TRANSNAV International Journal on Marine Navigation and Safety of Sea Transportation
Volume 5 Number 4 December 2011

Method of Evaluation of Insurance Expediency of Stevedoring Company's Responsibility for Cargo Safety

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ABSTRACT: The method of insurance expediency of stevedoring company's responsibility for safety of containers under their transshipment at port's terminal is proposed. This method is based on representation of terminal as a queueing system of GI/G/m type and on comparison of the stevedoring company's insurance expenditures and random value of transshipped containers' total damage (sum insured) for a given period of time.

1 INTRODUCTION

In operational activity of stevedoring companies, the many cases may occur related to the situations of risk. The main of them are listed below:

- damage of a ship's hull or equipment during the loading/unloading;
- cargo package's damage in result of violation of loading/unloading rules or rules of port's mechanisms exploitation;
- damage of cargo in result of violations of rules of its storage at warehouse;
- failures of port's equipment;
- exceeding of a ship's laytime.

Appearance of above events leads to some additional expenditure for the stevedoring company, charterer or cargo owner. As shows the international commercial practice, many stevedoring companies which operate in big seaports insure their responsibility for safe and qualitative transshipment of cargo (within the framework of contract responsibility) [1].

When the managers of stevedoring company make decision concerning an insurance of its responsibility for safe transshipment of cargo it is useful and even necessary to apply the methods of probability theory and actuarial mathematics [2, 3]. At the same time the standard methods of quantitative evaluation of risk proposed by mathematical risk theory are mainly aimed at insurance companies' profit but not at protection of commercial interests of insurants. Therefore, specifics of seaports operational activity and interrelations between stevedoring company's managers and its clients demand the special methods for actuarial calculations.

The purpose of our paper is working out a method of a risk evaluation of containers damage under their transshipment at a seaport terminal and substantiation of insurance expediency of this risk by a stevedoring company.

2 MAIN RESULTS

Our approach is based on representation of port's container terminal as a queueing system of GI/G/m type (m identical servers in parallel, infinite waiting room, service discipline is FIFO).

We denote:

$\omega(t)$ be a random number of served ships in time interval $(0,t)$;

γ_k be a random number of containers transshipped on/from the kth ship served in time interval $(0, t)$;

ν_k be a random number of damaged containers during loading/unloading of the kth ship;

Δ_{ki} be a random value of damage caused to the ith container loaded on or unloaded from the kth ship (estimated in money).

It is assumed that:

1 the random variables $\gamma_1, \gamma_2, \ldots$ are independent and identically distributed (i.i.d.) with the discrete distribution

$$\pi_M = \Pr\{\gamma_1 = M\}, M = 1,2,..,\quad \sum_{M \geq 1} \pi_M = 1; \tag{1}$$

2 $\nu_1, \nu_2, \ldots$ are the i.i.d. random variables with the conditional binomial distribution

$$\Pr\{\nu_k = n | \gamma_k = M\} = C_M^n p^n q^{M-n} \quad (q = 1 - p), \quad n = 0,1,\ldots,M, \tag{2}$$

where p is the probability that a damage is caused to arbitrary container through a stevedoring company's fault ;

3 $\Delta_{11}, \Delta_{12}, \ldots, \Delta_{21}, \ldots$ are the i.i.d. random variables with the distribution function (d.f.)

$$D(x) = \Pr\{\Delta_{11} \leq x\}; \tag{3}$$

4 the sequences of random variables $\nu_1, \nu_2, \ldots$ and $\Delta_{11}, \Delta_{12}, \ldots$ are mutually independent.

If τ denotes the constant loading/unloading time of one container, than service time of the kth served ship is the random variable $\gamma_k \tau$. We shall consider the steady-state regime of our queueing system functioning and assume that the following stability condition holds true

$$\lambda < m / (\tau \, \mathrm{E}\gamma_1), \tag{4}$$

where λ^{-1} is the mean interarrival time of the ships.

Let us evaluate the total damage in time interval (0, t) caused to containers by stevedoring company ($\Delta(t)$). Using the above designations we can write

$$\Delta(t) = \sum_{k=1}^{\omega(t)} \sum_{i=1}^{\nu_k} \Delta_{ki}. \tag{5}$$

The financial managers of a stevedoring company face the dilemma: to insure or not to insure the possible total damage (4) with the gross risk premium rate c (we assume that the sum insured is $\Delta(t)$). Note that t we consider as the period of insurance policy action.

The simplest criterion of insurance expediency is: the average profit of a stevedoring company in result of the total damage insurance must be positive, i.e.

$$\mathrm{E}(\Delta(t) - ct) > 0. \tag{6}$$

Taking into account relations (1)-(5) and applying to right-hand side of (5) theorem of total mathematical expectation, from (6) we have

$$\mathrm{E}\omega(t)\mathrm{E}\nu_1 \mathrm{E}\Delta_{11} > ct, \tag{7}$$

where

$$\mathrm{E}\Delta_{11} = \int_0^\infty x dD(x) < \infty, \mathrm{E}\nu_1 = p \sum_{n \geq 1} n\pi_n < \infty. \tag{8}$$

For ergodic queue (see (4)) $\mathrm{E}\omega(t) = \lambda t$ [4]. Therefore, from (7) we obtain

$$\lambda \, \mathrm{E}\nu_1 \mathrm{E}\Delta_{11} > c. \tag{8}$$

More precise criterion than (8) is

$$\Pr\{\Delta(t) > ct\} \geq 1 - \varepsilon, \tag{9}$$

where ε is a given small probability. For application of criterion (9) we need to determine the d.f. of stochastic process $\Delta(t)$.

For the sake of simplicity, we suppose that $\gamma_k = N, k = 1,2,\ldots,$ where N may be interpreted as hold capacity of a ship (in TEU). In other words, we assume that each ship arrives for loading/unloading of exactly N containers. Then by theorem of total probability, taking into account (5), mutual independence of $\omega(t)$ and $\nu_1, \nu_2, \ldots$, we can write

$$F(x,t) = \Pr\{\Delta(t) \leq x\} = \Pr\{\omega(t) = 0\} + \sum_{k=1}^{\infty} \Pr\{\omega(t) = k\} \times$$
$$\times \sum_{n_1=0}^{N} \ldots \sum_{n_k=0}^{N} \prod_{i=1}^{k} C_N^{n_i} p^{n_i} q^{N-n_i} D^{(n_1+\ldots+n_k)}(x), \tag{10}$$

where $D^{(n)}(x)$ is n-multiple convolution of d.f. $D(x)$ with itself, $D^{(0)}(x) \equiv 1$.

Due to the formula (10) the criterion (9) takes the form

$$F(ct,t) = \Pr\{\omega(t) = 0\} + \sum_{k=1}^{\infty} \Pr\{\omega(t) = k\} \times \tag{11}$$
$$\times \sum_{n_1=0}^{N} \ldots \sum_{n_k=0}^{N} \prod_{i=1}^{k} C_N^{n_i} p^{n_i} q^{N-n_i} D^{(n_1+\ldots+n_k)}(ct) \leq \varepsilon.$$

In practice, N may be considered as large and p as small quantities. Therefore, the binomial terms in

(11) may approximately be substituted for the Poisson distribution. From (11), it follows

$$\Pr\{\omega(t)=0\}+\sum_{k=1}^{\infty}e^{-ak}\Pr\{\omega(t)=k\}\times$$
$$\times\sum_{n_1=0}^{\infty}\ldots\sum_{n_k=0}^{\infty}\prod_{i=1}^{k}\frac{a^{n_i}}{n_i!}D^{(n_1+\ldots+n_k)}(ct)\le\varepsilon, \qquad (12)$$

where $a = Np$.

The Laplace-Stieltjes transform of d.f. (10) on variable x is given by

$$\int_0^{\infty}e^{-sx}d_xF(x,t)=\Pr\{\omega(t)=0\}+$$
$$+\sum_{k=1}^{\infty}\Pr\{\omega(t)=k\}\sum_{n_1=0}^{N}\ldots\sum_{n_k=0}^{N}\prod_{i=1}^{k}C_N^{n_i}[p\delta(s)]^{n_i}q^{N-n_i}=$$
$$=\Pr\{\omega(t)=0\}+\sum_{k=1}^{\infty}\Pr\{\omega(t)=k\}[p\delta(s)+q]^{kN}=$$
$$=\Phi([p\delta(s)+q]^N,t),\ \mathrm{Re}\,s\ge 0, \qquad (13)$$

where

$\delta(s)=\int_0^{\infty}e^{-sx}dD(x)$ and $\Phi(y,t)=\sum_{k=1}^{\infty}y^k\Pr\{\omega(t)=k\}$ is the generating function of stochastic process' $\omega(t)$ distribution, $|y|\le 1$.

In particular, from (13) we find

$$\mathrm{E}\Delta(t)=-\frac{\partial}{\partial s}\Phi([p\delta(s)+q]^N,t)\Big|_{s=0}=Np\mathrm{E}\Delta_{11}\mathrm{E}\omega(t),$$
$$\mathrm{Var}\Delta(t)=\frac{\partial^2}{\partial s^2}\Phi([p\delta(s)+q]^N,t)\Big|_{s=0}-(\mathrm{E}\Delta(t))^2=$$
$$=Np[\mathrm{E}\Delta_{11}^2-p(\mathrm{E}\Delta_{11})^2]\mathrm{E}\omega(t)+(Np\mathrm{E}\Delta_{11})^2\mathrm{Var}\omega(t).$$

One more simplification of criterion (9) may be done by application of the Chebyshev's inequality. Applying this inequality, taken in modified form [5], we obtain (under condition (6))

$$\Pr\{\Delta(t)>ct\}\ge\frac{(\mathrm{E}\Delta(t)-ct)^2}{\mathrm{E}\Delta^2(t)}. \qquad (14)$$

Hence, the criterion (9) may be reduced to the simple inequality

$$\frac{(\mathrm{E}\Delta(t)-ct)^2}{\mathrm{E}\Delta^2(t)}\ge 1-\varepsilon.$$

For application of the criteria (11),(12),(14) it is necessary to find the probabilistic distribution of process $\omega(t)$. It may be found by the methods of queueing theory [6]. Below, will be considered two particular cases of queue GI/G/m for which this distribution is known.

1 Queue of M/D/∞ type, i.e. with infinite number of servers, the Poisson input with the rate λ, and constant service time. Such queueing system is good approximation to multi-server queue if $\lambda\tau N \ll m$. As it was shown in [7], for such system (in equilibrium)

$$\Pr\{\omega(t)=k\}=\frac{(\lambda t)^k}{k!}e^{-\lambda t},\ k=0,1,2,\ldots \qquad (15)$$

and, consequently, $\mathrm{Var}\omega(t)=\mathrm{E}\omega(t)=\lambda t$. In this case the condition (11) takes the following form

$$e^{-\lambda t}\sum_{k=0}^{\infty}\frac{(\lambda t)^k}{k!}\sum_{n_1=0}^{N}\ldots\sum_{n_k=0}^{N}\prod_{i=1}^{k}C_N^{n_i}p^{n_i}q^{N-n_i}D^{(n_1+\ldots+n_k)}(ct)]\le\varepsilon. \qquad (16)$$

From (15), it follows also that

$$\Phi([p\delta(s)+q]^N,t)=\exp\{-\lambda t[1-(p\delta(s)+q)^N]\}.$$

For inversion of this expression the known numerical methods of the Laplace transform inversion may be used [8].

The criterion (16) is too complex for calculations. Note that in this case $\Delta(t)-ct$ is the compound Poisson process with the drift c [9]. Therefore if $t\to\infty$, we can apply the central limit theorem for such kind of stochastic processes [9]. Hence, instead of (9), we have as $t\to\infty$

$$\Pr\{\Delta(t)-ct\le 0\}\approx \mathrm{N}(R\sqrt{t})\le\varepsilon, \qquad (17)$$

where

$$R=(c/\mathrm{E}\Delta_{11}-\lambda Np)\times$$
$$\times\sqrt{\lambda Np[\mathrm{E}\Delta_{11}^2+(N-1)p(\mathrm{E}\Delta_{11})^2]};$$

N(x) is the standard normal distribution with zero mean and variance equals to unity.

2 One-server queue of M/D/1 type, i.e. with the Poisson input and constant service time. For such system the following result is valid [6]:

Np \ $c/\mathrm{E}\Delta_{11}$	0,15	0,20	0,25	0,30	0,35
0,10	0,0436	0,0721	0,1112	0,1635	0,2327
0,15	0,0092	0,0150	0,0244	0,0384	0,0582
0,20	0,0021	0,0035	0,0058	0,0092	0,0140
0,25	0,0006	0,0009	0,0015	0,0024	0,0037

$$\Phi^*(y,\theta) \equiv \int_0^\infty e^{-\theta t}\Phi(y,t)dt =$$

$$= \Phi_0^*(y,\theta)\left[1 + \frac{\lambda(1-z_0)(1-e^{-\theta N\tau})}{\theta(1-ye^{-\theta N\tau})}\right] - \frac{\lambda(1-e^{-\theta N\tau})}{\theta^2(1-ye^{-\theta N\tau})}\times$$

$$\times\left[(1-y)e^{-\theta N\tau} + \frac{1}{y}(1-\rho)(1-z_0)\frac{z_0 - ye^{-\lambda(1-z_0)N\tau}}{z_0 - e^{-\lambda(1-z_0)N\tau}}\right] +$$

$$+\frac{\rho}{\theta}, \qquad (18)$$

where $\rho = \lambda N\tau < 1$;

$$\Phi_0^*(y,\theta) = \frac{1-\rho}{\theta(\theta+\lambda-\lambda z_0)}\times$$

$$\times\left[\theta + \frac{\lambda(1-z_0)ye^{-\lambda(1-z_0)N\tau}(1-e^{-\theta N\tau})}{e^{-\lambda(1-z_0)N\tau} - z_0}\right];$$

z_0 is the unique root of the equation

$$z_0 = y\exp[-(\theta+\lambda-\lambda z_0)N\tau]$$

in the domain $|y| \le 1, \mathrm{Re}\,\theta > 0.$

With the help of relation (18) we can determine $\mathrm{Var}\,\omega(t)$ and then use the criterion (14).

3 NUMERICAL RESULTS

Let us demonstrate the application of criterion (17) for real initial data. Put $\lambda = 5$ ships per month, t = 25 months, $p = 10^{-3}$, and assume that $\mathrm{E}\Delta_{11}^2 = 2(\mathrm{E}\Delta_{11})^2$. The results of calculations of probability in the formula (17) for different values of Np and ratio $c/\mathrm{E}\Delta_{11}$ are given in the Table.

Table

From these results, it follows the expedience of insurance, for example, if Np = 0,1, $c/\mathrm{E}\Delta_{11} \le 0{,}2$ or $Np > 0{,}1$, $c/\mathrm{E}\Delta_{11} \le 0{,}3$ because probability in (17) is sufficiently small in these cases.

4 CONCLUSIONS

The real problems of risk-management concerning the port operator's (or stevedoring company's) activity may be formulated and solved with application of mathematical risk theory. The main feature of above problems is: first of all they must be aimed at the protection of financial state of stevedoring company but not an insurance firm. In most cases these problems may not be solved by standard theoretical methods and require the use of combination of different fields of applied probability, for example, ruin theory, queueing and reliability theories, theory of storage processes, etc. This is necessary for modeling the port's operational activity side by side with the corresponding financial processes [10].

For practical applications of results obtained it is necessary to use the corresponding statistical data concerning the cases of containers damage and values of damage, moments of ships' arrival, etc. for a previous period. Such information must be accumulated in the data base of a stevedoring company.

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