Monte Carlo Simulation Approach to Determination of Oil Spill Domains at Port and Sea Water Areas

E. Dąbrowska & K. Kołowrocki
Gdynia Maritime University, Gdynia, Poland

ABSTRACT: Monte Carlo simulation method of oil spill domains determination based on the probabilistic approach to the solution of this problem is proposed. A semi-Markov model of the process of changing hydro-meteorological conditions is constructed and its parameters are defined. The general stochastic model of oil spill domain movement for various hydro-meteorological conditions is described. Monte Carlo simulation procedure is created and applied to generating the process of changing hydro-meteorological conditions and the prediction of the oil spill domain movement impacted by these changes conditions.

1 INTRODUCTION

The probabilistic approach to determination of oil spill domains at port and sea water areas is proposed in (Dąbrowska & Kołowrocki 2019B). In this paper, the stochastic approach is supplemented by the Monte Carlo simulation approach (Dąbrowska 2019, Law & Kelton 2000, Zio & Marseguerra 2002) to the oil spill domain movement in changing hydro-meteorological conditions (Dąbrowska & Kołowrocki 2019A, 2019B). First, the model of the process of changing hydro-meteorological conditions is defined and its parameters are introduced. The identification methods of the unknown parameters of process of changing hydro-meteorological conditions is described in (Dąbrowska & Soszyńska-Budny 2018, 2019A). Under these assumptions, the process of changing hydro-meteorological conditions $A(t)$ is completely described by the following parameters (Dąbrowska & Soszyńska-Budny 2018, Kołowrocki & Soszyńska-Budny 2011):

- the vector of probabilities of its initial states at the moment $t = 0$

$$[p(0)] = [p_1(0), p_2(0),..., p_m(0)],$$ (1)
- the matrix of probabilities of its transitions between the particular states

\[
[p_i] = \begin{bmatrix}
p_{i1} & p_{i2} & \cdots & p_{in} \\
p_{2i} & p_{22} & \cdots & p_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
p_{ni} & p_{n2} & \cdots & p_{nn}
\end{bmatrix},
\] (2)

where

\[ p_{ii} = 0, \quad i = 1, 2, \ldots, m; \]

- the matrix of distribution functions of its conditional sojourn times \(0|0|\) at the particular states

\[
[W(t)] = \begin{bmatrix}
W_{11}(t) & W_{12}(t) & \cdots & W_{1m}(t) \\
W_{21}(t) & W_{22}(t) & \cdots & W_{2m}(t) \\
\vdots & \vdots & \ddots & \vdots \\
W_{n1}(t) & W_{n2}(t) & \cdots & W_{nm}(t)
\end{bmatrix},
\] (3)

where

\[ W_{ii}(t) = 0, \quad i = 1, 2, \ldots, m. \]

3 MODELLING OIL SPILL DOMAIN IN VARYING HYDRO-METEOROLOGICAL CONDITIONS

We assume that the process of changing hydro-meteorological conditions \(A(t)\) in succession takes the states

\[ k_1, k_2, \ldots, k_{n+1}, \quad i = 1, 2, \ldots, n+1. \]

For a fixed step of time \(\Delta t\), after multiple applying sequentially the procedure from Section 4.1 in (Dąbrowska & Kolowrocki 2019B):

- for

\[ t = 1\Delta t, 2\Delta t, \cdots, s_1\Delta t, \quad s_1 \leq \Delta t, \quad i = 1, 2, \ldots, n+1. \]

- at the process \(A(t)\) state \(k_i\);

- for

\[ t = (s_1 + 1)\Delta t, (s_1 + 2)\Delta t, \cdots, s_2\Delta t, \quad s_2 \leq 2\Delta t, \quad i = 1, 2, \ldots, n. \]

- at the process \(A(t)\) state \(k_j\);

... for

\[ t = (s_{n-1} + 1)\Delta t, (s_{n-1} + 2)\Delta t, \cdots, s_n\Delta t, \quad s_n \leq (n-1)\Delta t, \quad i = 1, 2, \ldots, n. \]

we receive the following sequence of oil spill domains:

\[ \overline{D}^h_{1\Delta t}, \overline{D}^h_{2\Delta t}, \ldots, \overline{D}^h_{s_1\Delta t}, \] (7)

\[ \overline{D}^h_{s_1\Delta t}, \overline{D}^h_{s_1 + 1\Delta t}, \ldots, \overline{D}^h_{s_2\Delta t}, \] (8)

... for

\[ \overline{D}^h_{(s_n - 1)\Delta t}, \overline{D}^h_{(s_n - 1) + 1\Delta t}, \ldots, \overline{D}^h_{s_n\Delta t}, \] (9)

where \(s_i, i = 1, 2, \ldots, n\), are such that

\[ (s-1)\Delta t < \sum_{j=1}^{s_i} \overline{t_{k_j}}, \quad s_i \Delta t \leq T, \] (10)

and

\[ \overline{t_{k_j}}, \quad j = 1, 2, \ldots, n, \] (11)

are the realizations of the process \(A(t), t \in (0,T)\), conditional sojourn times \(0|0|\) at the states \(k_i\), upon the next state is \(k_{i+1}\), \(j = 1, 2, \ldots, n\), \(k_i \in [1,2, \ldots, m]\), \(i = 1, 2, \ldots, n\) introduced in Section 2 and in (Dąbrowska & Kolowrocki 2019B).

Hence, the oil spill domain

\[ \overline{D}^h_{1\Delta t} \ldots k_n, \quad k_1, k_2, \ldots, k_n \in [1,2, \ldots, m], \]

is described by the sum of determined domains of the sequences (7)-(9), given by

\[ \overline{D}^h_{1\Delta t} \ldots k_n \equiv \bigcup_{i=1}^{n} \overline{D}^h_{i\Delta t} (s_i\Delta t) \]

\[ = \left[ \overline{D}^h_{1\Delta t} \cup \overline{D}^h_{2\Delta t} \cup \ldots \cup \overline{D}^h_{s_1\Delta t} \right] \]

\[ \cup \left[ \overline{D}^h_{s_1\Delta t} \cup \overline{D}^h_{s_1 + 1\Delta t} \cup \ldots \cup \overline{D}^h_{s_2\Delta t} \right] \]

... for \(k_1, k_2, \ldots, k_n \in [1,2, \ldots, m]\), \(s_0 = 0\)

(12)

The oil spill domain \(\overline{D}^h_{1\Delta t} \ldots k_n\) defined by (12) is determined for constant radiiuses

\[ r^h_i, \quad t \in (0,T), \quad k_i \in [1,2, \ldots, m], \quad i = 1, 2, \ldots, n. \]

If the radiuses are not constant, we define the sequence of domains for each state \(k_i, k_i \in [1,2, \ldots, m]\), \(i = 1, 2, \ldots, n\), in a way similar to that described in Remark 1 in (Dąbrowska & Kolowrocki 2019B), i.e. we define the sequence of domains

\[ \overline{D}^h_{1\Delta t} \ldots k_n \equiv \bigcup_{i=1}^{n} \overline{D}^h_{i\Delta t} (a_i\Delta t) \]

\[ = \left[ \overline{D}^h_{1\Delta t} \cup \overline{D}^h_{2\Delta t} \cup \ldots \cup \overline{D}^h_{s_1\Delta t} \right] \]

\[ \cup \left[ \overline{D}^h_{s_1\Delta t} \cup \overline{D}^h_{s_1 + 1\Delta t} \cup \ldots \cup \overline{D}^h_{s_2\Delta t} \right] \]

...
\[ \bigcup \overline{\mathbb{D}}^n((s_{n-1} + 1)\Delta t) \cup \overline{\mathbb{D}}^n((s_{n-1} + 2)\Delta t) \cup \ldots \cup \overline{\mathbb{D}}^n(s_n\Delta t) \]

for \( b_i = 1, 2, \ldots, s_i - s_{i-1}, \ k_i \in \{1, 2, \ldots, m\}, 

i = 1, 2, \ldots, n, 

(13)

where

\[ \overline{\mathbb{D}}^n_i(s_{i-1} + a_i\Delta t) = \overline{\mathbb{D}}^n_i(s_{i-1} + a_i\Delta t), \]

\[ a_i = 1, 2, \ldots, b_i, \ b_i = 1, 2, \ldots, s_i - s_{i-1}, \ k_i \in \{1, 2, \ldots, m\}, \]

\[ i = 1, 2, \ldots, n, \]

with the following, modified slightly in comparison that defined in (Dąbrowska & Kołowrocki 2019B), substitutions:

\[ m_i^h(t) := m_i^{h,i}(s_{i-1}\Delta t) + m_i^{h,i}(a_i\Delta t), \]

\[ m_i^h(t) := m_i^{h,i}(s_{i-1}\Delta t) + m_i^{h,i}(a_i\Delta t), \]

\[ \sigma_i^h(t) := \overline{\mathbb{D}}^n_i(s_{i-1} + a_i\Delta t), \]

\[ = \sigma_i^h(s_{i-1} + a_i\Delta t) + \sum_{j=1}^i \tau_j^i(b_j\Delta t), \]

\[ \sigma_i^h(t) := \overline{\mathbb{D}}^n_i(s_{i-1} + a_i\Delta t), \]

\[ = \sigma_i^h(s_{i-1} + a_i\Delta t) + \sum_{j=1}^i \tau_j^i(b_j\Delta t), \]

where \( m_i^{h,i}(s_{i-1}\Delta t) = 0, \ m_i^{h,i}(s_{i-1}\Delta t) = 0, \) for \( a = 1, 2, \ldots, b_i, \ b_i = 1, 2, \ldots, s_i - s_{i-1}, \ k_i \in \{1, 2, \ldots, m\}, \ i = 1, 2, \ldots, n, \)

\[ s_0 = 0. \]

4 MONTE CARLO SIMULATION PREDICTION OF THE OIL SPILL DOMAIN

4.1 Generating process of changing hydro-meteorological conditions at oil spill area

We denote by \( k_i = k_i(q), \ i \in \{1, 2, \ldots, m\}, \) the realization of the process’ \( A(t) \) initial state at the moment \( t = 0. \) Further, we select this initial state by generating realizations from the distribution defined by the vector \( [p(q)]_{v=0}^\infty, \) according to the formula

\[ k_i = \sum_{\xi=1}^{\psi} p_{i,\xi}, \]

\[ 0 < \psi < \sum_{\xi=1}^\infty p_i(0), \]

\[ \psi \in \{1, 2, \ldots, m\}, \]

(14)

where \( \psi \) is a randomly generated number from the uniform distribution on the interval \((0,1)\) and \( p(0) \) for \( \xi = 0 \) equals \( 0. \)

After selecting the initial state \( k_i, \ i \in \{1, 2, \ldots, m\}, \) we can fix the next operation state of the process of changing hydro-meteorological conditions at oil spill area. We denote by \( k_i = k_i(q), \ j \in \{1, 2, \ldots, m\}, \ i \neq j, \) the sequence of the realizations of the operation process’ consecutive states generated from the distribution defined by the matrix \([p_i](n)\). Those realizations are generated for a fixed \( i, \ i \in \{1, 2, \ldots, m\}, \) according to the formula

\[ k_i(g) = k_i, \sum_{\xi=1}^{\psi} p_{i,\xi} 

\[ \psi \in \{1, 2, \ldots, m\}, \psi \neq i, \]

where \( g \) is a randomly generated number from the uniform distribution on the interval \((0,1)\) and \( p_i(0) = 0. \)

We can use several methods generating draws from a given probability distribution, e.g., an inverse transform method, a Box-Muller transform method, Marsaglia and Tsang’s rejection sampling method (Dąbrowska 2019). The inverse transform method (also known as inversion sampling method) is convenient if it is possible to determine the inverse distribution function (Grabski & Jaźwiński 2009). This section will consider only this one sampling method, but the other methods are discussed in (Law & Kelton 2000, Rao & Naikani 2016, Zio & Marseguerra 2002).

We denote by \( t_{ij}^{(v)}, \ i, j \in \{1, 2, \ldots, m\}, \ i \neq j, \)

\[ v = 1, 2, \ldots, n, \] the realization of the conditional sojourn times \( \theta_{ij} \) of the process \( A(t), t \in <0,T>, \) generated from the distribution function \( W_{ij}(t), \) where \( v \) denotes the subsequent number of the sojourn times realizations and \( n \) is the number of those sojourn time realizations during the experiment time \( T. \) Thus, using the inverse transform method, the realization \( t_{ij}^{(v)} \) is generated from

\[ t_{ij} = W_{ij}^{-1}(h), \ i, j \in \{1, 2, \ldots, m\}, \ i \neq j, \]

(16)

where \( W_{ij}^{-1}(h) \) is the inverse function of the conditional distribution function \( W_{ij}(t) \) and \( h \) is a randomly generated number from the interval \((0,1)\).

Having the realizations \( t_{ij}^{(v)}, \ i, j \in \{1, 2, \ldots, m\}, \ i \neq j, \)

\[ v = 1, 2, \ldots, n, \] of the process \( A(t), \) it is possible to determine approximately the entire sojourn time as the sum of all sojourn time realizations during the experiment time \( T, \) applying the formula

\[ \tau_n = \sum_{v=1}^{\psi} t_{ij}^{(v)}, \ i, j \in \{1, 2, \ldots, m\}, n = 1, 2, \ldots. \]

(17)

The exemplary realization of the process \( A(t) \) and the entire sojourn time is presented in a figure below.
4.2 General procedure of Monte Carlo simulation application to determine the oil spill domain in varying hydro-meteorological conditions

The procedure of generating and estimating the parameters of the process of changing hydro-meteorological conditions at oil spill area characteristics is formed as follows.

First, we have to draw a randomly generated number from the uniform distribution on the interval $(0,1)$. Next, we can select the initial state $k_0$, $i \in \{1,2,...,m\}$, according to (14). Further, we draw another randomly generated number $g$ from the uniform distribution on the interval $(0,1)$. For the fixed $i$, $i \in \{1,2,...,m\}$, we select the next state $k_j$, $j \in \{1,2,...,m\}$, $j \neq i$, according to (15). Subsequently, we draw a randomly generated number $h$ from the uniform distribution on the interval $(0,1)$ for the fixed $i$ and $j$, we generate a realization $t_{ij}$ of the conditional sojourn time $\theta_{ij}$ from a given probability distribution, according to (16). Then, we compare the realization $t_{ij}$ of the conditional sojourn time with the experiment time $T$. If the realization $t_{ij}$ of the conditional sojourn time is less than the experiment time $T$, we write the sequence of domains

$$D_{t_{ij}, s_{i-s_{i-1}}} (b_i\Delta t),$$

for $b = 1,2,...,s_i - s_{i-1}$, $k \in \{1,2,...,m\}$, $i = 1,2,...,n$, using formula (13).

As the realisation $t_{ij}$ is the first one, we put $v = 1$ and consequently

$$\tau = t_{ij}^{(1)}.$$

Further, we substitute $i := j$ and repeat drawing another randomly generated numbers $g$ and $h$ (selecting the states $k_i$ and generating another realization $t_{ij}^{(v)}$, $v = 2$, of the conditional sojourn time. Having the realizations $t_{ij}^{(v)}$, $ij \in \{1,2,...,m\}$, $i \neq j$, $v = 1,2,$ of the process $A(t)$, we calculate the entire sojourn time $\tau$, $n = 1,2,...$, applying the formula (17), i.e. we have

$$\tau = t_{ij}^{(1)} + t_{ij}^{(2)}.$$

Further, we compare it with time $T$. If the sum $\tau$ is less than the experiment time $T$, we draw the sequence of domains using formula (13).

We repeat the procedure above until the sum $\tau$ of all generated realizations $t_{ij}^{(v)}$, $i,v = 1,2,...,n$, reach a fixed experiment time $T$. Consequently, we calculate the entire sojourn time $\tau$, according to (17) and draw the sequence of domains using formula (13).

Finally, we put together all the sequences of domains drawn before and we get the oil spill domain movement (Figure 2). In the interval $(0,\tau)$ the number of ellipses is $s_1 - s_0 = s_i$ in the next intervals $(\tau - \tau, \tau), \tau, \tau, \tau, (\tau - \tau, \tau), \tau$ the number of ellipses are respectively $s_2 - s_1, s_3 - s_2, s_n - s_{n-1}$, where $s_n, i = 1,2,...,n$, are defined by (10).

The general Monte Carlo simulation flowchart for generating and determination of a process of changing hydro-meteorological conditions at oil spill area is illustrated in Figure 3.

4.3 Monte Carlo simulation prediction of the oil spill domain in varying hydro-meteorological conditions

Using the procedures of the process of changing hydro-meteorological conditions at oil spill area prediction described in Sections 4.1-4.2 and the modified method of the domain of oil spill determination presented in Section 4.3 in (Dąbrowska & Kołowrocki 2019B) the Monte Carlo simulation oil spill domain prediction can be done.

The modified method of the domain of oil spill determination presented in Section 4.3 in (Dąbrowska & Kołowrocki 2019B) depends on changing the procedure (4)-(12) by replacing the conditions (10)-(12) by conditions:

The $s_i$, $i = 1,2,...,n$, existing in (4)-(9), according to (10), are such that

$$(s_i - 1)\Delta t < \sum_{j=1}^{v-1} t_{ij}^{v-1} = s_i\Delta t, i = 1,2,...,n, s_i\Delta t \leq T, \quad (18)$$

and

$$t_{ij}^{v_{max}}, j = 1,2,...,n - 1, \quad (19)$$

are the realizations of the process $A(t)$, $t \in <0,T>$, conditional sojourn times

$$\theta_{ij}^{v_{max}}, j = 1,2,...,n - 1$$

at the states $k_0$ upon the next state is $k_{v_{max}}, j = 1,2,...,n - 1,$ $k_0$, $k_{v_{max}} \in \{1,2,...,m\}$, $j = 1,2,...,n - 1$, defined in Section 4.1.
5 CONCLUSIONS

The proposed Monte Carlo simulation approach allows us for the determination of oil spill domains at port and sea water areas in changing hydro-meteorological conditions. It is a new supplementary method to the probabilistic methods of the oil spill domains determination in the varying hydro-meteorological states presented in (Dąbrowska & Kołowrocki 2019A, 2019B), (Kim et.al. 2013) and (Chen, Li & Li 2007). The comparison of results of these methods’ applications in real conditions should lead to the selection one of them with the best accuracy and develop in the future research.

REFERENCES


Kuligowska, E. 2018. Monte Carlo simulation of climate-weather change process at maritime ferry operating area, Technical Sciences, University of Warmia and Mazury in Olsztyn, 1(21), 5-17.


