Model of Interaction of Water and Tank's Structure in Sloshing Phenomenon

P. Krata
Gdynia Maritime University, Gdynia, Poland

ABSTRACT: The paper presents the model of interaction between ship's tank structure and water contained in the tank. The experimental investigation of sloshing loads in ships' tank is characterized and the need for description of mutual influence is underlined. The model of interaction is made up for rectangular ship's tanks and it is assumed to be in time-domain and nonlinear. The solution is based on the data obtained by the experimental research and numerical simulation. The study may be the contribution to more sophisticated estimation of ship's stability than it takes place nowadays.

1 INTRODUCTION

The dynamic behavior of the vessel at the sea is greatly affected by the dynamics of moving masses existing onboard. The cargo securing procedures ensure avoiding the loose cargo moving, but the liquids contained in partly filled tanks cannot be avoided at all. The modeling of interaction between water sloshing inside the ship's tank and the tank's structure is very important with regards to the safety of transportation system, human's life and environment. The sloshing loads should be also taken into consideration in the process of designing of tank's structure and ship's hull structure. Regardless the strength calculation the effects of sloshing should be also taken into consideration in course of vessel's sea keeping prediction and transverse stability assessment.

2 EQUATIONS OF SHIP'S MOVEMENT

The most precise analysis of ship's movements should be based on the differential equations of movement. The most convenient attitude towards this task is to make the assumption of symmetrical mass distribution and steady values of moments of mass inertia. These equations referred to the center of gravity G are following (Dudziak 1988):

$$\frac{d\overline{P}}{dt} + \overline{\omega} \times \overline{P} = \overline{F}$$

$$\frac{d\overline{K_G}}{dt} + \overline{\omega} \times \overline{K_G} + \overline{V} \times \overline{P} = \overline{M_G}$$

$$(1)$$

where: $\overline{P}$ - momentum of the ship and added masses [kg·m/s]; $\overline{K_G}$ - angular momentum of the ship and added masses referred to the center of gravity G [kg·m²/s]; $\overline{V}$ - velocity of point G [m/s]; $\overline{\omega}$ - ship's angular velocity [1/s]; $\overline{F}$ - resultant external force [N]; $\overline{M_G}$ - resultant moment of external forces about center of gravity G [N·m].

When formula (1) is divided into components and momentum and angular momentum is derived, then movement complex has the form of six differential equations (Dudziak 1988):

$$m\frac{dV_x}{dt} + m(\omega_y V_z - \omega_z V_y) = F_x$$

$$m\frac{dV_y}{dt} + m(\omega_z V_x - \omega_x V_z) = F_y$$

$$m\frac{dV_z}{dt} + m(\omega_x V_y - \omega_y V_x) = F_z$$

$$I_{xx}\frac{d\omega_x}{dt} - I_{zx}\frac{d\omega_z}{dt} + \omega_y\omega_z(I_{zz} - I_{yy}) - \omega_x\omega_y I_{zx} = M_x \qquad (2)$$

$$I_{yy}\frac{d\omega_y}{dt} + \omega_z\omega_x(I_{zz} - I_{xx}) - (\omega_z^2 - \omega_x^2)I_{zx} = M_y$$

$$I_{zz}\frac{d\omega_z}{dt} - I_{zx}\frac{d\omega_x}{dt} + \omega_x\omega_y(I_{yy} - I_{xx}) + \omega_y\omega_z I_{zx} = M_z$$

where: I_{ii} - moment of mass inertia about appropriate axes (i = x, y, z) [kg·m²]; I_{ij} - moment of mass deviation about appropriate axes (i ≠ j) [kg·m²]; m – mass of the ship and added masses [kg]; and the rest of the symbols like in formula (1).

The solution of such general formulated movement's equations is impossible at present state of the art. By neglected coupling, for the sake of simplicity, the ship's rolling is usually analyzed by the single degree-of-freedom system. The governing differential equation of motion, as the result of equilibrium of moments, is (Senjanovic I. & Parunov J. & Cipric G. 1997):

$$I\ddot{\varphi} + D(\dot{\varphi}) + R(\varphi) = M(t) \qquad (3)$$

where: I - moment of inertia of ship and added masses [kg·m²]; D - damping moment [N·m]; R - restoring moment [N·m]; M - excitation moment [N·m]; φ - angle of heel [rad]; $\dot{\varphi}$ - angular velocity of heel [1/s]; $\ddot{\varphi}$ - angular acceleration [1/s²].

The resultant excitation moment $M(t)$ consists of as many components, as many influences swing the ship. The main components are waves and wind. Anyway when the prediction of ship rolling should comprise the effects of water sloshing in partly filled tanks, then the moment of water-ship interaction have to be included as a component of $M(t)$ in formula (3). The moment due to sloshing water should be obtained in time-domain.

3 CHARACTERISTIC OF SLOSHING PHENOMENON

Liquid sloshing phenomenon is the result of partly filled tank motions. As the tank moves, it supplies energy to induce and sustain the fluid motion (Akyildiz & Unal 2005). Both: liquid motion and its effects are called sloshing (Warmowska 2005).

The effect of water motion inside the tank is the pressure on the tank's structure. The hypothetical pressure distribution on the side wall of rectangular tank is shown in figure 1. The dimensions of the wall are y by z.

The interaction between ship's and tank's structure and the water sloshing inside the tank consists in the permanent transmission of the energy. As the ship rolls, the walls of the partly filled tank induce the movement of water. Then the water press against the opposite situated tank's walls and return the energy to the ship, taking simultaneously the next portion enabling the counter-direction movement. The mass and the energy are conserved within the cycle.
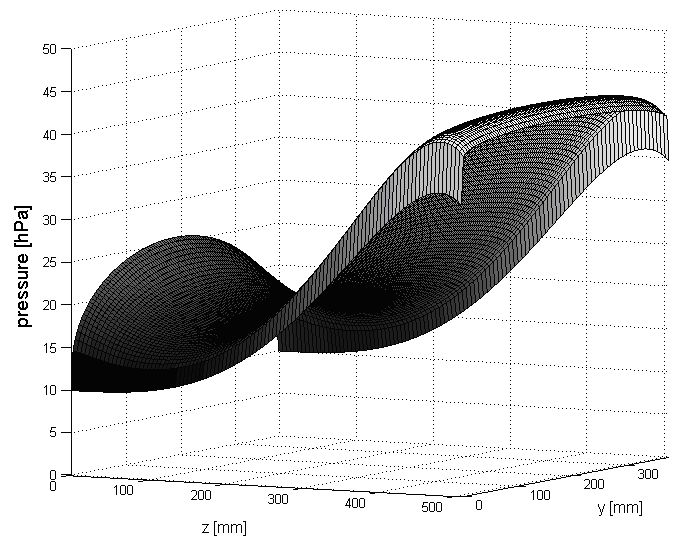

Fig. 1. Pressure distribution on the side wall of rectangular tank (source (Krata 2007))

If the influence of sloshing water dynamics is to be taken into consideration when ship's sea keeping predicted, then the moment of sloshing loads have to be obtained and put into formula (3) as a component of $M(t)$. The calculation of time-domain history of heeling moment due to sloshing is based on the dynamic pressure distribution and its integration. The pressure distributions have to be determined on both side walls of the considered tank, on its roof and the bottom. The assumption of simplified formula (3) is getting rid of any coupling of movements, so the time changing pressure distributions on fore and aft tank's wall are not required.

4 PRESSURE DISTRIBUTION DUE TO SLOSHING IN SHIP'S TANK

4.1 „Tank" program - assumptions and simulation

The computer program "Tank", used for estimation of dynamic pressure distribution due to sloshing, is developed in Polish Register of Shipping. The computation algorithm is based on the Euler equation (Jankowski & Warmowska 1997). The sloshing problem was described by two-dimensional model (Warmowska & Jankowski 2005). It was also assumed that the liquid is non-viscid, incompressible, of constant density. The additional assumption refers to the liquid boundary. It is assumed that (Warmowska & Jankowski 2005):

− the liquid particles slide on the free surface and on the wetted surface of the tank walls;
− the particles in the corners are not moving;
− pressure on the liquid free surface is equal to the atmospheric pressure.

All these assumptions allow for application of the potential theory to solve the problem (the flow is irrotational).

The numerical simulation of sloshing phenomenon, made by "Tank" program, was performed for the oscillation and tank's geometry corresponding with the suitable geometric parameters of experimental investigation. The angle amplitudes of tank's oscillations were 18°, 30° and 40° and the height of water level in tank varied from 150 mm to 450 mm. The program allows to compute time history of dynamic pressures in ninety points around the tank's model. The control points are situated along vertical walls, the bottom and the tank's roof.

4.2 Experimental investigations

The experimental investigation on determining the pressure distribution due to sloshing requires the generation of sloshing phenomenon. After that the dynamic pressure time history in selected places is to be measured and recorded. To achieve this, the test apparatus was designed and built (Krata 2006).

The main part of the apparatus is the tank. It is equipped with six pressure transducers and one inclinometer. The tank is forced to oscillating movements that excite the water movement inside it. The dimensions of the model tank are following:

− length - 1040 mm;
− width - 380 mm;
− depth - 505 mm.

The tank is hanged on the shaft by the bearings and forced to the oscillation by the driving mechanism.

The drive mechanism is based on the electric motor, the transmission reducing revolution velocity and the crank mechanism. The view of testing apparatus and localization of dynamic pressure sensors is shown in figure 2.
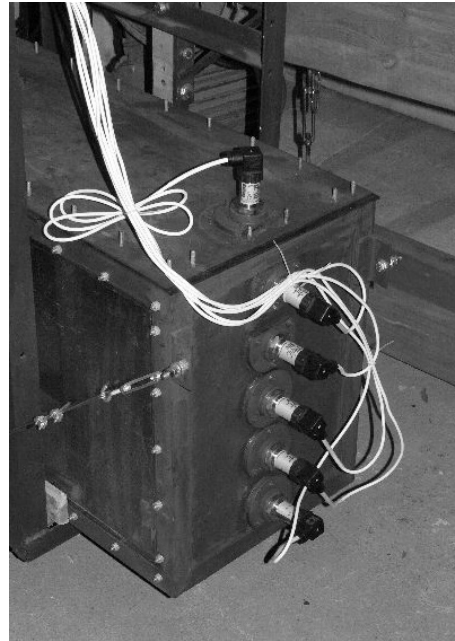

Fig. 2. Picture of the tank and pressure gauges

The oscillating movement which induces the sloshing phenomenon is described fair enough by the harmonic function. The amplitude of tank's rotary motion assumed to be 18°, 30° and 40°. It reflects the heavy seas conditions and enables to make the conclusions for worst possible condition at the sea. The water depth in tank (tank filling level) assumed to vary from 50 mm to 450 mm. The period of the oscillation was equal 2,6 s.

The assumption of plane tank's oscillation and the neglected water viscosity, resulted the two-dimensional character of water flow inside the tank (Warmowska & Jankowski 2005). It allowed to equip the tank with one set of pressure transducers, fixed in the middle line of the tank. The pressure transducers were installed evenly alongside the vertical wall of the tank (5 sensors) and one in the roof of the tank close to the upper corner.

The pressure signal, measured by the transducer, consists of two components. One of them is called non-impulsive dynamic pressure and the second one impulsive pressure or impact pressure (Akyildiz & Unal 2005). The non-impulsive dynamic pressure is slowly varying. It is the result of global movement of liquid in the tank (CTO 1998). The impact pressure is usually short lasting, local and may be of very high value. It is caused by hydraulic jump during the impact stroke of liquid's free surface against the solid surface of the tank construction (wall). The assumption of the experiment was to measure and

record both components of dynamic pressure. The measure appliances have to be fast enough.

All the signals received from the sensors were verified and the measuring instruments were calibrated. All pressure sensors were hydrostatically calibrated and the gain coefficient and shift coefficient were determined. The inclinometer was calibrated by geometric formulas. The calibration procedure allowed to deem the experimental measurements to be correct and reliable (Krata 2006).

The analog signals received from the sensors were sampled and transformed into discrete digital signals by the 12-bit A/D card and then they were recorded in the text format files. The maximum working frequency of measuring device was 1000 Hz. Thus the aliasing distortions of the signal were avoided, because the measuring instruments were much faster than the required Nyquist rate for sloshing phenomenon (Zieliński 2002).

The further digital signal processing was carried out. The main operation was low pass filtering for high frequency noise reduction. The filtering enabled to decompose recorded digital signal an emerged the non-impulsive dynamic pressure component (Zieliński 2002).

The example of pressure distribution estimated on the basis of experimental investigations is shown in figure 3.
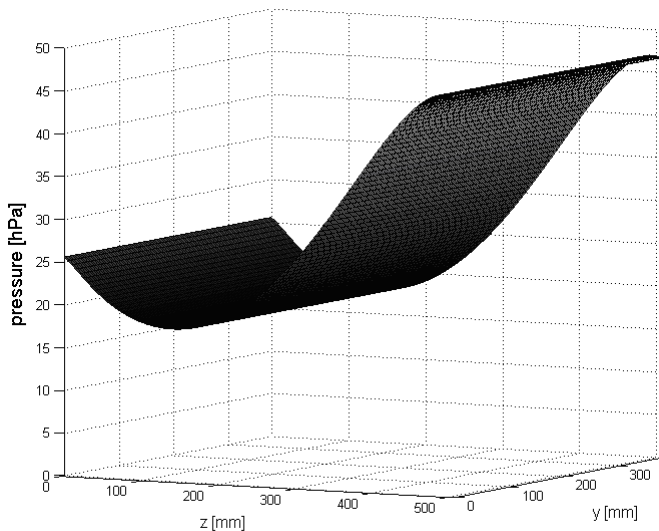

Fig.3. Estimation of pressure distribution on the side wall of model tank (source (Krata 2007))

5 MODEL OF INTERACTION OF SLOSHING WATER AND TANK'S STRUCTURE

5.1 *Assumptions for heeling moment calculations*

The mathematical model of interaction between water sloshing inside the ship's tank and the hull's structure is prepared for rectangular tank. It corresponds with the shape of the tank, which was used during the numerical simulations described in point 4.1. and the experimental research described in 4.2. The example of tank's localization and sloshing forces is shown in figure 4.
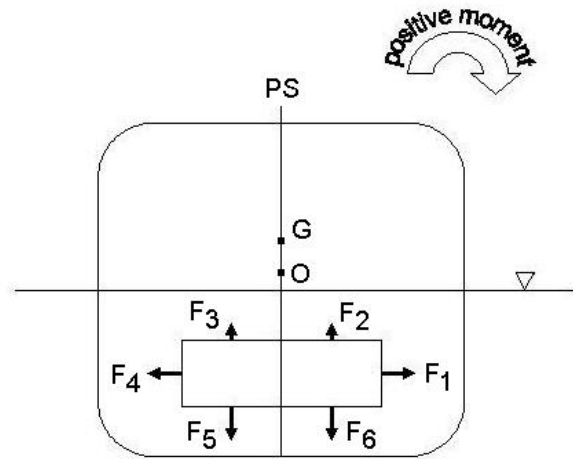

Fig. 4. The arrangement of forces affecting tank's structure

It is assumed, that the rolling motions of the ship and the tank take place about the rolling axe which is perpendicular to the plane of figure 4 and contains point O. The forces F_1 to F_6 are local values and they acts on both side walls of the tank and its roof and bottom, as shown in figure 4.

The local value of the force acting on the tank's structure can be obtained from the formula:

$$F = p \cdot ds \tag{4}$$

where: F - local force [N]; p - local pressure [Pa]; ds - infinitesimal segment of tank's wall area [m^2].

Taking into consideration the two-dimensional character of water flow inside the rectangular tank, described in point 4.2. the force may be calculated from two formulas:

$$F_V = p_{(z)} \cdot l \cdot dz \tag{5}$$

$$F_H = p_{(y)} \cdot l \cdot dy \tag{6}$$

where: F_V - local value of force due to sloshing acting on vertical side walls of the tank [N]; F_H - local value of force due to sloshing acting on horizontal roof and bottom of the tank [N]; l - length

of the tank (along x-dimension) [m]; z - vertical co-ordinate [m]; y - transverse co-ordinate [m].

The local value of moment of force due to sloshing is the simple product of multiplying: force by the lever of acting the force about the rolling axe. The lever can be defined as the distance between the line of force acting and the point O, as shown in figure 5 for horizontal force direction and in figure 6 for vertical force direction.
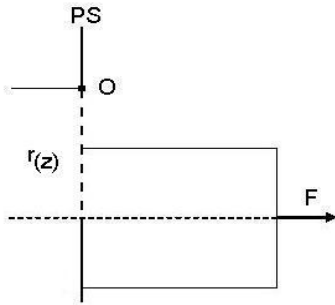
Fig. 5. The lever of heeling moment of sloshing force acting on the tank's side wall
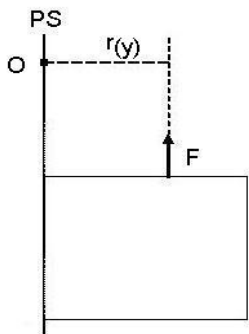
Fig. 6. The lever of heeling moment of sloshing force acting on the tank's roof

The localization of side walls of the tank and its roof and bottom is fixed by the vertical and transverse co-ordinates. It is assumed that the bottom of the tank is situated at the height z_{min} and its roof at the height z_{max}. The port side wall of the tank has the transverse co-ordinate y_{min} and the starboard side wall y_{max}. The symmetry plain has the y co-ordinate equal zero. Taking all the assumptions into consideration, the total values of force moments on the individual walls numbered 1 to 6 (see Fig. 4) can be calculated as the following integrals:

$$M_1 = - \int_{z_{min}}^{z_{max}} p_{1(z)} \cdot r_{(z)} \cdot l \cdot dz$$

$$M_2 = - \int_{0}^{y_{max}} p_{2(y)} \cdot r_{(y)} \cdot l \cdot dy$$

$$M_3 = \int_{y_{min}}^{0} p_{3(y)} \cdot r_{(y)} \cdot l \cdot dy$$

$$M_4 = \int_{z_{min}}^{z_{max}} p_{4(z)} \cdot r_{(z)} \cdot l \cdot dz \qquad (7)$$

$$M_5 = - \int_{y_{min}}^{0} p_{5(y)} \cdot r_{(y)} \cdot l \cdot dy$$

$$M_6 = \int_{0}^{y_{max}} p_{6(y)} \cdot r_{(y)} \cdot l \cdot dy$$

where: M_i - force moment on i-numbered tank's wall [N·m]; p_i - local pressure on i-numbered tank's wall [Pa]; rest of symbols same as in formula (5) and (6).

The total value of heeling moment due to sloshing inside the ship's tank is calculated as the sum:

$$M = \sum_{i=1}^{6} M_i \qquad (8)$$

The heeling moment is obtained from the formula (8) for one time-step only. The time domain calculation of heeling moment, which is required to be put into formula (3) governing ship's rolling, has to be performed for at least one rolling period. Thus the pressures p_1 to p_6 have to be investigated for the time of one rolling period as well.

5.2 Localization of rolling axe

The rolling motion of the ship's hull takes place about the non-steady positioned axe. The area, where the rolling axe occurs, depends mainly on the shape of the hull, its damping moments in the water, character of its external excitation, coupling with motions in any other degree-of-freedom. In course of practical computations of ship's rolling at the sea, the fixed placement of the roll axe is assumed.

The most often used approximation of the rolling axe localization is the ship's centre of gravity G (Dudziak 1988). The more exact approximation of the rolling axe localization is described by Balcer (Balcer 2000). The damping coefficients of roll and sway were taken into consideration. The values of added masses were obtained with respect of strip theory for simplified ship's shapes. The computations were made for numerous existing vessels of different size and shapes (Balcer 2000).

The final formula obtained in course of such reasoning is following:

$$z_O = 0{,}57 \cdot T + 0{,}43 \cdot z_G - 0{,}1 \cdot B \qquad (9)$$

where: z_O - height of the rolling axe above the base line [m]; T - mean draught [m]; z_G - height of the centre of gravity above the base line [m]; B - breadth of the ship [m]. The formula (9) enable to obtain the height of rolling axe and the axe is situated in the symmetry plain of the ship, what fix the axe clear-cut within the hull's body.

5.3 Results of computation

The computations of time-domain heeling moment due to sloshing were performed for two localizations of rolling axe (beneath the tank and above of it) and three angular amplitudes for each case: 18, 30 and 40 degrees. The pressure distributions on the tank's bottom and the roof (except of the upper corner) were obtained by numerical simulation described in point 4.1. The distributions of water pressures affecting the side walls of the tank and its upper corner were investigated in course of experimental research described in point 4.2.

The moments of forces acting on any of the tank's walls were obtained from the formula (7). The total value of heeling moment due to sloshing inside the ship's tank was calculated from the formula (8). The example of resultant heeling moment obtained in time-domain is shown in figure 7. The moment was calculated for angular amplitude 30°, water depth 400 mm and the rolling axe situated 1220 mm above the tank's roof.
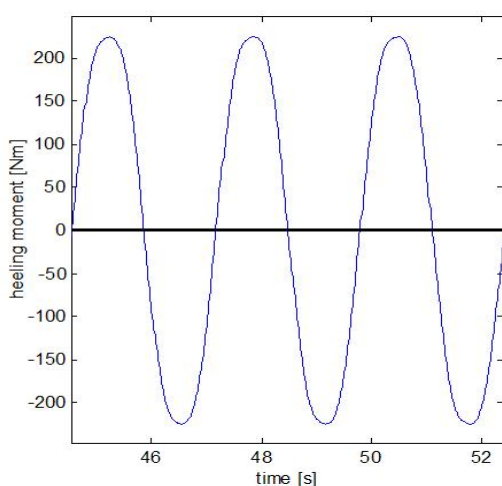

Fig. 7. The time-domain heeling moment due to sloshing

The results of presented computations may comprise a part of compound estimation of ships rolling movement including the sloshing phenomenon. Thanks to the time-domain presenta-tion, presented in the paper, computations may be also considered regarding to the phase shift of the heeling moment and the external moment of ship's roll excitation.

6 CONCLUSIONS

The model presented in the paper comprise the complete procedure of the time-domain calculation of heeling moment due to sloshing in ship's tanks. It is based on the investigated sloshing phenomenon. The experimental research and numerical simulation were performed.

The model enables the determining of reliable interaction between water and ship's hull, including the influence of tank's localization. This aspect is the step ahead in comparison with free surface effect considered in course of standard procedure of stability evaluation nowadays. The proposed model improves the credibility of estimation of the dynamical free surface effect. It is much more precise than quasi-static attitudes towards sloshing phenomenon. The dynamic behavior of the water sloshing inside the partly filled tank was passed over so far.

There are available brand new publications covering the problem of coupling between the sloshing phenomenon and ship's movement. The Polish Register of Shipping developed the computer program, which enables the time domain calculation of ship's rolling when carrying the liquid cargo (Warmowska & Jankowski 2006). Anyway the procedure of dynamic heeling moment calculation is not published and the results of numerical simulation are showed only. The model used by PRS is based on the potential theory of fluid dynamics and there isn't any possibility to put into the calculation any experimental data. Moreover the simulations presented during the National Conference of Fluid Dynamics in 2006, were performed for one tank's filling level only and one loading condition of the ship. Anyway such publication as mentioned above shows the pending direction of the research in the marine industry.

The time domain estimation of heeling moment due to sloshing can be used in any ship's movement equation, for instance like formulas (2) or (3). It makes the model universal part of time domain considerations regarding vessel's sea keeping.

REFERENCES

Akyildiz H., Unal E., „Experimental investigation of pressure distribution on a rectangular tank due to the liquid

sloshing", *Ocean Engineering 32 (2005)*, www.sciencedirect.com

Balcer L., "Model matematyczny kołysań statku z biernym zbiornikiem stabilizacyjnym", rozprawa doktorska, Wydział Oceanotechniki i Okrętownictwa Politechniki Gdańskiej, Gdańsk 2000 (in Polish).

Centrum Techniki Okrętowej, „Porównanie wyników eksperymentu modelowego z wynikami obliczeń numerycznych przy użyciu programu LR FLUIDS", Raport Techniczny Nr RK-98/T-055, CTO, Ośrodek Mechaniki Konstrukcji Okrętu, Gdańsk 1998 (in Polish).

Dudziak J., „*Teoria okrętu*", Wydawnictwo Morskie, Gdańsk 1988 (in Polish).

Jankowski J., Warmowska M., „Development of computer program describing the flow in partly filled tank", PRS, Technical Report No.27/97, Gdańsk, 1997.

Krata P., „Experimental determination of the areas of maximum liquid sloshing loads in moving rectangular ship's double bottom tank", Proceedings of the XV-th International Scientific and Technical Conference "*The Role of Navigation in Support of Human Activity on the Sea*", Gdynia, Poland 2006 (in Polish).

Krata P., "Determining of maximum water thrust affecting ship's tank structure based on the experimental research of sloshing phenomenon", *Scientific Works of Navigational Faculty of Gdynia Maritime University* No 20, Gdynia 2007 (in Polish).

Senjanovic I., Parunov J., Cipric G., "Safety Analysis of Ship Rolling in Rough Sea", *Chaos, Solitons and Fractals, Vol. 8, No 4*, Elsevier Science Ltd, Great Britain 1997.

Warmowska M., Jankowski J. „Ship motion affected by moving liquid cargo in holds", *Krajowa Konferencja Mechaniki Płynów, Bełchatów 2006*.

Warmowska M., Jankowski J. „Simulation of water movement in partly filled ship tanks", *HYDRONAV '2005*, Ostróda 2005.

Warmowska M., „Określenie parametrów ruchu cieczy w zbiorniku okrętowym metodą rzutowania z uwzględnieniem zjawisk nieliniowych - Ocena dokładności opracowanej metody za pomocą istniejących w literaturze wyników obliczeń dla problemów szczególnych, posiadanych wyników eksperymentu; stabilność metody obliczeniowej", PB nr 5 T12C 057 24, Politechnika Gdańska, Gdańsk 2005 (in Polish).

Zieliński P.T., „*Od teorii do cyfrowego przetwarzania sygnałów*", Wydział EAIiE AGH, Kraków 2002 (in Polish).