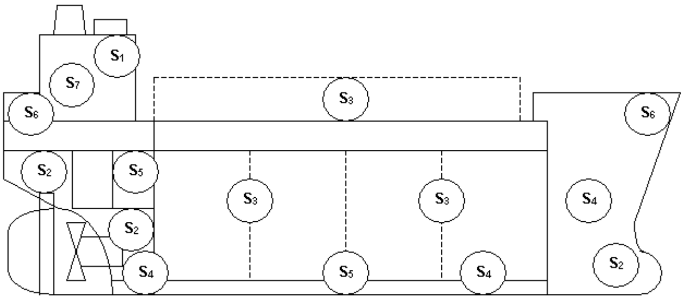

Fig. 3. Subsystems having an essential influence on ship's safety

On the scheme of the ship presented in Figure 3, there are distinguished her following subsystems:

S_1 – a navigational subsystem,

S_2 – a propulsion and controlling subsystem,

S_3 – a loading and unloading subsystem,

S_4 – a hull subsystem,

S_5 – a protection and rescue subsystem,

S_6 – an anchoring and mooring subsystem,

S_7 – a social subsystem.

In our further ship safety analysis we will omit the social subsystem S_7 and we will consider its technical subsystems S_1, S_2, S_3, S_4, S_5 and S_6 only.

According to *Definition 1*, we mark the safety functions of these subsystems respectively by vectors

$$s_i(t\,,\cdot\,) = [s_i(t,0),\, s_i(t,1),\dots,\, s_i(t,z)],\ t \in <0,\infty),$$

$$i = 1,2,\dots,6,$$

with co-ordinates

$$s_i(t,u) = P(S_i(t) \geq u \mid S_i(0) = z) = P(T_i(u) > t)$$

for $t \in <0,\infty)$, $u = 0,1,\dots,z$, $i = 1,2,\dots,6$, where $T_i(u)$, $i = 1,2,\dots,6$, are independent random variables representing the lifetimes of subsystems S_i in the safety state subset $\{u,u+1,\dots,z\}$, while they were in the state z at the moment $t = 0$ and $S_i(t)$ is a subsystem S_i safety state at the moment t, $t \in <0,\infty)$.

Further, assuming that the ship is in the safety state subset $\{u,u+1,\dots,z\}$ if and only if all its subsystems are in this subset of safety states and considering *Definition 4*, we conclude that the ship is a series system of subsystems S_1, S_2, S_3, S_4, S_5, S_6 with a scheme presented in Figure 4.

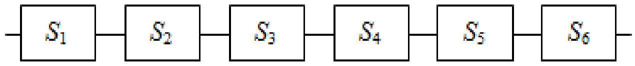

Fig. 4. The scheme of a structure of ship subsystems

Therefore, the ship safety is defined by the vector

$$\bar{\mathbf{s}}_6(t,\cdot) = [\,\bar{\mathbf{s}}_6(t,0),\ \bar{\mathbf{s}}_6(t,1),\dots,\ \bar{\mathbf{s}}_6(t,z)\,],\ t \in <0,\infty),$$

with co-ordinates

$$\bar{\mathbf{s}}_6(t,u) = \mathrm{P(S(t) \geq u \mid S(0) = z) = P(T(u) > t)}$$

for $t \in <0,\infty)$, $u = 0,1,\dots,z$, where $T(u)$ is a random variable representing the lifetime of the ship in the safety state subset $\{u,u+1,\dots,z\}$ while it was in the state z at the moment $t = 0$ and $S(t)$ is the ship safety state at the moment t, $t \in <0,\infty)$, and according to *Corollary 2*, is given by the formula

$$\bar{\mathbf{s}}_6(t,\cdot) = [1,\ \bar{\mathbf{s}}_6(t,1),\dots,\ \bar{\mathbf{s}}_6(t,z)\,],\ t \in <0,\infty), \tag{9}$$

where

$$\bar{\mathbf{s}}_6(t,u) = \prod_{i=1}^{6} s_i(t,u)\,,\, t \in <0,\infty),\ u = 1,2,\dots,z. \tag{10}$$

Applying (9)-(10), we can find

– the mean value of the system lifetime in the safety state subset $\{u, u+1,\dots, z\}$,

$$m(u) = \int_0^\infty \bar{\mathbf{s}}_6(t,u)dt,\ u = 1,2,\dots,z,$$

– the standard deviation of the system lifetime in the state subset $\{u, u+1,\dots, z\}$

$$\sigma(u) = \sqrt{n(u) - [m(u)]^2}\,,\ u = 1,2,\dots,z,$$

where

$$n(u) = 2\int_0^\infty t\bar{\mathbf{s}}_6(t,u)dt,\ u = 1,2,\dots,z,$$

– the mean values of the ship lifetimes in the particular states

$$\overline{m}(u) = m(u) - m(u+1),\ u = 0,1,\dots, z-1,$$

$$\overline{m}(z) = m(z).$$

Moreover, if the safety critical state is r, $r \in \{1,\dots,z\}$, then the ship risk function is given by

$$\boldsymbol{r}(t) = 1 - P(S(t) \ge r \mid S(0) = z)$$

$$= 1 - \bar{\boldsymbol{s}}_6(t,r),\ t \in <0,\infty), \tag{11}$$

and, if τ is the moment when the risk exceeds a permitted level δ, $\delta \in <0,1>$, then

$$\tau = \boldsymbol{r}^{-1}(\delta),$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of $\boldsymbol{r}(t)$ given by (11).

5 SHIP OPERATION PROCESS

Technical subsystems S_1, S_2, S_3, S_4, S_5, S_6 indicated in Figure 3 are forming a general ship safety structure presented in Figure 4. However, the ship safety structure and the ship subsystems safety depend on her changing in time operation states.

Considering basic sea transportation processes the following operation ship states have been specified:

z_1 – loading and unloading of cargo,

z_2 – route planning,

z_3 – leaving and entering the port,

z_4 – navigation at restricted water areas,

z_5 – navigation at open sea waters.

In this case the ship operation process $Z(t)$ may be described by (Dziula, Jurdzinski, Kolowrocki & Soszynska 2007):

- the vector of probabilities of the process initial operation states $[p_b(0)]_{1x5}$,
- the matrix of the probabilities of the process transitions between the operation states $[p_{bl}]_{5x5}$, where $p_{bb}(t) = 0$ for $b = 1,2,...,5$,
- the matrix of the conditional distribution functions $[H_{bl}(t)]_{5x5}$ of the lifetimes θ_{bl}, $b \ne l$, of the process lifetimes θ_{bl}, $b \ne l$, in the operation state z_b when the next operation state is z_l, where $H_{bl}(t) = P(\theta_{bl} < t)$ for $b, l = 1,2,...,5$, $b \ne l$, and $H_{bb}(t) = 0$ for $b = 1,2,...,5$.

Under these assumptions, the lifetimes θ_{bl} mean values are given by

$$M_{bl} = E[\theta_{bl}] = \int_0^\infty t dH_{bl}(t),\ b, l = 1,2,...,5,\ b \ne l. \tag{12}$$

The unconditional distribution functions of the lifetimes θ_b of the ship operation process $Z(t)$ at the operation states z_b, $b = 1,2,...,5$, are given by

$$H_b(t) = \sum_{l=1}^{5} p_{bl} H_{bl}(t),\ b = 1,2,...,5.$$

The mean values $E[\theta_b]$ of the unconditional lifetimes θ_b are given by

$$M_b = E[\theta_b] = \sum_{l=1}^{v} p_{bl} M_{bl},\ b = 1,2,...,5,$$

where M_{bl} are defined by (12).

Limit values of the transient probabilities at the operation states

$$p_b(t) = P(Z(t) = z_b),\ t \in <0,+\infty),\ b = 1,2,...,5,$$

are given by

$$p_b = \lim_{t \to \infty} p_b(t) = \frac{\pi_b M_b}{\sum_{l=1}^{5} \pi_l M_l},\ b = 1,2,...,5,$$

where the probabilities π_b of the vector $[\pi_b]_{1x5}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{5} \pi_l = 1. \end{cases}$$

6 SHIP SAFETY IN VARIABLE OPERATION CONDITIONS

We assume as earlier that that the ship is composed of $n = 6$ subsystems S_i, $i = 1,2,...,6$, and that the changes of the process $Z(t)$ of ship operation states have an influence on the ship subsystems safety and on the ship safety structure as well (Dziula, Jurdzinski, Kolowrocki & Soszynska 2007). Thus, we denote the conditional safety function of the ship subsystem S_i while the ship is at the operational state z_b, $b = 1,2,...,5$, by

$$s_i^{(b)}(t,\cdot) = [1,\ s_i^{(b)}(t,1),\ s_i^{(b)}(t,2),\ ...,\ s_i^{(b)}(t,z)], \tag{13}$$

where for $t \in <0,\infty)$, $b = 1,2,...,5$, $u = 1,2,...,z$,

$$s_i^{(b)}(t,u) = P(T_i^{(b)}(u) > t \mid Z(t) = z_b), \tag{14}$$

and the conditional safety function of the ship while the ship is at the operational state z_b, $b = 1,2,...,5$, by

$$\boldsymbol{s}_{n_b}^{(b)}(t,\cdot) = [1,\ \boldsymbol{s}_{n_b}^{(b)}(t,1),\ \boldsymbol{s}_{n_b}^{(b)}(t,2),\ ...,\ \boldsymbol{s}_{n_b}^{(b)}(t,z)], \quad (15)$$

where for $t \in <0,\infty)$, $b = 1,2,...,5$, $n_b \in \{1,2,3,4,5,6\}$, $u = 1,2,..., z$,

$$\boldsymbol{s}_{n_b}^{(b)}(t,u), = P(T^{(b)}(u) > t | Z(t) = z_b). \quad (16)$$

The co-ordinate $s_i^{(b)}(t,u)$ defined by (14) of the safety function (13) is the conditional probability that the subsystem S_i lifetime $T_i^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is not less than *t*, while the process *Z*(*t*) is at the ship operation state z_b. Similarly, the co-ordinate $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ defined by (16) of the safety function (15) is the conditional probability that the ship lifetime $T^{(b)}(u)$ in the state subset $\{u, u+1,..., z\}$ is not less than *t*, while the process *Z*(*t*) is at the ship operation state z_b.

In the case when the ship operation time is large enough, the unconditional reliability function of the system is given by

$$\bar{\boldsymbol{s}}_6(t,\cdot) = [1,\ \bar{\boldsymbol{s}}_6(t,1),\ \bar{\boldsymbol{s}}_6(t,2),\ ...,\ \bar{\boldsymbol{s}}_6(t,z)],\ t \geq 0,$$

where

$$\bar{\boldsymbol{s}}_6(t,u) = P(T(u) > t) \cong \sum_{b=1}^{5} p_b \boldsymbol{s}_{n_b}^{(b)}(t,u) \quad (17)$$

for $t \geq 0$, $n_b \in \{1,2,3,4,5,6\}$, $u = 1,2,..., z$, and $T(u)$ is the unconditional lifetime of the ship in the safety state subset $\{u, u+1,..., z\}$.

The mean values and variances of the ship lifetimes in the safety state subset $\{u, u+1,..., z\}$ are

$$m(u) = E[T(u)\} \cong \sum_{b=1}^{5} p_b m^{(b)}(u),\ u = 1,2,..., z, \quad (18)$$

where

$$m^{(b)}(u) = \int_0^\infty \boldsymbol{s}_{\boldsymbol{n_b}}^{(b)}(t,u)dt,$$

for $n_b \in \{1,2,3,4,5,6\}$, $u = 1,2,..., z$, and

$$D[T(u)] = 2\int_0^\infty t\bar{\boldsymbol{s}}_6(t,u)dt - [m(u)]^2,\ u = 1,2,..., z,$$

and $\boldsymbol{s}_{n_b}^{(b)}(t,u)$ is given by (16) and $\bar{\boldsymbol{s}}_6(t,u)$ is given by (17).

The mean values of the system lifetimes in the particular safety states u, are

$$\overline{m}(u) = m(u) - m(u+1),\ u = 1,2,..., z-1,$$

$$\overline{m}(z) = m(z),$$

where $m(u)$, $u = 1,2,..., z$, are given by (18).

Moreover, if the safety critical state is *r*, *r* $\in \{1,...,z\}$, then the ship risk function is given by

$$\boldsymbol{r}(t) = 1 - \bar{\boldsymbol{s}}_6(t,r),\ t \in <0,\infty), \quad (19)$$

where $\bar{\boldsymbol{s}}_6(t,r)$a is given by (17), and if τ is the moment when the risk exceeds a permitted level δ, $\delta \in <0,1>$, then

$$\tau = \boldsymbol{r}^{-1}(\delta),$$

where $\boldsymbol{r}^{-1}(t)$ is the inverse function of $\boldsymbol{r}(t)$ given by (19).

7 CONCLUSIONS

In the paper the multi-state approach to the analysis and evaluation of systems' safety has been considered. Theoretical definitions and preliminary results have been illustrated by the example of their application in the safety evaluation of a ship system. The ship safety structure used in the application is very general and simplified and the subsystems safety precise data are not know at the moment and therefore the results may only be considered as an illustration of the proposed methods possibilities of applications in ship safety analysis. However, the obtained evaluation may be a very useful example in simple and quick ship system safety characteristics evaluation, especially during the design and when planning and improving her operation processes safety and effectiveness.

The results presented in the paper can suggest that it seems reasonable to continue the investigations focusing on the methods of safety analysis for other more complex multi-state systems and the methods of safety evaluation related to the multi-state systems in variable operation processes (Soszynska 2005, 2006) and their more adequate applications to the ship transportation systems and processes (Dziula, Jurdzinski, Kolowrocki & Soszynska 2007).

REFERENCES

Aven T., 1985. Reliability evaluation of multi-state systems with multi-state components. *IEEE Transactions on Reliability* 34, 473-479.

Dziula P., Jurdzinski M., Kolowrocki K. & Soszynska J., 2007. On multi-state approach to ship systems safety analysis. *Proc. 12th International Congress of the International Maritime Association of the Mediterranean, IMAM 2007.*

A.A. Balkema Publishers: Leiden - London - New York - Philadelphia - Singapore.

Dziula P., Jurdzinski M., Kolowrocki K. & Soszynska J., 2007. Multi-state safety analysis of ships in variable operation conditions. *Proc. 12th International Congress of the International Maritime Association of the Mediterranean, IMAM 2007*. A.A. Balkema Publishers: Leiden - London - New York - Philadelphia - Singapore.

Grabski F., 2002. *Semi-Markov Models of Systems Reliability and Operations.* Warsaw: Systems Research Institute, Polish Academy of Science.

Hudson J. & Kapur K., 1985. Reliability bounds for multi-state systems with multi-state components. *Operations Research* 33, 735- 744.

Jurdzinski M., Kolowrocki K. & Dziula P. 2006. *Modelling maritime transportation systems and processes*. Report 335/DS/2006. Gdynia Maritime University.

Kolowrocki K. 2004. *Reliability of large Systems*. Elsevier: Amsterdam - Boston - Heidelberg - London - New York - Oxford - Paris - San Diego - San Francisco - Singapore - Sydney - Tokyo.

Lisnianski A. & Levitin G., 2003. *Multi-state System Reliability. Assessment, Optimisation and Applications.* World Scientific Publishing Co., New Jersey, London, Singapore , Hong Kong.

Meng F., 1993. Component - relevancy and characterisation in multi-state systems. *IEEE Transactions on reliability* 42, 478-483.

Soszynska J., 2005. Reliability of large series-parallel system in variable operation conditions. *Proc. European Safety and Reliability Conference, ESREL 2005*, 27-30 June, 2005, Tri City, Poland. *Advances in Safety and Reliability*, Edited by K. Kolowrocki, Volume 2, 1869-1876, A.A. Balkema Publishers: Leiden - London - New York - Philadelphia - Singapore.

Soszynska J., 2006. Reliability evaluation of a port oil transportation system in variable operation conditions. *International Journal of Pressure Vessels and Piping*, Vol. 83, Issue 4, 304-310.

Xue J. & Yang K., 1995. Dynamic reliability analysis of coherent multi-state systems. *IEEE Transactions on Reliability* 4, 44, 683-688.