

Modeling of Ship Collision Risk Index Based on Complex Plane and Its Realization

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ABSTRACT: Ship collision risk index is the basic and important concept in the domain of ship collision avoidance. In this paper, the advantages and deficiencies of the various calculation methods of ship collision risk index are pointed out. Then the ship collision risk model based on complex plane, which can well make up for the deficiencies of the widely-used evaluation model proposed by Kearon.J and Liu ruru is proposed. On this basis, the calculation method of collision risk index under the encountering situation of multi-ships is constructed, then the three-dimensional image and spatial curve of the risk index are figured out. Finally, single chip microcomputer is used to realize the model. And attaching this single chip microcomputer to ARPA is helpful to the decision-making of the marine navigators.

1 INTRODUCTION

Ship collision refers to the physical event that two or more ships occupy the same point or area on the ocean surface at the same time, it is the overall embodiment of the temporal and spatial effects of the marine traffic. And ship collision risk index, which is the elementary and important concept related with ships, is the researching focus of the marine traffic field and is the key technology for the realization of ship automation (Chen, 2011). To improve the navigation security level and realize the economic and green navigation, since the end of 1970s, researchers have been researching the automatic collision avoidance system and have proposed a mass of related calculation methods. However, there always have been divergences on the calculation of collision risk index in the encountering situation. In this paper, the model of the ship collision risk index space (generalized space) based on the unification of temporal and spatial effects is proposed, which can quickly realize the risk degree calculation in the

encountering situation of two or more ships, and solve the time-series causal relationships in the two-dimensional temporal and spatial space. And using the collision risk index on the two-dimensional temporal and spatial space, ship trajectories can be tracked, which has obvious physical significance.

2 REVIEW OF THE RESEARCHING METHODS OF SHIP COLLISION RISK INDEX

There are several quantitative calculation methods of ship collision risk index, which are as follows:

2.1 *Macroscopic ship collision risk index*

According to the theory of marine traffic engineering, by analyzing the ship encountering probability and historical collision incidents within certain time period in specific water area, the ship collision

probability in the researched area is determined and the collision risk index of ships navigating in the water area is figured out. (Wang, 1998)

2.2 DCPA and TCPA

With the development of computer technology and the application of ARPA, the two parameters of DCPA (Distance to Closest Point of Approach) and TCPA (Time to Closest Point of Approach) can be easily acquired. (Zhao et al., 1999)

2.2.1 Direct weighting method of DCPA and TCPA

In 1977 foreign scholar Kearon.J firstly proposed the evaluation criteria and evaluation model of ship collision risk in the international seminar of mathematic problems in international marine navigation (Kearon, 1979). In this model, the ship collision risk index is calculated as: $\rho = (aDCPA)^2 + (bTCPA)^2$, a 、 b are the weighting coefficients of DCPA and TCPA, when the coming ship is from the starboard, then $a = 5$ 、 $b = 0.5$; when the coming ship is from the portside, then $a = 5$ 、 $b = 1$. ρ represents the collision risk index in the encountering situation of the own-ship and target ship, and smaller ρ is, it will be more dangerous.

DCPA and TCPA of the target ship can be acquired using current ARPA, and caution circle of the own-ship can be established by setting the secure DCPA (called DCPAs for short) and secure TCPA (called TCPAs for short). However, determining the danger alarm only by dividing the value range of DCPA and TCPA has great limitations. In the evaluation model of ship collision risk index proposed by Kearon.J, the collision risk index is quantitatively calculated as: $\rho = (aDCPA)^2 + (bTCPA)^2$, but the model has severe deficiencies, even mistakes, which are as follows:

- 1 The most severe deficiency of the model is the dimension disunity of variables. DCPA is the dimension of length, TCPA is the dimension of time, however, weighting summation of them only involves the value, without consideration of the dimensions, which are in accordance with the actual situations.
- 2 The determination of the values of a 、 b only considered which side the coming ship is from, however, once the direction of the target ship is determined, the value of a 、 b is determined. Therefore, different weighting relationships of TCPA and DCPA, that is, the contribution of TCPA and DCPA to the value of ρ , under different conditions of the same direction can't be reflected
- 3 The evaluation criteria is that the smaller ρ , the more dangerous, which will easily cause confusion. Because it is usually considered that the larger the risk value is, the more dangerous will be.
- 4 Wrong conclusions will be acquired by this model in some situations. Supposing that for ship A, its DCPA is zero n mile and TCPA is 11min; for ship B, its DCPA is 2 n mile and TCPA is 1 min (Zheng, 2010). And for both ship A and B, $a = 5$, $b = 1$. Then according to the evaluation criteria of Kearon, $\rho_A = 121$, $\rho_B = 101$, and $\rho_A > \rho_B$,

which indicates that ship B is more dangerous than ship A. While in the navigation practice, ship A is much more dangerous than ship B. Therefore, this evaluation model has its deficiencies, and mistakes, and needs to be improved.

2.2.2 Weighting of DCPA, TCPA and subjective factors

A new equation was proposed to calculate the collision risk index of the own-ship, which is as follows (Liu and Hu, 2012):

$$\rho_0 = \frac{R_0 - S_{DCPA}}{R_0} \times \frac{T_0 - t_{TCPA}}{T_0} \quad (1)$$

In Eq.1, R_0 is the radius of the safe domain of the own-ship; T_0 depends on the last opportunity of steering rudder to avoid collision.

Using the product of $\frac{R_0 - S_{DCPA}}{R_0}$ and $\frac{T_0 - t_{TCPA}}{T_0}$ to define the collision risk index is not reasonable. For an example, when $R_0 = S_{DCPA}$, $T_0 = t_{TCPA}$, the risk index is zero; when $S_{DCPA} = 0$, $t_{TCPA} = T_0$, the risk index is zero as well, though the values of the risk index are the same, these two conditions are totally different. At the same time, when $S_{DCPA} < R_0$, $t_{TCPA} > T_0$, the risk index ρ_0 is negative; and when $S_{DCPA} > R_0$, $t_{TCPA} < T_0$, the risk index is negative as well, however, both these two cases are wrong. S_{DCPA} reflects the spatial urgency degree, t_{TCPA} reflects the temporal urgency degree, and smaller t_{TCPA} is, the more important this factor will be, therefore, there should be a changing weighting coefficient.

In general, when $DCPA > DCPAs$, or $DCPA > MINDCPA$, the ship is secure, just as the district I in fig.1 shows.

When $DCPA < DCPAs$ (or $DCPA < MINDCPA$) and $TCPA \geq TCPAs$ (or $TCPA \geq MINTCPA$), the ship is in danger, just as the district II in fig.1 shows. At this moment, the ships are in close-quarters situation. When $DCPA < DCPAs$ (or $DCPA < MINDCPA$) and $TCPA < TCPAs$ (or $TCPA < MINTCPA$), just as the district III in fig.1 shows, then the ships are in immediate danger, and actions need to be taken at once by navigators.

When DCPA is in the range of 2-3n miles, and TCPA is in the range of 15-30min, then the changing of collision danger includes three processes, they are security, close-quarters situation and immediate danger. Only by making full preparation in the close-quarters situation, the probability of the occurrence of immediate danger can be effectively reduced.

Multiplying $\frac{R_0 - S_{DCPA}}{R_0}$ with $\frac{T_0 - t_{DCPA}}{T_0}$ can't

fully reflect the two-dimensional temporal and spatial physical properties.

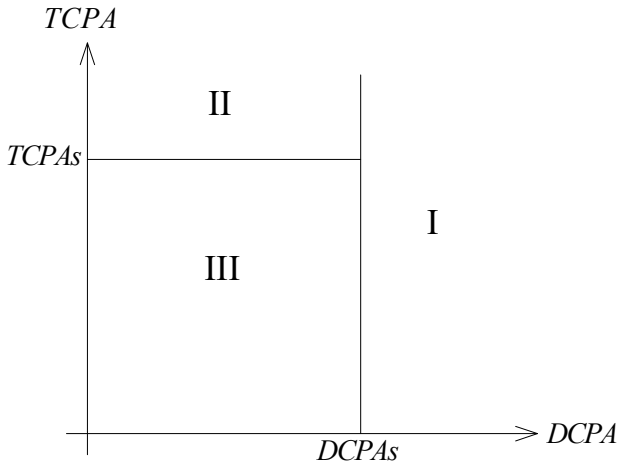


Figure 1. Ships in security, in danger and in immediate danger

2.3 Fuzzy mathematical method

When using fuzzy mathematical method to determine the ship collision risk index, factor set, evaluation set and evaluation indexes needs to be established, and the subordinating degree function of all the parameters in the evaluation set needs to be established. All these should be described using fuzzy language, for the factors considered are not comprehensive enough, and human factors such as the determination of evaluation value and selection of subordinate function will affect the results. (Yan, 2002)

2.4 Method of artificial neural work

This method extensively connects simple artificial neurons to simulate human brain behavior and function. This method proposed that the artificial neural model of ship collision risk index can be established using multi indexes. Usually, three indexes are used, they are S_{DCPA} , t_{TCPA} and whether the coming ship is from the portside or starboard, and when the coming ship is from the portside, then the input value is 0.1; when the coming ship is from the starboard, then the input value is 0.9. However, if the collision risk index model established using this method is inappropriate, then the results are lack of credibility. (Zhou and Wu, 2004)

3 NEW MODEL OF SHIP COLLISION RISK INDEX BASED ON COMPLEX PLANE

3.1 Security vector, security level and ship collision risk degree

DCPA represents the distance and its unit is n mile, TCPA represents the time and its unit is minute. DCPA reflects the spatial effects and urgency degree of collision, and TCPA reflects the temporal effects and urgency degree of collision. Therefore, security

vector S is introduced in the two-dimensional spatial and temporal space, and $S = DCPA + i\eta V_0 TCPA$, i is the imaginary unit, V_0 is the speed, and $V_0 = DCPAs / TCPAs$, it is related with factors such as size, maneuvering performance, loading condition of the ships and the navigating water area and atmospheric condition. η is the weighting coefficient of DCPA and $V_0 TCPA$, it has different values under different conditions of the same direction of the ship coming. And η is in inverse proportion to the relation speed of two ships, that is, larger the relative speed is, smaller η will be, then the security vector S will be smaller and risk degree will be higher. At this time, the weighting coefficient of temporal effect to spatial effect will increase, which can make up for the deficiency of the evaluation criteria proposed by Kearon that the weighting coefficient is invariable.

The module of security vector is defined as:

$$s = \sqrt{DCPA^2 + (\eta V_0 \times TCPA)^2} / \sqrt{DCPA_s^2 + (\eta V_0 \times TCPA_s)^2} \quad (2)$$

The collision risk degree is defined as:

$$d = 1/s = \sqrt{DCPA_s^2 + (\eta V_0 \times TCPA_s)^2} / \sqrt{DCPA^2 + (\eta V_0 \times TCPA)^2} \quad (3)$$

We can see from the definition that smaller the security coefficient S is, larger d will be, and it will be more dangerous, which corrected the cognitive trap of the evaluation criteria proposed by Kearon that smaller ρ is, it will be more dangerous.

3.2 Ship collision risk model based on complex plane

DCPA and TCPA of the target ship are acquired using the ARPA of the own ship, then the security vector of the target ship can be represented as:

$S = DCPA + i\eta V_0 \times TCPA$. η is the weighting

coefficient between DCPA and $V_0 TCPA$, and it is related with the relative speed of two ships. η also varies with the encounter situation, when the ship is coming from the starboard,

then $\eta = 0.5 \times \frac{TCPA - TCPAs}{TCPAs}$; and when the ship is

coming from the portside, then

$\eta = 0.55 \times \frac{TCPA - TCPAs}{TCPAs}$. Generally speaking, the

condition when the ship is coming from the starboard is more dangerous than the condition when the ship is coming from the portside. Therefore, when the ship is coming from the portside, η will increase, then the security vector will increase and it will be less dangerous.

Ship collision risk index is defined as:

$$d = 1/s = \sqrt{DCPAs^2 + (\eta V_0 \times TCPAs)^2} / \sqrt{DCPA^2 + (\eta V_0 \times TCPA)^2} \quad (4)$$

In Eq.4, s is the security degree, η is the weighting coefficient between DCPA and $V_0 \times TCPA$, and $V_0 = DCPAs / TCPAs$, $DCPAs$ is the secure approaching distance, $TCPAs$ is the secure approaching time. When $TCPA < TCPAs$ and $DCPA < DCPAs$, it is under urgent risk condition, which is not discussed here.

4 COMPARISON OF TWO EVALUATION CRITERIA AND MODELS

4.1 Comparison of Kearon model and the new evaluation model

For ship A, DCPA=0.1 n mile, TCPA=11min; for ship B, DCPA=2.1 n mile, TCPA=1min. Let $a=5$, $b=1$, $\rho = (aDCPA)^2 + (bTCPA)^2$, and smaller ρ is, it will be more dangerous, then $\rho_A = 121.25$, $\rho_B = 111.25$, and $\rho_B < \rho_A$, therefore, risk degree between ship B and the own ship is larger than that between ship A and the own ship. While in fact, the risk degree of ship A to the own ship is higher. According to the new evaluation criteria and model, the following equation can be acquired:

$$S = DCPA + i\eta V_0 \times TCPA = DCPA + i(\eta * DCPAs / TCPAs) \times TCPA \quad (5)$$

According to Eq.5, let $DCPAs=2$ n mile, $TCPAs = 15$ min, Then $S_A = 0.05$, $S_B = 1.05$, $d_A = 20$, $d_B = 0.9524$, $d_A > d_B$, larger d is, it will be more dangerous, therefore, the risk degree of ship A to the own-ship is larger than that of ship B, which is in accordance with the fact.

4.2 Comparison of these two modes under different conditions

- 1 When $DCPA_A = DCPA_B$ and $TCPA_A < TCPA_B$, larger TCPA is, it will be more secure, and the results of these two evaluation models are in accordance with the fact.
- 2 When $TCPA_A = TCPA_B$ and $DCPA_A < DCPA_B$, larger DCPA is, it will be more secure, and the results of these two evaluation models are in accordance with the fact.
- 3 When $DCPA_A < DCPA_B$ and $TCPA_A < TCPA_B$, larger TCPA and DCPA are, it will be more secure, and the results of these two evaluation models are in accordance with the fact.
- 4 The condition is different when the DCPA of one ship is smaller than that of the other ship, while TCPA of it is larger than that of the other ship. Table1 shows the countering condition of ship A, B and the own-ship.

Table 1. Countering condition of ship A, B and the own-ship

	Ship A	Ship B
DCPA	0.5nmile	1.5nmile
TCPA	18min	15min

The risk degree of which ship is higher cannot be directly judged under this condition, therefore it must be realized by computer, using JAVA to compare the risk degree of one ship whose DCPA is larger and

TCPA is smaller and another ship whose DCPA is smaller and TCPA is larger.

The calculated results of the evaluation model proposed by Kearon is $\rho_A = 330.25$, $\rho_B = 281.25$; the calculated results of the evaluation model proposed in this paper is $d_A = 2$, $d_B = 0.67$. According to the traditional model proposed by Kearon, $\rho_A > \rho_B$, therefore, ship B is more dangerous than ship A; while according to the new evaluation criteria, $d_A > d_B$, so ship A is more dangerous than ship B. The fact is that ship A is more dangerous than ship B, so the evaluation criteria proposed in this paper is more reasonable and effective.

5 DISCUSSION

- 1 The ship collision risk index proposed by Kearon.J is represented as: $\rho = (aDCPA)^2 + (bTCPA)^2$, as this model considers DCPA and TCPA respectively, without establishing the temporal and spatial relationship between them, therefore, the temporal causal relationship and the precedence order cannot be described. The mathematical model proposed in this paper, of which the results are in accordance with the fact, is established on the generalized space of the two-dimensional time and space, and has great physical significance. In fig.2, the equation of the two lines are: $DCPA - DCPA_0 = \pm \eta V_0 (TCPA - TCPA_0)$, and the shade area represents the absolute significance of the generalized space of two-dimensional time and space. Point A represents the earlier moment, point P_0 represents the current moment, and point B represents the later moment. Different η and V_0 are corresponding to two different intersecting lines, then system of intersecting straight lines is generated. To certain η and V_0 , the shade area constructed by their corresponding intersecting lines is call as the causal cone in time series. As fig.2 shows, event A is absolutely earlier than event P_0 , because event A can have an effect on event P_0 ; event B is absolutely later than event P_0 , because event B can be influenced by event P_0 .

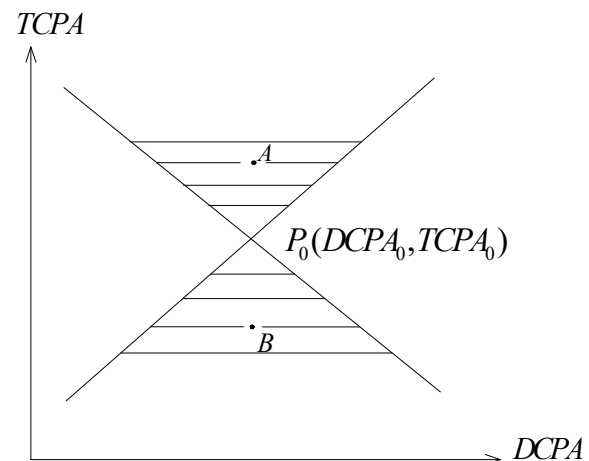


Figure 2. Spatial and temporal precedence order

- 2 The contributions of DCPA and TCPA to risk degree should not be invariable. The traditional evaluation model proposed by Kearon considered that the weighted values are invariable, when the coming ship is from the starboard, then $a=5$, $b=0.5$; when the coming ship is from the portside, then $a=5$, $b=1$, which is not in accordance with the actual situations. The weighting coefficient in the new evaluation model varies with the magnitude relationship of TCPA and TCPAs, which is logical and corresponding to the actual situations.
- 3 In the traditional evaluation model proposed by Kearon, when ρ is a constant, the risk degree curve is an ellipse in the first quadrant. While in the new evaluation model, the risk degree curve has different shapes when the values of η and V_0 are different, which can provide more physical connotation and analysis of the risk index space.
- 4 Calculation of risk index under the collision avoidance situation of multi-ships

Assuming that the encountering situation involves 6 ships, and DCPA and TCPA of them at the same moment are measured by ARPA, the results are showed in table 2.

Table 2. DCPA and TCPA of different ships

	Ship 1	Ship 2	Ship 3	Ship 4	Ship 5	Ship 6
DCPA (n mile)	1.6	1.7	1.7	1.6	1.4	1.5
TCPA (min)	24	28	18	19	29	22

Using JAVA to calculate the collision risk degree, the results are as follows: $d_1=0.5802$, $d_2=0.4663$, $d_3=0.5882$, $d_4=0.6173$, $d_5=0.5135$, $d_6=0.6542$, and the sequencing result is $d_6 > d_4 > d_3 > d_1 > d_5 > d_2$, which shows that the ship 6 is the most dangerous one, and the ship 2 is the most secure one.

This function is further realized in single chip microcomputer (STM32F107) and extended TFT display screen. If it is installed to ARPA, the function of APRA will be greatly improved. Under the encountering condition of multi-ships, the quantitative value of the collision risk index between the own-ship and other ships can be acquired using the calculation model established in this paper, which can help navigators to have a more accurate and deeper understanding of the encountering situation in advance, and has great significance to the collision avoidance and ship security. Different weighting coefficients are given under different situations, the weighting coefficient under the head-on situation is the largest, that under the crossing encounter situation is the second largest, and that under the overtaking situation is the smallest. It can be predicted that this model will have promising application is the intelligent collision avoidance system, and can be extensively applied to the civilian ships, military aircraft, naval vessel, and etc.

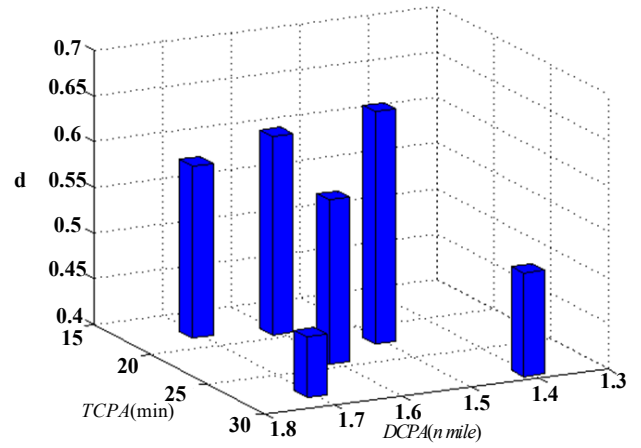


Figure 3. Three-dimensional image of the risk degree of ship 1 to 6

Fig.4 shows that when TCPA is in the range of 25 to 30 minutes, the increasing of collision risk degree is slow; when TCPA is in the range of 20 to 25 minutes, the increasing of collision risk degree is fast; and when TCPA is in the range of 15 to 20 minutes, the collision risk degree increases rapidly. When $TCPA = TCPAs$, the collision risk degree is the largest; when $TCPA < TCPAs$, then the ship is in immediate danger, as district III in fig.1 shows.

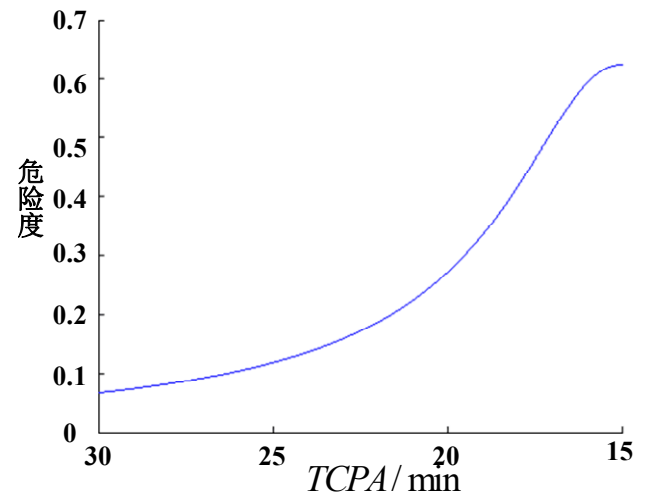


Figure 4. Variation curve of risk degree of ship 1 with the risk degree

6 CONCLUSIONS

The evaluation model proposed by Kearon, Liu ruru and et al. has great limitations and deficiencies, even wrong conclusions. In this paper, $i\eta V_0 TCPA$ is taken as a coordinate of the space of ship collision risk index, and security vector is introduced in the two-dimensional spatial and temporal space, which can be expressed in the complex plane as: $S = DCPA + i\eta V_0 TCPA$. In this new model, the reasonable part of the evaluation model proposed by Kearon is maintained, and the unreasonable part, even wrong part is corrected. As the same time, it has effectively made up for the limitations of current ARPA that only DCPA is considered when giving the collision risk alarm, and the quantitative value of

collision risk index can't be acquired. In the new evaluation model of collision risk index, single chip microcomputer is used to calculate the risk index under the encountering condition of multi-ships, and sequencing is made, which can help navigators to better understand the encountering situation and made reasonable decisions of collision avoidance. Therefore, the model proposed in this paper has promising application in the field of marine navigation.

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