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Multi-criteria Optimization of Multi-step Matrix Game in Collision Avoidance of Ships

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ABSTRACT: This research article formulates a mathematical model of the matrix game of the safe ship control process containing: state variables and control, collision risk definition and the form of a collision risk matrix. Multicriteria optimization of the matrix game was introduced, leading to non-cooperative and cooperative game control algorithms and non-game control. Simulation safe trajectories of own ship for various types of control were compared to the example of the real situation at sea.

1 INTRODUCTION

The safe conduct of a commercial ship in situations of passing by with more of the ships they meet is to keep the least risk of collision with each of them. The source of information on the current state of the vessel traffic control process and the collision risk assessment is the Automatic Radar Plotting Aid (ARPA) radar anti-collision system, which enables the operator of the vessel through the TRIAL MANOEUVRE function to determine the anti-collision manoeuvre in relation to the most dangerous vessel, including the rules of the International Regulations for Preventing Collision at Sea (COLREGs) maritime route according to International Maritime Organization (IMO). The COLREGs rules apply only to two ships met, separately for the conditions of good and restricted visibility at sea.

Over 80% of ship collisions are caused by the human factor, during the subjective assessment of the navigational situation and maneuvering decision in the game environment (Engwerda 2005, Kun 2001, Lebkowski 2018, Millington & Funge 2009).

It is assumed that about half of these losses can be avoided using computer-aided navigator decision support software using artificial intelligence, game theory and optimization methods (Bist 2000, Isaacs 1965, Lazarowska 2017, Miloh 1974, Perez 2005, Szlapczynski & Szlapczynska 2016).

The aim of this article is to present a multi-step matrix game that contain the risk of ship collisions, determining the safe trajectory of the ship in terms of multi-criteria optimization, allowing for the degree of cooperation in avoiding collisions (Kouemou, 2009, Lisowski 2016, Olsder & Walter 1977).

This article proposes a new mathematical formulation model of the collision-risk index, depending on ships proximity parameters and the distance between them.

2 THE MATHEMATICAL MODEL OF A MATRIX GAME

Figure 1 presents a matrix game of the process of controlling your own ship in the situation of passing by with the ships at sea.

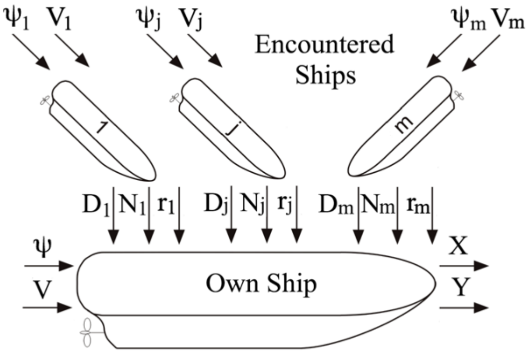

Figure 1. Matrix game of ships.

2.1 *State and control variables*

Process state variables are represented by the distance between the encountered ships D_j, bearing N_j and the risk of collision r_j and the coordinates of the own-ship position (X,Y) on the reference trajectory of movement (Kazimierski & Stateczny 2013).

The control variables of the own-ship are: the course ψ and the speed V, and the control quantities of the j-th encountered ship in question are its course ψ_j and the speed V_j (Tomera 2012).

State variables in the ship control process in collision situations are measurable by means of the on-board ARPA anti-collision system.

2.2 *Collision-risk*

The forms of the collision-risk r_j of own-ship with the j-th encountered ship as the mean square reference of a safe situation are determined by the value of the assumed safe distance D_s and safe approach time T_s with the current situation of the ships approximation which are determined by the smallest expected distance $D_{j,min}$ and time to the largest approximations of $T_{j,min}$ and the distance between ships D_j:

$$r_j\left(s_w,s_j\right)=\sqrt{a_1\left(\frac{D_s}{D_{j,\min}^{\left(s_w,s_j\right)}}\right)^2+a_2\left(\frac{T_s}{T_{j,\min}^{\left(s_w,s_j\right)}}\right)^2+\left(\frac{D_s}{D_j}\right)^2} \tag{1}$$

where the coefficients a_1 and a_2 determine: the type of visibility at sea - good or restricted and the type of navigation area - open or closed (Modarres 2006, Xu 2014) (Fig. 2).

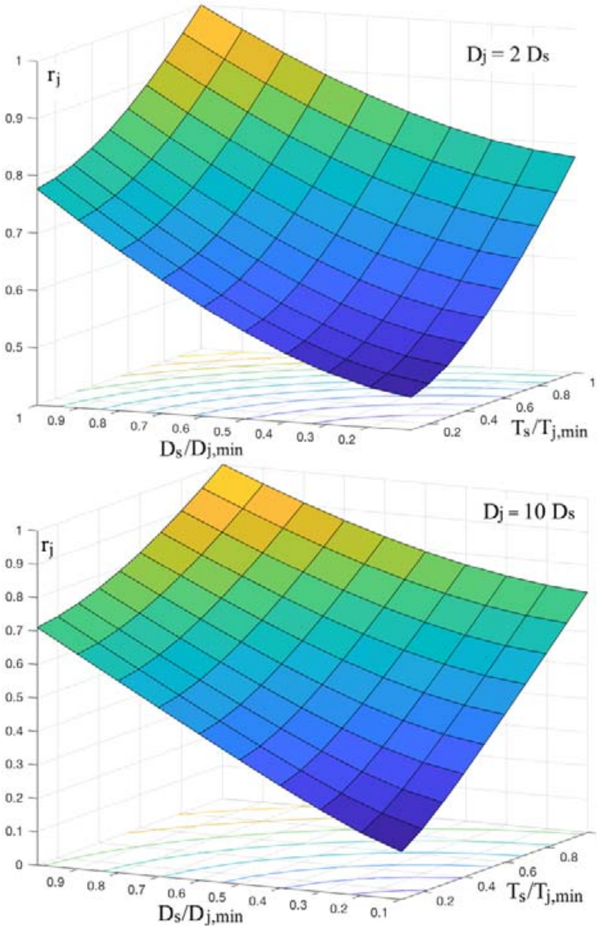

Figure 2. Collision-risk characteristics in the situation of concentrated ship traffic at: $a_1 = a_2 = 0.4$, $D_s/D_j = 0.5$ (above) and in the situation of larger distances between ships at: $a_1 = a_2 = 0.5$, $D_s/D_j = 0.1$ (below).

The mathematical collision-risk model proposed here was taken into account not only the approach parameters of $D_{j,\min}$ and $T_{j,min}$, but also the distances between ships D_j. The value of collision-risk also depends on the density of the position of the encountered ships (Lisowski 2014).

2.3 *Matrix of collision-risk*

The game matrix $\boldsymbol{R}$, in which own ship with clean s_w strategies and m encountered ships with clean strategies, can each be presented each in the following equation form:

$$R=\left[r_j\left(s_w,s_j\right)\right]=\begin{vmatrix} r_1^{\left(s_{w1},s_{11}\right)} & r_1^{\left(s_{w1},s_{12}\right)} & \ldots & r_j^{\left(s_{w1},s_{j1}\right)} \\ r_1^{\left(s_{w2},s_{11}\right)} & r_1^{\left(s_{w2},s_{12}\right)} & \ldots & r_j^{\left(s_{w2},s_{j1}\right)} \\ \ldots & & & \\ r_1^{\left(s_{wW},s_{11}\right)} & r_1^{\left(s_{wW},s_{12}\right)} & \ldots & r_j^{\left(s_{wW},s_{j1}\right)} \end{vmatrix} \tag{2}$$

The numbers of rows are corresponded to the amount W of the acceptable own-hip's strategy in the S_w set, in the form of changes in its course $\Delta\psi$ and the speed ΔV to avoid collision (Mesterton-Gibbons 2001, Polak et al. 2016):

$$S_w\left(s_{wi}\right)=\left(s_{w1}=0, s_{w2}=\pm\Delta\psi, s_{w3}=\pm\Delta V, \ldots\right) \quad (3)$$
$$i=1,2,\ldots,W$$

The numbers of columns were corresponded to the total amount of $K = mW_j$ admissible strategies for all m ships in the set S_j, in the form of changes in their courses $\Delta\psi_j$ and speeds ΔV_j in order to avoid collisions:

$$S_j\left(s_{ji}\right)=\left(s_{j1}=0, s_{j2}=\pm\Delta\psi_j, s_{j3}=\pm\Delta V_j, \ldots\right) \quad (4)$$
$$j=1,2,\ldots,W_j$$

The Constraint-set $C(s_w,s_j)$ of the permissible strategies result from the rules of the right of the COLREGs sea route are shown in Figures 3 and 4.

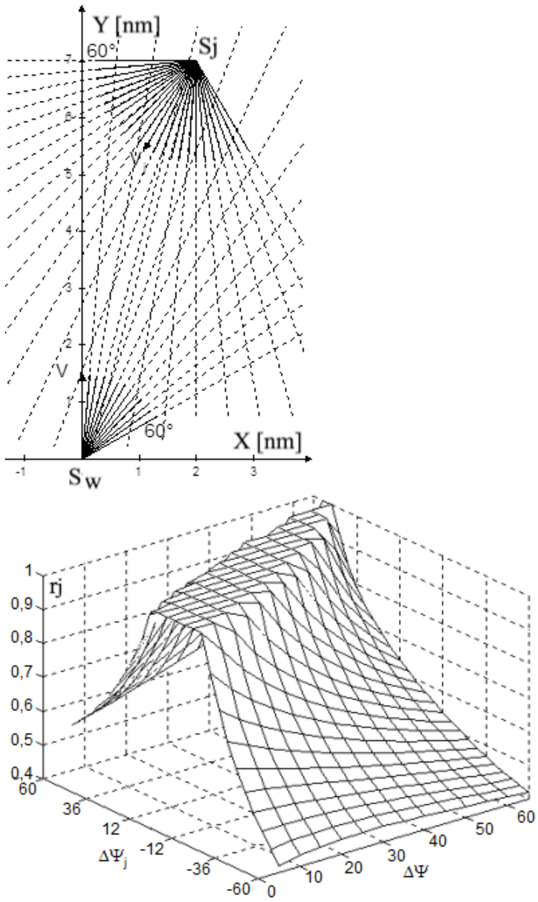

Figure 3. Met j-th ship on the starboard side: the sets of acceptable game strategies of own-ship S_w and j-th encountered ship S_j (above), and hyperplane of collision-risk r_j depending on the $\Delta\psi$ own-ship course changes and the encountered j-th ship $\Delta\psi_j$ (below).

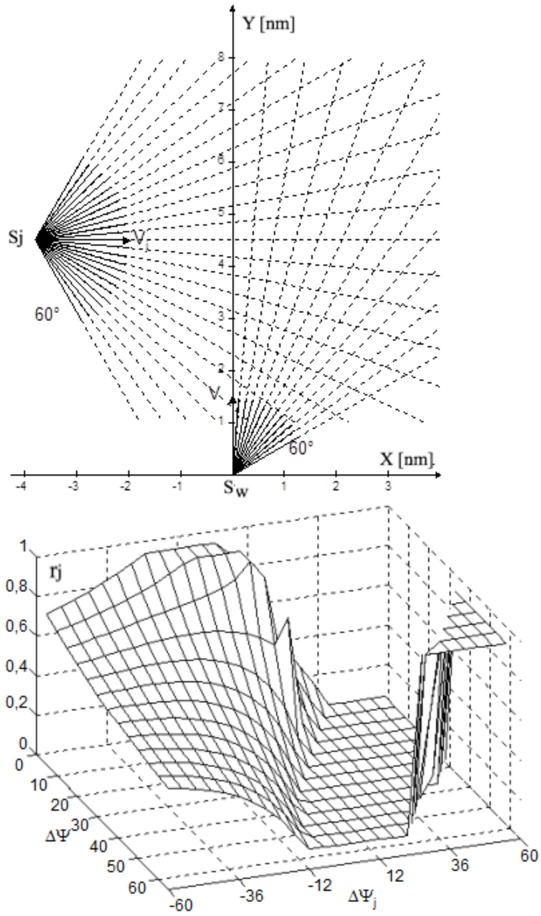

Figure 4. Met j-th ship on the port side: the sets of acceptable game strategies of own-ship S_w and j-th encountered ship S_j (above), and hyperplane of collision-risk r_j depending on the $\Delta\psi$ own-ship course changes and the encountered j-th ship $\Delta\psi_j$ (below).

3 TYPES OF OPTIMIZATION OF THE MULTI-STEP MATRIX GAME

In most real transport and logistics processes, the matrix game does not reach the saddle point and then has no solution when using pure strategy in game theory. Therefore, an approximate solution to the game is the use of the mixed strategy chain as the probability of using pure strategies (Ehrgott 2005, Ehrgott & Gandibleux 2002).

First, the probability matrix for using pure strategies is determined as shown in equation 5:

$$\left[p_j\left(s_w,s_j\right)\right]=\left|\begin{matrix} p_1^{\left(s_{w1},s_{11}\right)} \ldots & p_j^{\left(s_{w1},s_{j1}\right)} \\ p_1^{\left(s_{w2},s_{11}\right)} \ldots & p_j^{\left(s_{w2},s_{j1}\right)} \\ \ldots & \\ p_1^{\left(s_{wW},s_{11}\right)} \ldots & p_j^{\left(s_{wW},s_{j1}\right)} \end{matrix}\right| \quad (5)$$

Then, the most probable strategy is the optimal control used of the own-ship (Eshenauer et al. 1990, Osborne 2004):

$$u_w^* = u_w \left[\left(p_j^{(s_w, s_j)} \right)_{\max} \right] \tag{6}$$

The game ends after bringing the collision-risk of each ship to the value of zero and reaching a certain value of the final payment p_f, in the form of the final deviation of the safe trajectory from its initial value.

3.1 *The criterion of a non-co-operative matrix game*

The algorithm of *MMG_nc* multi-step non-co-operative matrix game uses the following optimization criterion (Nisan et al. 2007, Straffin 2001):

$$Q_{nc}^* = \min_{s_w} \max_{s_j} p_j^{(s_w, s_j)} \tag{7}$$

3.2 *The criterion of the co-operative matrix game*

The MMG_c algorithm of multi-step co-operative matrix game uses the following optimization criterion (Basar & Bernhard 2008, Wells 2013):

$$Q_c^* = \min_{s_w} \min_{s_j} p_j^{(s_w, s_j)} \tag{8}$$

3.3 *The criterion of non-game control*

The *MNG* algorithm of multi-step non-game control uses the following optimization criterion:

$$Q_{ng}^* = \min_{s_w} p_j^{s_w} \tag{9}$$

The *MMG_nc*, *MMG_c* and *MNG* algorithms for determining a safe own-ship trajectory in a collision situation were developed using the *linprog* function from the Matlab Optimization Toolbox package (Breton & Szajowski 2010).

The method of entering the initial data for calculations describing the navigational situation is shown in Figure 5, and the form of calculations results of the safe trajectory of the ship is illustrated in Figure 6.

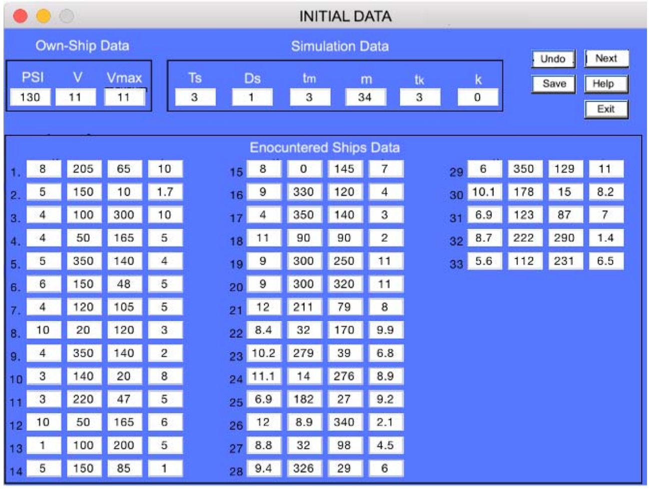

Figure 5. Algorithm window with initial data of the navigational situation.

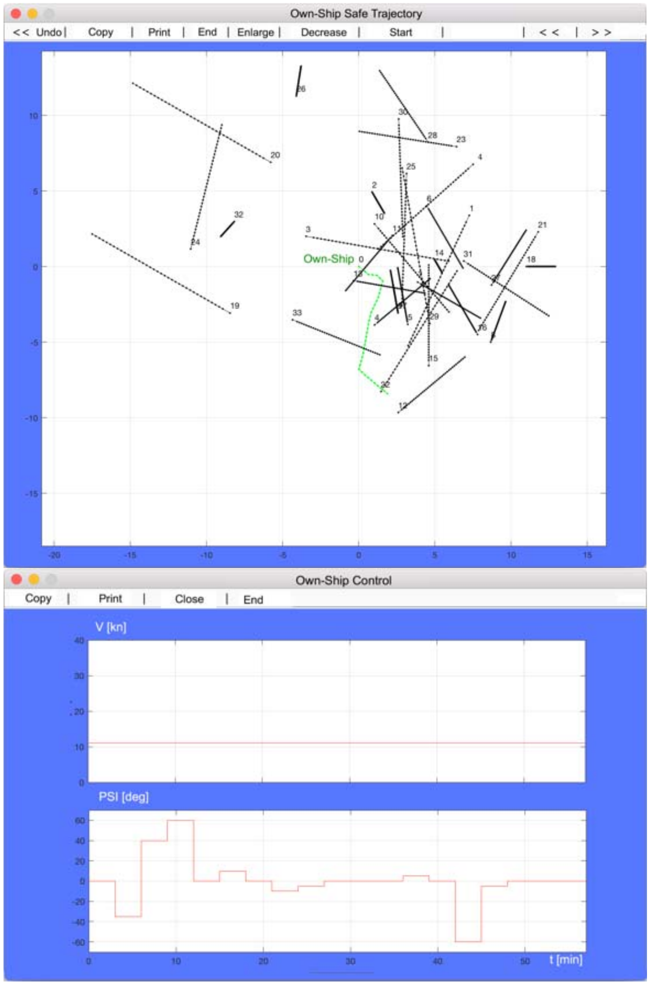

Figure 6. The algorithm window with the results of calculations of the safe own-ship trajectory and own-ship control by change of speed V or course ψ.

4 COMPUTER SIMULATION OF GAME CONTROL ALGORITHMS

The safe trajectories of own ship in the situation of 33 ships in the Kattegat Strait, in conditions of good and restricted visibility of the sea, are determined according to algorithms of multi-criteria optimization:

MMG_nc, *MMG_c* and *MNG*, and this are shown in Figures 7-13.

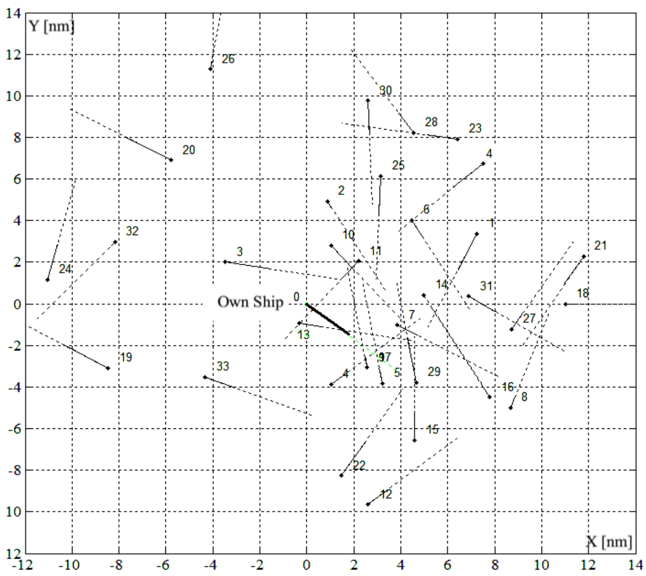

Figure 7. The six-minute speed vectors of own ship and 33 encountered ships in a navigational situation in Kattegat Strait.

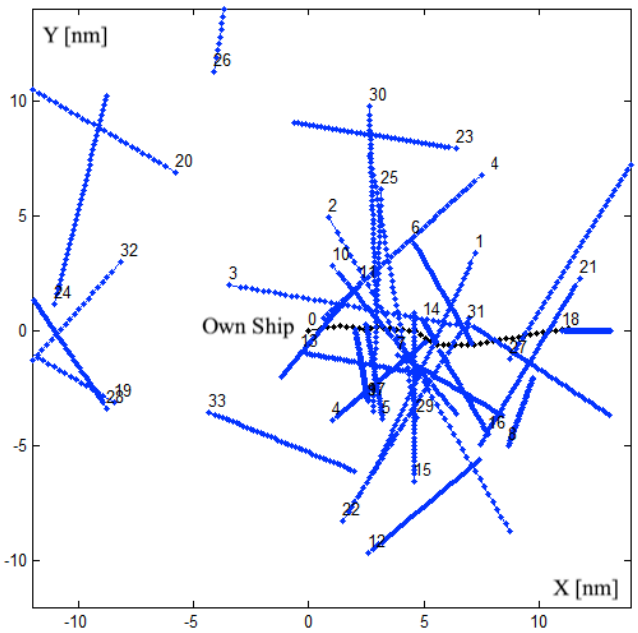

Figure 8. A safe trajectory of own-ship in non-co-operative matrix game, in conditions of good visibility in the sea, for D_s = 1.0 nm, final payment p_f = 1.88 nm (nautical mile).

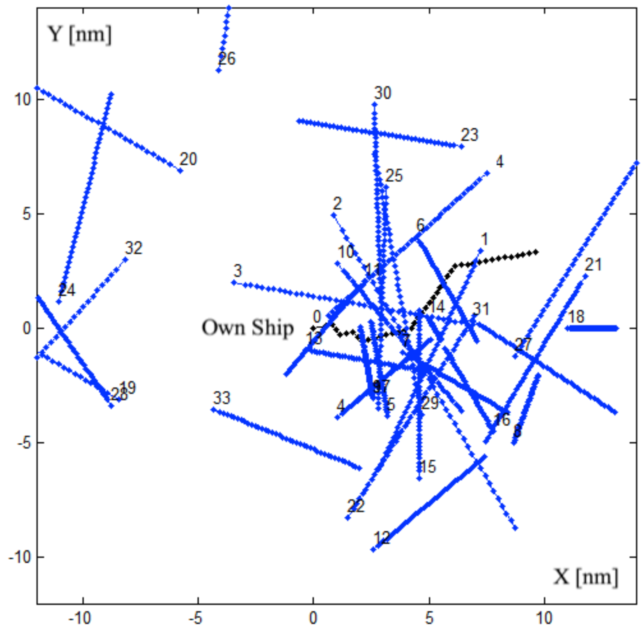

Figure 9. A safe trajectory of own-ship in non-co-operative matrix game, in conditions of restricted visibility of the sea, for D_s = 2.6 nm, p_f = 2.18 nm (nautical mile).

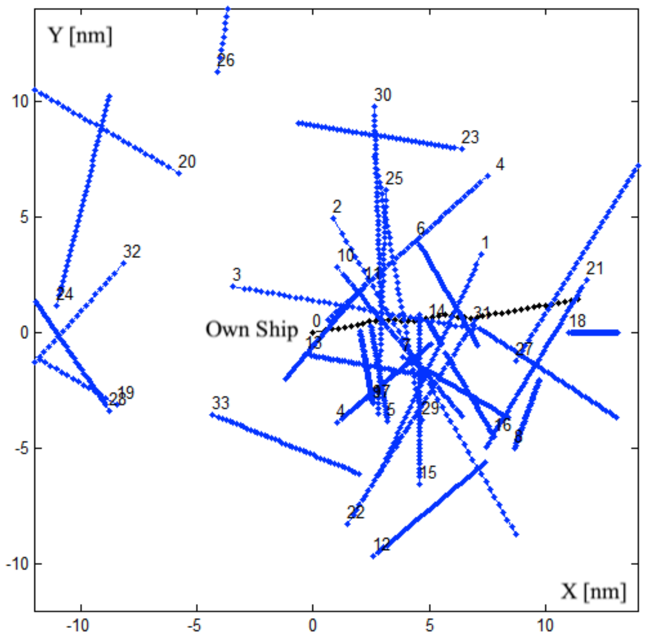

Figure 10. A safe trajectory of own-ship in co-operative matrix game, in conditions of good visibility in the sea, for D_s = 1.0 nm, p_f = 0.57 nm (nautical mile).

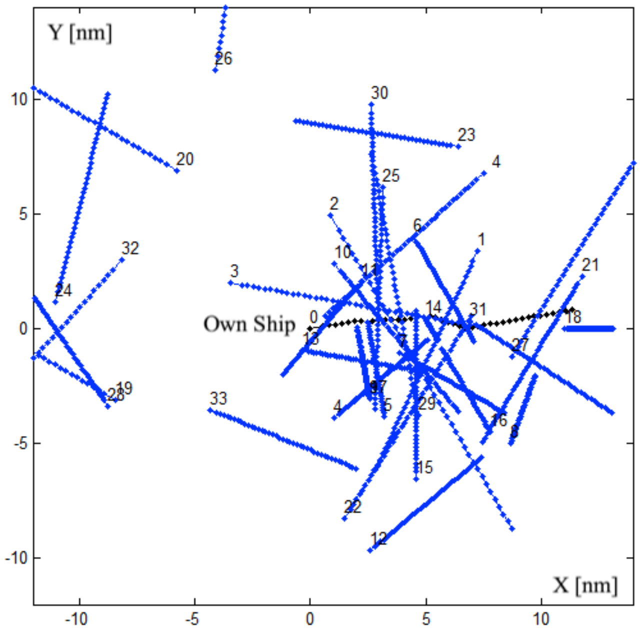

Figure 11. A safe trajectory of own-ship in co-operative matrix game, in conditions of restricted visibility of the sea, for Ds = 2.6 nm, pf = 1.19 nm (nautical mile).

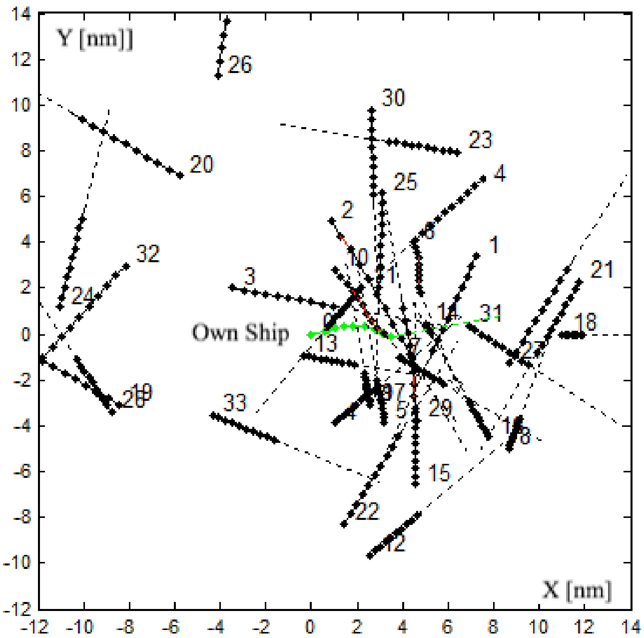

Figure 12. A safe trajectory of own-ship in non-game control, in conditions of good visibility on the sea, for D_s = 2.6 nm, p_f = 0.69 nm (nautical mile).

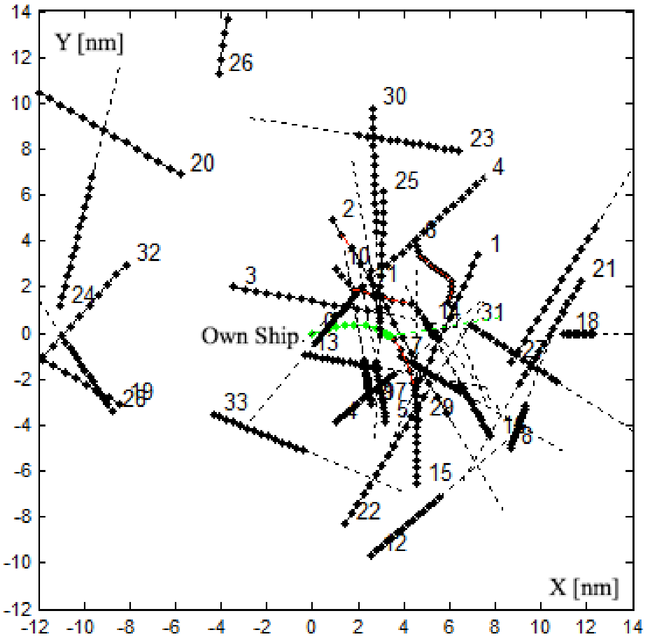

Figure 13. A safe trajectory of own-ship in non-game control, in conditions of restricted visibility on the sea, for D_s = 2.6 nm, p_f = 0.71 nm (nautical mile).

5 CONCLUSIONS

The use of a matrix game with collision-risk for the synthesis of algorithms for computer-aided navigating maneuvering decision makes it possible to take into account the degree of indeterminacy of the navigational situation caused by the imperfection of the law of the sea route and the subjectivity of the navigator making the maneuvering decision to avoid a collision.

The multi-criteria approach to the task of optimizing the safe control of the ship's movement allows taking into account of both the non-co-operative and co-operative game control and the control of the non-target ships.

Obtained safe ship trajectories differ mainly in the value of the final deviation from the set trajectory of the movement.

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