

Modeling of Ionospheric Delay for SBAS Using Spherical Harmonics Functions

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ABSTRACT: In SBAS (satellite-based augmentation system), it is important to estimate ionospheric delay accurately to guarantee user's accuracy and integrity. Grid based ionospheric models are generally used to estimate ionospheric delay for SBAS. In grid based model, SBAS broadcasts vertical ionospheric delays at the grid point, and users get their ionospheric delay by interpolating those values. Ionospheric model based on spherical harmonics function is another method to estimate ionospheric delay. This is a function based approach and spherical harmonics function is a 2-D fourier series, containing the product of latitude dependent associated Legendre functions and the sum of the longitude dependent sine and cosine terms. Using ionospheric delay measurements, coefficients for each spherical harmonics functions are estimated. If these coefficients are known, user can reconstruct ionospheric delay. In this paper, we consider the spherical harmonics based model and propose a ionospheric delay estimation strategy for SBAS that can be used to mitigate ionospheric delay estimation error, especially in storm condition. First, coefficients are estimated under initial order and degree. Then residual errors for each measurement are modeled by higher order and degree terms, then coefficients for these terms are estimated. Because SBAS message capacity is limited, in normal condition, initial order terms are only used to estimate ionospheric delay. If ionospheric storm is detected and there is need to mitigate the error, higher order terms are also used and error can be decreased. To compare the accuracy of spherical harmonics based model with grid based model, some post-processing test results are presented. Raw observation data is obtained from RINEX format and the root mean square(RMS) and max value of residual errors are presented.

1 INTRODUCTION

The ionosphere is the major error source that affects positioning accuracy of GPS users. GPS satellites broadcast the Klobuchar parameters to correct the ionospheric delay. But its accuracy is not enough to meet the required navigation performance in many cases. Many countries have been developed augmentation systems such as GBAS, SBAS to achieve higher navigation performance. In SBAS(satellite-based augmentation system), like WAAS, they estimate local ionospheric delays more accurately

using spare network of reference stations and provide estimated result to users. To estimate the local ionospheric delays, grid-based models are generally used. Grid-based model estimates the ionospheric delays at the pre-defined grid points and provide the values to user. Then users get ionospheric delays by interpolating received values close to their ionospheric pierce point. This grid model usually works well. But, in ionospheric storm condition, the ionospheric spatial gradient becomes higher and local ionospheric delays have irregularity structures. Because of its interpolating method, grid-based

models have problem estimating those irregularity structures and modeling errors are increased.

Spherical harmonics model is another option for modeling ionospheric delay. In contrast to grid-based model, Spherical harmonics model is analytic model[1]. Spherical harmonics are used in many theoretical and practical applications such as gravitational fields modeling and geoids modeling. There are some researches that use spherical harmonics for ionosphere modeling [1, 2, 5, 6]. In this paper, we carried out post-processing test to compare the performance of spherical harmonics model and grid model. In this test, we only conducted the accuracy performance test.

2 MEASUREMENT OUTLIER DETECTION

SBAS collects dual-frequency GPS observations from reference stations. Sometimes, observed data have outliers and these outliers affect the modeling performance. To improve the performance, it must be detected and removed. In this paper, poly nominal fit method is used to detect the outliers [3]. For each satellite, previous 30 epoch data are fitted in second order. Residuals are calculated from fitted value and current epoch data. If calculated residuals exceed threshold, these data are classified as outlier. Figure1 shows raw vertical ionospheric delay of PRN 16 satellite and figure 2 shows vertical ionospheric delay after removing outliers.

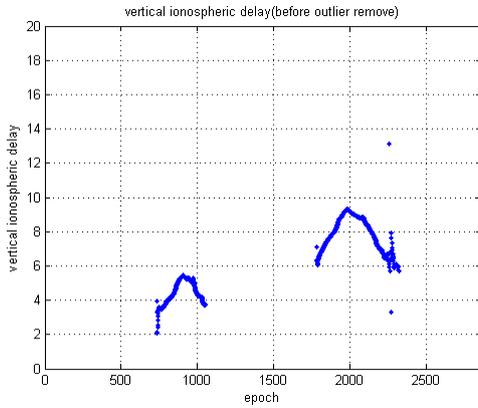


Figure 1. vertical ionospheric delay of PRN 16 satellite (before removing outlier)

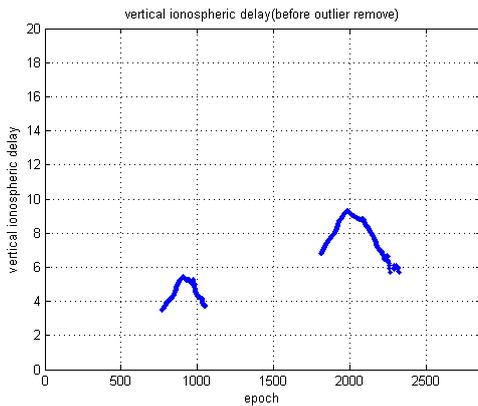


Figure 2. vertical ionospheric delay of PRN 16 satellite (after removing outlier)

3 IONOSPHERIC DELAY MODELING WITH SPHERICAL HARMONICS FUNCTION

3.1 Spherical harmonics function

Spherical harmonics function is based on the Fourier series expansion in longitude and Legendre function in latitude as the orthogonal basis functions[2]. The mathematical expression of vertical ionospheric delay using spherical harmonics is as follows.

$$I_v(f, \lambda) = \sum_{n=0}^n \sum_{m=0}^m \bar{P}_{nm}(\sin f)(C_{nm} \cos(m\lambda) + S_{nm} \sin(m\lambda)) \quad (1)$$

ϕ is geomagnetic latitude of an IPP

λ is geomagnetic longitude of an IPP

n, m are integer degree and order of Legendre function

C_{nm}, S_{nm} are unknown spherical harmonics coefficients

\bar{P}_{nm} are normalized associated Legendre functions

Like the grid model, this approach also assumes that electron density is concentrated in a very thin shell at 350km altitude. The degree and order of the Legendre function defines the resolution of the model. Using above equation, local ionospheric delays can be represented by spherical harmonics coefficients.

3.2 Coefficient estimation using least square method

To model the local ionospheric delay with spherical harmonics functions, vertical ionospheric delay measurements are collected. Then the least square method is used to estimate the unknown coefficients. The system can be represented as these matrices.

$$z = Hx \quad (2)$$

z is vertical ionospheric delay measurement

H is geometry matrix

x is unknown coefficients

$$H = \begin{bmatrix} P_{00}(\sin \phi_1) \cdot \cos(0 \cdot \lambda_1) & P_{00}(\sin \phi_1) \cdot \sin(0 \cdot \lambda_1) & \cdots & P_{nm}(\sin \phi_1) \cdot \cos(m \cdot \lambda_1) & P_{nm}(\sin \phi_1) \cdot \sin(m \cdot \lambda_1) \\ P_{00}(\sin \phi_2) \cdot \cos(0 \cdot \lambda_2) & P_{00}(\sin \phi_2) \cdot \sin(0 \cdot \lambda_2) & \cdots & P_{nm}(\sin \phi_2) \cdot \cos(m \cdot \lambda_2) & P_{nm}(\sin \phi_2) \cdot \sin(m \cdot \lambda_2) \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ P_{00}(\sin \phi_n) \cdot \cos(0 \cdot \lambda_n) & P_{00}(\sin \phi_n) \cdot \sin(0 \cdot \lambda_n) & \cdots & P_{nm}(\sin \phi_n) \cdot \cos(m \cdot \lambda_n) & P_{nm}(\sin \phi_n) \cdot \sin(m \cdot \lambda_n) \end{bmatrix}$$

$$x = [C_{00} \quad S_{00} \quad \cdots \quad C_{nm} \quad S_{nm}]^T$$

In general, this equation is over determined and can be solved. The unknown coefficients are computed as follows

$$x = (H^T H)^{-1} H^T z \quad (3)$$

Estimated coefficients will be broadcasted to user, then user can reconstruct the local ionospheric delay using these coefficients.

4 TEST RESULTS AND DISCUSSION

4.1 Data

GPS RINEX format data taken from 32 stations are considered for analysis. These data were obtained via the ftp servers of CORS (ftp://www.ngs.noaa.gov/cors/rinex). To investigate the performance in ionospheric storm condition, we select the data observed in such condition. On October 29th, 2003, there is a severe ionospheric storm[4] and we used these data. For example, figure 3 shows the vertical ionospheric delays on that day. In this figure, 2-D cubic poly nominal function in matlab is used to represent the raw measurements. The maximum vertical delay reach 30 meters and spatial gradient is very high in some region.

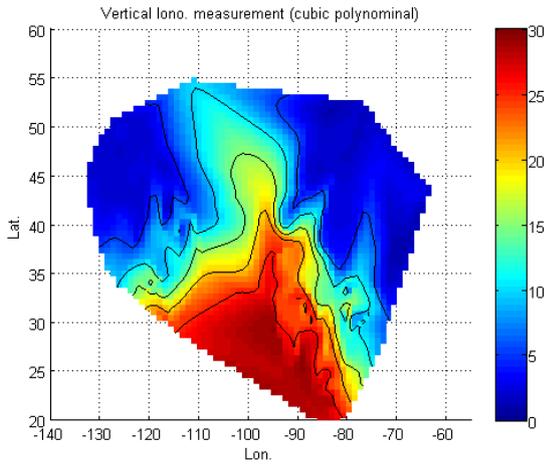


Figure 3. Vertical Ionospheric delay on October 29th, 2003. (GPS time 334320, 2-D cubic polynomial)

4.2 Methodology

At each epoch, vertical ionospheric delays are estimated using grid model, and spherical harmonics model. Three methods are implemented, one is grid model and others are spherical model.

Method 1 just used Grid model and 64 grid points are used in this method.

Method 2 used spherical harmonics model. When the spherical harmonics model is used, the total number of coefficients will be fixed depending on pre-determined order and degree. Because message capacity is limited, it is important to determine proper number of parameters considering both accuracy and message capacity. In this method , both order and degree are set to 5 and total coefficients are 36.

Method 3 used spherical harmonics model with 81 coefficients(both order and degree are set to 8) to estimate the ionospheric delay more accurately in severe condition. In contrast to Method 2, estimation process is divided into two steps in this method. First step is repetition of Method 2. In second step, the residuals are calculated from the measurements and estimated value. Then higher order and degree coefficients are estimated from these residuals. At this time, the geometry matrix is same as in first step and equations are as follows.

$$r = z - Hx_L \quad (4)$$

$$= Hx_H$$

$$x_H = (H^T H)^{-1} H^T r \quad (5)$$

z is vertical ionospheric delay measurement

r is residual

x_L is lower degree and order coefficients

x_H is higher degree and order coefficients

Rather than estimating all coefficients at once, we proposed this method to provide correction information to user more faster maintaining accuracy performance. When this method is used to estimate spherical harmonics coefficients, users can roughly compute ionospheric delay after they only receive first 36 coefficients. After receiving all coefficients, user can improve the accuracy. In this method, our proposed broadcasting strategy is as follows. Check the residuals and if the accuracy of the first step is enough, broadcast the only first 36 coefficients to user. If residual errors are grow and there is a need to estimate the ionospheric delay more accurately, higher order and degree terms will also be broadcasted to user.

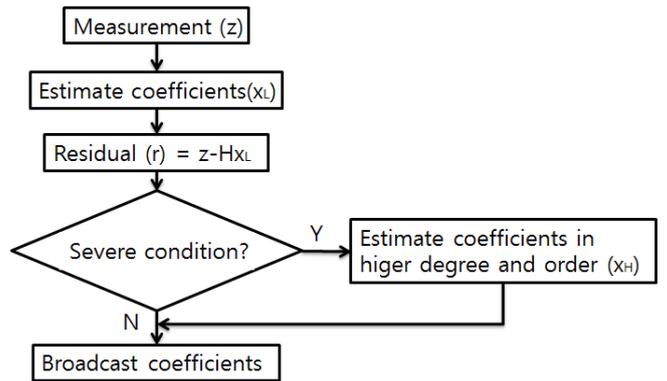


Figure 4. Diagram for method 3

Method	Model	Number of Parameters
1	Grid	64
2	SHF	36
3	SHF	81

Figure 5, 6, 7 respectively shows the estimated vertical ionospheric delays for each method.

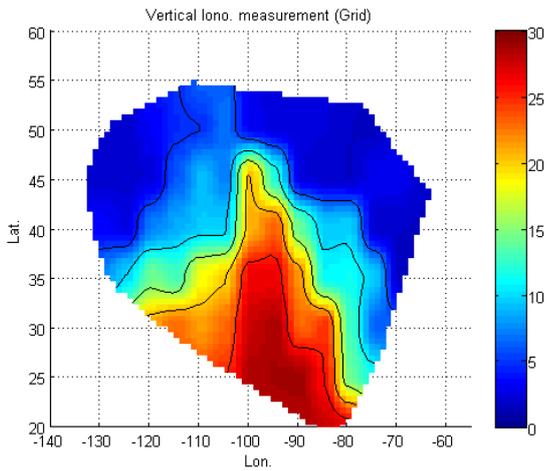


Figure 5. Estimated Vertical Ionospheric delay'' (GPS time 334320, Method 1)

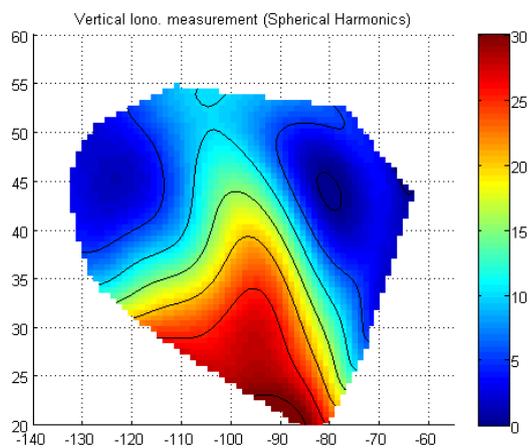


Figure 6. Estimated Vertical Ionospheric delay (GPS time 334320, Method 2)

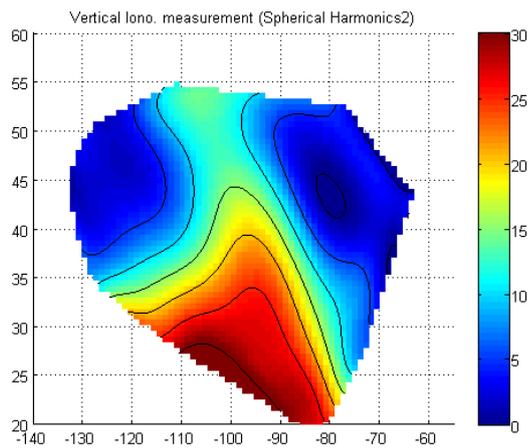


Figure 7. Estimated Vertical Ionospheric delay (GPS time 334320, Method 3)

4.3 Accuracy performance test

To compare the accuracy performance, residuals are calculated for every measurement. Then maximum value and root mean square(rms) value of residuals are computed for each epoch. This test is conducted

for five hours. In this interval, ionosphere condition gets worse as time flows.

Figure 8 and figure 9 shows the time history of maximum and rms value of residual. When the spherical harmonics model is used rather than grid model, accuracy performance is improved. Especially, method 2 shows slightly better performance compared with method 1, even though the total number of parameters used in method 2 is about a half of method 1. The average improvement is 1.42 meters in maximum value and 0.24 meters in rms value for method 2, 1.60 meters in maximum value and 0.31 meters in rms value for method 3. The improvement is more noticeable in latter part of the figure corresponding to severe ionosphere condition.

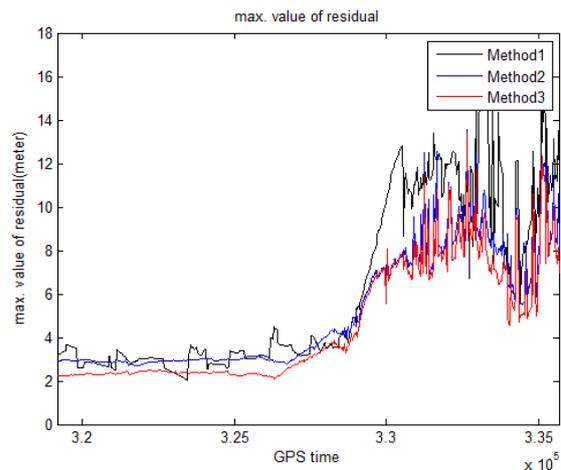


Figure 8. Time history of maximum value

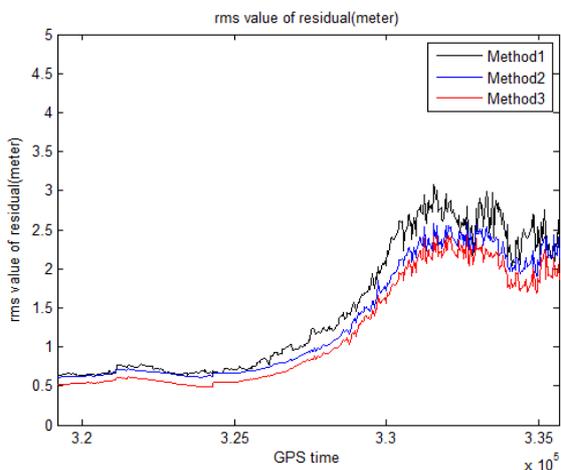


Figure 9. Time history of rms value

5 CONCLUSIONS

In this paper, we modeled the local ionospheric delay using spherical harmonics functions. In ionospheric storm condition, the vertical ionospheric delays are collected from 32 stations. Then outlier detection and removal is conducted on these measurements. Unknown spherical harmonics coefficients are estimated by least square method. Using these coefficients, residuals for every measurement are estimated to test the accuracy performance. These

results are compared with the estimated residuals of grid model. Spherical harmonics model shows better accuracy performance with lower number of parameters especially in more severe condition. We also proposed conceptual estimation and broadcasting strategy that can be used to provide correction information more faster when we use spherical harmonics model for SBAS.

ACKNOWLEDGEMENT

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