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Nautical Knowledge Extraction and Decision Making

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ABSTRACT: One frequently encountered decision-making problem is the evaluation that boils down to judging hypotheses. Typically, we determine whether they are true or false, although we may also have doubts. Hypotheses can be statements of various kinds. For example, we may wish to classify a given student as belonging to the category of good students. Mentioned hypotheses are related to different disciplines, quite often seemingly uncorrelated. To confirm this hypothesis, we would most often refer to the subjective opinions of their teachers. A similar issue arises in nautical science; for instance, consider the problem of identifying a particular location as the most probable one where an observer is situated. Accompanied establishing ranges of the true, false and uncertain statement might be subjective. Objectivity could be also considered provided stored sets of instances are available. Expected are adequate functionalities of software tools at hand. Functional aspects tends to increase nowadays. Random observations are usually accompanied by methods rectifying knowledge regarding their behaviour and quality. Available data are explored in order to extract necessary parameters required within the inference schemes of evaluating the hypothesis truth.

1 INTRODUCTION

One can examine statements like; “student X is a good one” or “point (x, y) is a true location of the ship”. Evaluating the first, we should consult the student’s teacher for his subjective opinion. Belief, uncertainty and plausibility measures are expected as an educator answer. Second hypothesis is of the same type but appears not so easy. Although one can ask nautical experts for their help, nonetheless this way of conduct seems inadequate. Primarily one has to point at reference data. Statement changes a bit and takes the form “is the point (x, y) a true location of the ship given a set of indications delivered by various accompanied navigational observations”. Two-dimensional problem can be solved considering each coordinate separately.

In navigation problem of assessing the belief, uncertainty and presumption of the x coordinate as the true location in proximity of a given abscissa. The assessment here is not subjective. The opinion should be based on the indications of a certain positioning system. The necessary knowledge available to an experienced bridge officer is usually based on available samples of observed, random indications of such a system – see Figure 1 for illustration. It should be added that the interesting location here is approximately the point, not exactly in it. In the latter case, having density of distributions, inference in relation to a specific point is not possible.

Proposed way of verifying the statement exploits collected sets of instances generated by the navigational aids. Uses fuzzy systems and methods of multi-attribute decision-making. Contribution from belief theory results in ability of the data informative context enrichment. The proposed approach refers to belief structures and the mechanism of evidence aggregation. The association of encoded fragments of evidence is a little-known and often undervalued mechanism. It allows for the enrichment of the informational context of the components. It provides a formalized framework through which we can obtain an answer regarding the cumulative assessment of a given hypothesis in light of opinions originating from various sources, often differing in reliability.

2 UNCERTAINTY MODEL

Binary logic rules for the two-state distinction of values [true, false]. Fuzzy systems of diversification switches in understanding these values. The statement can be true/false to some extent. A widely possible interval model for introducing belief, plausibility and uncertainty of the hypotheses being true. Suppose we ask a teacher for an opinion about a certain student. Only in a few cases will we hear an opinion that clearly states a high grade for a student. The evaluator will more likely state that based on the exams conducted, the student is good, but while direct contacts and meetings during practical classes, he believes that, the opinion expressed is not entirely true. The teacher has doubts reaching a certain value that allows him to assess the student as good. Above this level, he would not say that the student belongs to this category. In the example given, we have three ranges of values: conviction or belief, uncertainty and presumption or plausibility, above which we have impossibility. The state where the truth of the statement is not allowed. In practice, interval notation [a, b] is used. The value of a denotes the level of conviction, 1-b is the impossibility interval, and b-a is the range of uncertainty. Determining the appropriate interval involves objectively assessed exams, as well as a subjective evaluation of the entirety of achievements. From an axiological point of view, the most important thing is conviction. The range of uncertainty, although significant, is of lesser importance. The construction of a hierarchy in a set of elements should be based on the values of belief. This problem is of particular importance during the aggregation of assessments or opinions.

Figure 1 shows two sets of instances distributions, their conventional and modern histograms with examples of selected abscissas. Considering each coordinate of a location separately thus solving two single-dimensional problems is an approach enabling verification nautical hypothesis referring to the observer location. Moreover relying on density functions we should modify the hypothesis, finally it takes the form of: "is the neighbourhood of given abscissa a range where the true x-coordinate of the observer position is located given a set of indications delivered by various accompanied navigational observations".

It should be stressed that the area of interest is the neighbourhood of a point, not the point itself. In the latter case, if we rely on histograms representing distribution densities, reasoning about a specific point is not feasible.

Data for a certain navigation system are shown in Figure 1. The history of the system's readings is shown as a set of points located around the identified location. The distribution of the x-values of these locations are shown as histogram, although modern continuous density graph is also included. The figures included next to the drawing represent vertical and horizontal assessments of the distribution.

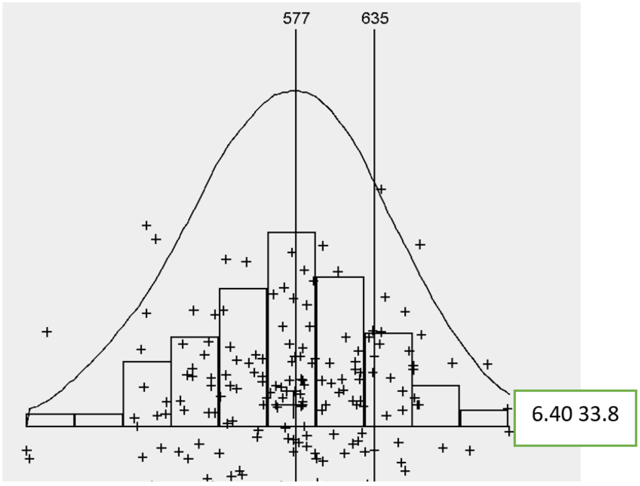

Figure 1. The set of instances distribution, its conventional and modern histogram with examples of selected abscissas and quality parameters

Included figures refer to assessment of the distribution. The first indicator reflects the variation in the heights of the histogram bars. A higher value is preferred, as it indicates a better-formed structure. The second value refers to the width of the histogram, which in the presented case is the width of a single bin. Systems with a smaller range of dispersion are favoured, as they indicate lower uncertainty in readings. The two indicators for evaluating a given system are of different types: the first is qualitative—higher values indicate better assessments. The second is cost-type—lower values are preferred.

MADM (Multi-Attribute Decision Making) is a field of knowledge where such contrasting cases are recognized. Unified methods for treating such values are proposed [9]. It is often that block of the characteristics of observations made at the same time is available. Thus upgrading hierarchy among its elements is of primary importance. Deferring further detail for now, it should be known what the given system indication quality is. At the same time, the relative belief in the system's reliability is assessed. It is at this point that the fundamental difference appears between the problem of evaluating a student and determining location based on readings from various systems. The second case involves geometric relationships. What is sought is belief and plausibility regarding the neighbourhood of a given x-coordinate as the area of the most likely observer location. These attributes result from the analysed reading but also depend on geometry, the relative position of the reading and the considered coordinate [1]. Belief is therefore variable. Figure 1 also contains two lines associated with two example x-coordinates. The proposed method for calculating plausibility refers to continuous functions. Since histograms do not satisfy this condition, they require transformation—a concept previously proposed by the author [2][3].

For a system, the maxima of belief and plausibility regarding the location of the coordinate around which the observer's position is most likely are identified with converted histograms. As the distance from the point of maximal values, increases, both of these indicators decrease. A reduction in belief has been assumed with increasing distance from the maximum of the continuous histogram. The value of uncertainty is a characteristic feature of the distribution and remains constant, regardless of the considered abscissa. The sum of belief and uncertainty gives the value of plausibility, which therefore decreases. In contrast, the value describing the impossibility of supporting the considered hypothesis increases. This implies that the true location coordinate should be sought elsewhere.

2.1 *Simple Belief Structures and Their Aggregation*

A practical problem belonging to the decision-making category may look as follows. Two sources provide evaluations of the same student. One of the student's teachers claims to be convinced that the student is good. Based on exam results, he assesses his belief level on a [0, 1] scale at 0.45. Based on personal interactions, he concludes that the student could be considered good up to a level of 0.85. However, beyond that threshold, the student should no longer be regarded as this category. The second evaluator is more sceptical. He ultimately sets his belief at 0.20 and suggests a wider uncertainty interval, which in this case should be 0.65.

In practice, a single assessment is required—one that consolidates all available opinions. Such a cumulative evaluation provides a more accurate and complete picture of the student being assessed. The parameters of this resulting evaluation might be calculated using non-null generating conjunctive-like aggregation of the available belief structures.

Each opinion allows for the definition of an individual, simple belief distribution. The two simple belief structures, based on the available evaluations of the student, are shown in Table 1. The column labelled {T} contains the belief values for the statement; the student is good. The second column {¬T} includes data regarding the impossibility that the student is good. The last column {T, ¬T} contains the uncertainty, i.e., doubts regarding the assessment.

Table 1. Two belief structures on statements regarding student evaluations

Structure	{T}	{¬T}	{T, ¬T}
I	0,450	0.400	0,150
II	0,200	0,150	0,650

{T} belief values for the statement; the student is good
{¬T} impossibility that the student is good
{T, ¬T} uncertainty that the student is good

Two belief structures of the presented forms, referring to the same domain of discourse, can be combined. [6][7]. This leads to an enrichment—compared to the original assignments—of informational content. The aggregation (association) of two structures results in a belief distribution with cumulative content, characterized by a richer informational context.

The elements of the resulting structure are calculated from the intersections of each pair of subsets from the aggregated structures. The intersection of two sets may be empty. The subsets {T} and {¬T} share no common elements; in such a case, the aggregation operation yields the union of the arguments—i.e., the set {T, ¬T}. In the context of this application, this corresponds to uncertainty. All of this is consistent with a method proposed by Hau and Kashyap [5]. This type of association can be described as conjunctive/disjunctive, characterized by the absence of empty values. The masses associated with the resulting sets are obtained as products of the masses of each paired element being combined. This basic scheme of conjunctive association is most easily implemented using a two-dimensional representation—an example is shown in Table 2.

Table 2. Conjunctive-like combination of two simple belief structures on statements regarding student evaluations

		Structure II			
	set mass	{T}	{¬T}	{T, ¬T}	result
		0,200	0,150	0,650	
Structure I	{T}	{T}	{T, ¬T}	{T}	{T}
	0,45	0,090	0,068	0,293	0,413
	{¬T}	{T, ¬T}	{¬T}	{¬T}	{¬T}
	0,4	0,080	0,060	0,260	0,343
	{T, ¬T}	{T}	{¬T}	{T, ¬T}	{T, ¬T}
	0,15	0,030	0,023	0,098	0,245

The number of columns in such a table corresponds to the number of elements in one of the structures, and the number of rows equals the number of elements in the second assignment. Each element corresponds to an event or option with a non-zero mass. Elements assigned a mass of zero do not affect the aggregation results, and for this reason, they should not appear in the respective structure [4][7].

Table 2 presents the aggregation of the belief structures based on the student evaluations. The first few columns show data from the first evaluator. The second evaluator's opinions are shown in the first rows. The table's interior contains the intermediate results of the conjunctive/disjunctive aggregation for each pair of elements. The final result, obtained by summing the values assigned to identical sets, is presented in the last column. The probabilistic-possibilistic diagrams of the student assessments and the result of their assembly are shown in Figure 2.

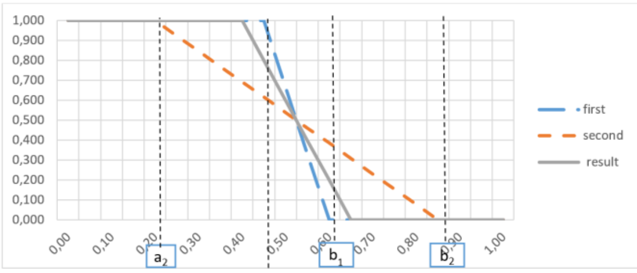

Figure 2. Diagrams of the student's evaluations and the combination result

Compared to the more favourable evaluation, we observe a slight decrease in the belief that the student is good (relative to the first evaluator's value). The uncertainty range increases (compared to the smaller value), while the indicator suggesting that the student should not be considered good decreases (relative to the higher original value). It is worth noting that the result confirms the subjective, common-sense assessment of the student being evaluated.

2.2 *Nautical belief assignments*

Let us return to the nautical example. The problem lies in seeking support for the hypothesis that the coordinate of the observer's true position lies in the vicinity of a given x-value based on density diagrams. Thus two sort of models are used. Student's assessment rely on possibilistic-probabilistic approach while reasoning on position fixing exploits possibilistic-density one. The agreement between the two approaches is obtained by assuming a unit width of the neighbourhood of a given abscissa. What is sought is the total, cumulative support, derived from two independent observations.

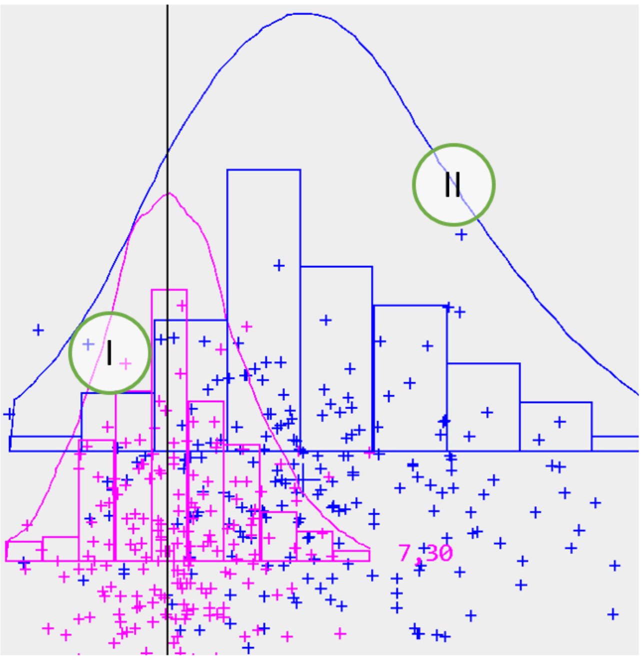

Figure 3. Distributions of the x-coordinates of the instances of two systems, their densities in the form of histograms and continuous functions, and an exemplary abscissa in the vicinity of which the location of the true position of the observer is determined

The situation is illustrated in Figure 3, which presents density distributions for two systems of differing precision. It can be observed that the first system (I) dominates the second (II) in terms of the narrower spread of its distribution area.

The goal is to determine the support for the hypothesis that the true coordinate of the observer's position lies near the marked x-value. The dataset characterizing the two simple belief structures is shown in Table 3. The characteristic values of the respective belief structures were calculated for the abscissa marked in Figure 3, based on the continuous graphs shown there. The distributions are labelled I and II, and they differ primarily in the spread of the observed instances. Vertical shape of the step-wise histogram (I) also dominates over the second one (II).

Table 3 Two simple belief structures defined from the histories of two independent systems

Structure	{T}	{¬T}	{T, ¬T}
I	0,456	0,430	0,114
II	0,138	0,524	0,338

Structure I corresponds to a distribution with less dispersion. Furthermore, the distance between the considered coordinate and the centre of dispersion of this system is very small. This results in significantly higher belief—derived from this observation—in support of the hypothesis that the neighbourhood of the given x-value represents the true coordinate location, compared to the belief derived from the second observation. Let us compare the relevant belief values: 0.456 versus 0.138. At the same time, the uncertainty ranges for each case display opposite magnitudes. These are shown in the last column: the uncertainty induced by the first observation is significantly lower than that from the second, namely: 0.114 versus 0.338. The method for calculating such values in the case of simultaneous indications from systems of varying accuracy will be presented in the following section.

Table 3. Conjunctive-like combination of two simple belief structures from Table 2

	set mass	Structure II {T}	{¬T}	{T, ¬T}	result
		0,138	0,524	0,338	
Structure I	{T}	{T}	{T, ¬T}	{T}	{T}
	0,456	0,063	0,239	0,154	0,233
	{¬T}	{T, ¬T}	{¬T}	{¬T}	{¬T}
	0,43	0,059	0,225	0,145	0,430
	{T, ¬T}	{T}	{¬T}	{T, ¬T}	{T, ¬T}
	0,114	0,016	0,060	0,039	0,337

Possibilistic-density diagrams of supporting the hypothesis of the true coordinate location approximately the considered x-value (Figure 3) are shown in Figure 4.

3 DETERMINING THE COMPONENTS OF BELIEF STRUCTURES FOR DATA SETS

In nautical science, we often work with so-called simultaneous observations—data sets whose element distributions indicate different degrees of uncertainty. Their practical application requires the definition of universal rules of procedure. Evaluating quality and building a hierarchy within the available data sets are fundamental tasks.

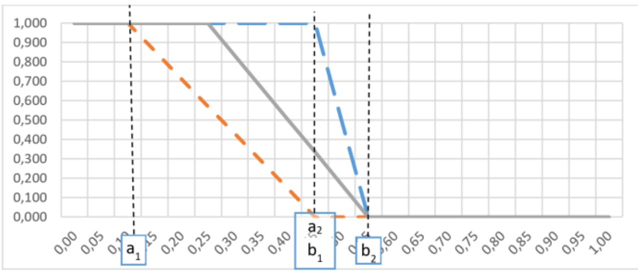

Figure 4. The graphs of the support for the hypothesis of locating the true coordinate of the observer's position in the vicinity of the abscissa marked in Figure 3

Figure 5 presents the density distributions of instances for four observations, shown as both histograms and continuous functions, along with sample x-values near which support is calculated for the hypothesis of the observer's true location. Each system is described using the data gathered in Table 5, which defines a multi-criteria decision-making problem (shaded columns), as well as its solution using the SAW (Simple Additive Weighting) method [9].

The first of the marked columns contains data labelled d_{i1}/v_{i1}. The values before the slash di1 correspond to the horizontal spreads of the x-values of instance sets for systems I, II, III, and IV. These represent the bin widths [in pixels] of the individual histograms. Smaller values are favoured, as they indicate systems with greater precision. For this reason, this attribute is considered cost-type. The importance weight for this attribute has been arbitrarily set to 0.75. The second marked column contains data labelled d_{i2}/v_{i2}. The d_{i2} values reflect the vertical spreads of the histograms. These are calculated based on the differences in histogram bin heights (i.e., the number of cases) across each system's structure. Larger values are favoured, as they indicate histograms with better shaping. Therefore, this attribute is considered qualitative. Its importance weight was arbitrarily set to 0.25. This means that the horizontal spread of the instances (i.e., the system's precision) is considered more important than the vertical shaping of the histogram.

The v_{ij} values placed after the slashes are the normalized values corresponding to each attribute. The calculation methods for these values are shown in Table 6. The appropriate formula is applied depending on the attribute type: cost or qualitative.

Table 5. Example characteristic data of the x-coordinate distributions of the instances observed for the four positioning systems

System	horizontal expansion d_{i1}/v_{i1} (cost 0.75)	vertical expansion d_{i2}/v_{i2} (quality 0.25)	ranking value	maximum density/belief
I	20.9/1.00	6.5/1.00	1.00	0.46/0.28
II	44.3/0.00	6.4/0.89	0.22	0.66/0.06
III	22.7/0.92	5.9/0.33	0.78	0.50/0.21
IV	33.2/0.47	5.6/0.00	0.36	0.60/0.10

Table 6. Methods of transforming attributes from the decision table of a multi-criteria problem

qualitative attribute	$v_{ij} = \dfrac{d_{ij} - d_j^{min}}{d_j^{max} - d_j^{min}}$ (1)
cost attribute	$v_{ij} = \dfrac{d_j^{min} - d_{ij}}{d_j^{max} - d_j^{min}}$ (2)

Table 7. Set of basic factors and proposed formulae for their estimation

	factor	formula of evaluation	meaning
1	$fd_{max}=pl_{max}$	read from the graph	plausibility for the centre of the graph, maximum of the continuous density function (see line 1 at Figure 5)
2	bel_{max}	$C*fd_{max}$	maximum belief obtained from the maximum of the continuous density function (see line 2 at Figure 5)
	$uncrt_{min}$	fd_{max} - bel_{max}	minimum uncertainty of the system
	fd_i	read from the graph	the value of the continuous density function for a given abscissa (see line 3 at Figure 5)
	bel_i	$bel_{max}*fd_i/fd_{max}$	the belief measure evaluated from the continuous density function for a given abscissa (see line 4 at Figure 5)
	$uncrt_i$	fd_{max} – bel_i	the uncertainty measure for i-th abscissa

C constant ratio, assumed as reduced system's ranking value (see Table 5)

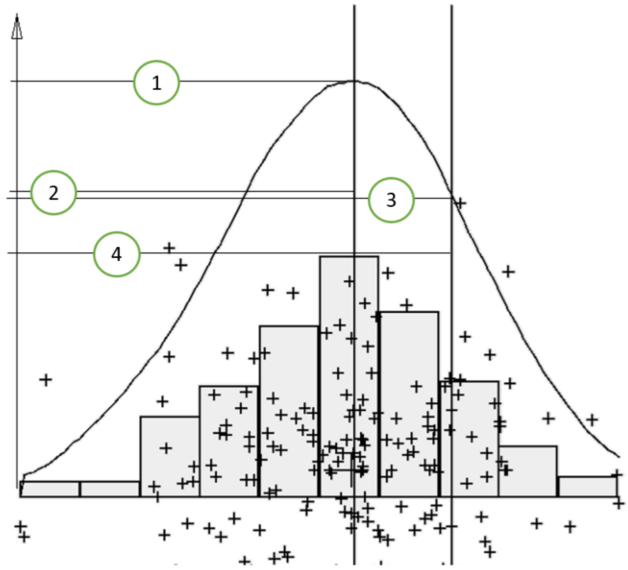

Figure 5. The set of instances distribution, its conventional and modern histogram with examples of data for selected abscissas

Ranking values are obtained by multiplying matrix V by the transposed weight vector. The resulting values are placed in the fourth column of Table 5. The best system turned out to be System I, with a ranking value of 1.00. The worst is System II, with the value of 0.22. The ranking values determine the quality hierarchy of the evaluated systems. They define the membership function shapes when using fuzzy systems, and consequently, they determine the shape of the transformed density distributions [4]. The ranking-based transformations influence the belief values regarding the truth of hypotheses in the central regions of the distributions. The dataset, ordered according to the ranking list, reveals an increasing trend in uncertainty across the systems. In such conditions, the belief values at the central coordinates of the distributions tend to decrease progressively.

The last column of Table 5 shows the maximum value of the continuous density function of each distribution. It also lists the belief value associated with the likelihood that the observer's coordinate lies near the x-value of the highest distribution density. The highest belief corresponds to the best system. This value depends on the ranking function and reaches a minimum for the lowest-rated distribution. In this way, an important practical principle is realized: that the estimated position should depend primarily on the most reliable data sources.

The set of calculated coefficients necessary for the construction of belief structures is shown in Table 7. It shows parameters characterizing the distributions of instances of individual navigation aids, but also ways of evaluating the measures of belief and presumption for any given abscissa.

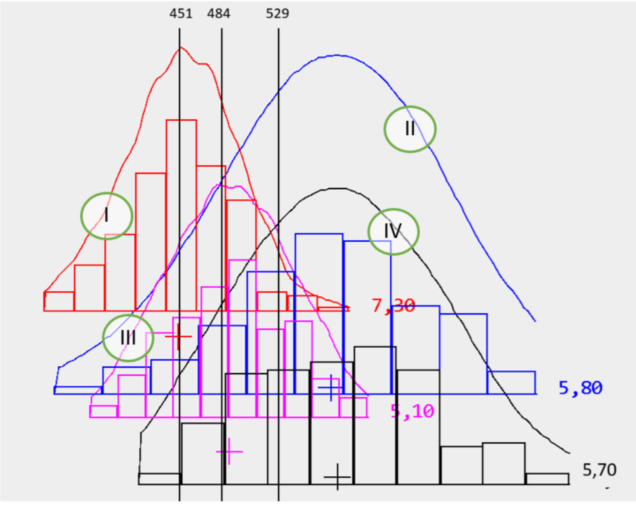

Figure 6. Density distributions of instances for four observations in the form of histograms and continuous functions, and sample abscissas, in whose neighbourhoods the support for the hypothesis about the location of the true position of the observer is determined

To illustrate the above-mentioned issues, the following section presents distributions reflecting the levels of support for the hypothesis that the true coordinate lies near the specified x-values shown in Figure 6. The dataset provides the foundation for defining the relevant belief structures.

Table 8. The ordered belief structures for the abscissas shown in Figure 6 along with the partial results of their sequential association

	Structure	{T}	{¬T}	{T, T¬}	{T}c	{¬T}c	{T, T¬}c
Abscissa	I	0.388	0.540	0.072	intermediate results 1		
451	III	0.244	0.501	0.255	0.211	0.444	0.345
	IV	0.042	0.402	0.556	0.141	0.564	0.295
	II	0.015	0.344	0.641	0.097	0.657	0.246
Abscissa	I	0.308	0.54	0.152	intermediate results 2		
484	III	0.323	0.501	0.176	0.203	0.442	0.355
	IV	0.069	0.402	0.529	0.146	0.554	0.300
	II	0.022	0.344	0.634	0.102	0.645	0.253
Abscissa	I	0.081	0.54	0.379	intermediate results 3		
529	III	0.248	0.501	0.251	0.134	0.596	0.270
	IV	0.099	0.402	0.499	0.107	0.645	0.248
	II	0.031	0.344	0.625	0.078	0.711	0.212

Table 8 shows three sets of belief structures for each of the x-values illustrated in Figure 6. Each set is sorted according to decreasing system-ranking value. The calculations of the presented elements are based on the maximum values for each system, taking into account the offset of the x-values from the peak density location of the respective system. It is assumed that as the distance between the x-value and the peak increases, the corresponding belief decrease. In the last three columns of Table 8, the results of successive associations of the four distributions shown alongside are presented. The outcome of this operation is a sequence of three belief structures, labelled as "intermediate result x". The first structure in each trio is the result of aggregating the two best distributions from the respective set of four. Initially, in the first two cases, these are structures I and III. The following results involve aggregating the previous result with structure IV, and then combining that new result with the last distribution. Within each trio of results, the belief values decrease, the uncertainty levels tends to increase, while the indicator corresponding to the rejection of the hypothesis—that the correct coordinate lies near the x-value—remains the same.

An interesting case is that of a belief distribution in which the uncertainty reaches a value of 1. This structure then takes the form $m_\emptyset = \{0, 0, 1\}$, and it serves as a neutral element in the aggregation process. This means that the aggregation operation $\otimes$ in the form $m_\emptyset \otimes m_i$ results in m_i.

4 SUMMARY

This paper presents a method for encoding opinions that include an element of doubt. The proposed approach uses an interval model, which allows for the creation of belief structures that are elements of the Mathematical Theory of Evidence (MTE), also known as belief theory. This scheme provides a mechanism for aggregation that enriches the informational context of arguments. Two sort of models were used; possibilistic-probabilistic and possibilistic-density one. The agreement between the two approaches is obtained by assuming the neighbourhood of a given abscissa of unit width.

The available opinions come from various sources and may be more or less objective. From a nautical perspective, an important task is the exploration of data regarding instances and observations, leading to the extraction of opinions on hypotheses about the observer's correct location. The available sets of observations represent different levels of reliability, making it necessary to define a hierarchy among these elements so that the implementation of the concept produces solutions that meet quality standards.

Preparing a ranking list is a multi-criteria decision-making problem. This field provides a range of methods for building hierarchies within a set of alternatives. One of the simplest is the additive method known by the acronym SAW (Simple Additive Weighting). Determining a ranking for a set of simultaneous observations is a challenge addressed in this work. The proposed approach utilizes sets of instances observed for each system used to determine the observer's position.

The computed utility values define the hierarchy of relative quality among the systems under consideration. These values determine the forms of membership functions in the context of using fuzzy sets. Consequently, they allow for defining the shapes of density distribution functions, usually perceived in the form of "step-like" histograms. The transformed, continuous form of the density distribution employs an approach assuming that histogram bins are fuzzy sets [2][3].

The ranking function values form the basis for calculating belief in the truth of the considered hypotheses, especially in the central regions of the distributions. The data set, sorted in descending order according to the ranking list, organizes the systems from best to worst. The differences in ranking values make it possible to determine the degree of qualitative dominance of one system over another.

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