Integration of Elliptical Ship Domains and Velocity Obstacles for Ship Collision Candidate Detection

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ABSTRACT: The maritime shipping industry has been making significant contributions to the development of the regional and global economy. However, maritime accidents and their severe consequences have been posing an incrementing risk to the individuals and societies. It is therefore important to conduct risk analysis on such accidents to support maritime safety management. In this paper, a modified ship collision candidate detection method is proposed as a tool for collision risk analysis in ports and waterways. Time-Discrete Velocity Obstacle algorithm (TD-NLVO) is utilized to detect collision candidates based on the encounter process extracted from AIS data. Ship domain model was further integrated into the algorithm as the criteria for determination. A case study is conducted to illustrate the efficacy of the improved model, and a comparison between the existing method and actual ship trajectories are also performed. The results indicate that with the integration of ship domain, the new method can effectively detect the encounters with significant collision avoidance behaviours. The choice of criteria can have a significant influence on the results of collision candidate detection.

1 INTRODUCTION

With the development of global economy, maritime transport system has been playing an important role in the world trading system. However, accidents, especially ship collision and grounding (EMSA, 2017), have been imposing a threat to society and individual in terms of multiple aspects. It is therefore of great necessity to conduct research on collision risk analysis to facilitate maritime safety administration to improve the safety level and reduce the occurrence of collision accident.

To quantitatively analyse the risk of ship collision accident, various methods have been proposed, see Li et al (Li et al., 2012). Among them, the framework proposed by Fujii (Fujii and Shiobara, 1971) and Macduff (Macduff, 1974) has been widely applied in regional collision risk analysis and management. The framework is shown in Eq. (1):

$$P_{collision} = P_{geometric} \times P_{causation} \qquad (1)$$

where $P_{geometric}$ denotes the number of collision candidate, also known as the geometrical probability of collision, which indicates the frequency of ship encounters that have the potential of collision. $P_{causation}$ indicates the probability of collision caused by accident contributing factors, e.g. human and organisational factors, extreme weather conditions, and mechanical failures, etc. Such framework provides a concise approach to estimate the risk of collision and both the maritime traffic situation and accident causations can be considered as one integrity.

To obtain $P_{geometric}$, generally, there are two major categories of approaches: 1) indicator-based approach and 2) safety boundary approach (Chen et al., 2018). The indicator-based approach determines the encounter situation of ships based on certain indicators that can reflect their spatiotemporal proximity, e.g. DCPA (Distance to Closest Point of Approach), TCPA (Time to Closest Point of Approach), relative position, relative speeds and bearing, etc. Zhang et al. (Zhang et al., 2017) proposed Vessel Conflict Risk Operator (VCRO) and its variations facilitate identification of collision candidate using AIS data. Li et al (Li et al., 2015) also utilized the distance between ships, relative speeds, course difference, etc. to formulate the mathematical function to evaluate the emergent level of encounters. The safety boundary approach, on the other hand, determines the encounter situation based on the violation of certain safety boundary, e.g. Collision diameter, ship domain, Minimum Distance to Collision (MDTC) (Montewka et al., 2010), etc. Compared with the indicator-based approach, this approach considers spatial proximity using the concept of the boundary. Fujii and Shiobara (Fujii and Shiobara, 1971) first proposed collision diameter as the boundary to determine which encounter is dangerous, and such a concept was mathematically proposed by Pedersen (Pedersen, 1995). Following such an approach, many similar models have been developed, see (Ylitalo, 2010). Christian and Kang (Christian and Kang, 2017) introduced the COWI model (COWI, 2008) to estimate the probability of collision of the ship which transports spent nuclear fuel, and Cucinotta et al (Cucinotta et al., 2017) utilized a similar approach to obtain the frequency of ship collision in Messina Strait. Montewka et al (Montewka et al., 2012). established a probabilistic model for the marine accident where MDTC is utilized as criteria of collision candidate. Szlapczynski et al (Szlapczynski and Szlapczynska, 2016) introduced ship domain as the criteria of collision candidate and proposed the degree of domain violation (DDV) and time to domain violation (TDV) as indices to reflect the emergent degree of the encounter.

Although various methods have been proposed to obtain the number of collision candidate, there is possibility which could cause over/underestimation of the results. The reason caused such issues is that traditional methods do not consider encounter as a process, instead of the instant information of encounter, either using indicator or safety boundary, are introduced to determine the situation. In (Chen et al., 2018) the authors have changed this perspective, to consider the encounter as a process and determine collision candidate using Time Discrete Non-line Velocity obstacle algorithm (TD-NLVO). The results of this paper indicate that compared with traditional methods, the new results of this new algorithm show high reliability. However, due to the simplification in this work, the safety boundary was set to be a circular shape, which could lead to overestimation to a certain extent. Therefore, in this paper, this issue is improved with the integration of ship domain model.

In this paper, the previous Time Discrete Non-linear Velocity obstacle algorithm is modified with the integration of ship domain model, to further improve the accuracy of the results. Firstly, the Non-linear velocity obstacle algorithm is introduced as the basic tool to assess encounter situation from the perspective of the process; Then, the elliptical ship domain model is integrated into the algorithm to act as criteria of candidate determination. A case study using actual AIS (Automatic Information System) data is conducted, together with compassion between the old and new algorithm. The arrangement of the article is as follows: Section 2 illustrates the methodology of this paper, followed by the design of the algorithm in Section 3. A case study is performed in section 4 to show the results of the algorithm and the comparison. Section 5 concludes the paper.

2 METHODOLOGY

According to the definition in (Chen et al., 2018), collision candidate is the pair of ships in an encounter process where their spatiotemporal relationships satisfy certain criteria that has the potential for collision. This definition provides an open framework that can integrate the selected criteria of geometric collision probability into account. Therefore, in this paper, the objective is to design a collision candidate detection algorithm that can determine the encounter to be dangerous according to the violation of ship domain of own ship through the process of the encounter using historical AIS data in the certain region. To do so, TD-NLVO algorithm is adopted as the basic framework for collision detection, and elliptical ship domain model is integrated as the criteria.

3 COLLISION CANDIDATE DETECTION MODEL

3.1 TD-NLVO algorithm

Velocity obstacle algorithm is a type of algorithm that determines the potential of collision by projecting the spatiotemporal relationship between own object and target, e.g. relative position, velocity, etc. into the velocity space of own object and then checking whether own velocity falls into the velocity obstacles induced by the target. Such methods have been widely applied in collision detection in robotics (Fiorini and Shiller, 1998), meanwhile, it is still a relatively new angle to assess ship collision risk. In maritime transport field, Degre and Lefevre (Degré and Lefèvre, 1981) first proposed the idea that checking the danger of collision using the velocities between own ship and target. Such method was further developed and mathematically formulated by Lenart (Lenart, 1983), which is defined as Collision Threat Parameter Area (CTPA). Since these methods assume that the kinematic status of both own ship and target ship remain constant during the encounter, they are also defined as Linear Velocity Obstacle (LVO), which is proved to be identical to CPA analysis by Huang, et al (Huang et al., 2017). Due to this assumption, the result based on LVO could be over/estimated since it cannot consider the changes of both ships' kinematics during the encounter. To improve the deficiency of LVO, the constraint of LVO that the velocities of ships remain constant during the

encounter is loosen to that velocity of target ship is flexible yet known to own ship in Large et al (Large et al., 2002). During the encounter process, the kinematic information of both ships can be updated, hence their influence on the velocity obstacle induced by the target. Therefore, in (Chen et al., 2018) and this work the non-linear velocity obstacle algorithm was applied as the fundamental tool for collision candidate detection. The basic theory of Non-linear velocity obstacle algorithm is shown in Fig. 1:
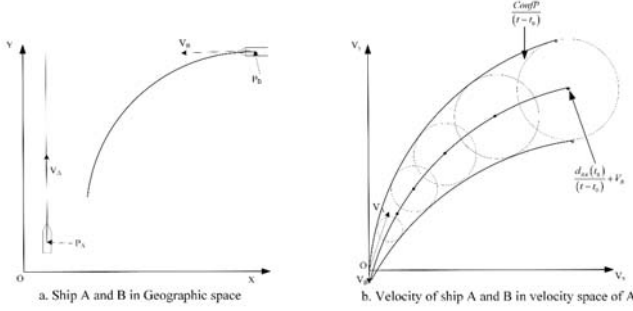

Figure 1 Basic illustration of Non-linear Velocity Obstacle algorithm (Chen et al., 2018)

Suppose that ship A and B in Fig. 1 are in an encounter situation. The kinematic information of both ships can be expressed as $A\{L_A, P_A(t), V_A(t)\}$ and $B\{L_B, P_B(t), V_B(t)\}$, respectively. L, $P(t)$, $V(t)$ are their length, position and velocities at time instance t. Through certain transformation, such spatiotemporal relationship can be transformed into the velocity space of A with the size of A shrinking into a point and B expands to a larger area indicated by Fig. 1 (b) with radius R of the area. This area is the collection of all potential position of ship A when the collision happens and is also defined as "conflict position $(ConfP)$"(Huang et al., 2018). The $ConfP$ is obtained according to Eq. (2):

$$ConfP = \left\{ \left\| P_A(t) - P_B(t) \right\| \le R \right\} \tag{2}$$

Eq. (2) is considered as the criteria of collision candidate, i.e. if the distance between two ships falls into $ConfP$, collision is then likely to happen in time t. considering the kinematic information of own ship, Eq. (2) can be rewritten as Eq. (3) :

$$P_A(t_C) \in P_B(t_C) \oplus ConfP \tag{3}$$

Eq. (3) is an equivalent form of the criteria for collision candidate. Consider the kinematics of both ships are known and deterministic and set $VO_{A|B}$ as the variable of velocity obstacle of ship A induced by target ship B, Eq. (3) can be rewritten in another form as Eq. (4) illustrates:

$$VO_{A|B} = \bigcup_t^{\infty} \left(\frac{P_B(t) - P_A(t_0)}{(t - t_0)} \right) \oplus \frac{ConfP}{(t - t_0)} \tag{4}$$

where $VO_{A|B}$ is the set of velocities of own ship that could lead to collision.

3.2 Elliptical ship domain

In the previous section, the $ConfP$ is defined as a circular area with radius R. Such definition is similar to the definition of Collision Diameter proposed by Fujii (Fujii and Tanaka, 1971) and Pedersen (Pedersen, 1995), hence it also inherits the similar issues when in practices: the area is too small that any violation of such an area would be physical contact (Montewka et al., 2010). Since collision candidate denotes the pair of ships in encounter situation that has the potential of the collision, it is reasonable to expand such area to some extent. In our previous work, the radius was arbitrarily set to simplify the modelling complexity, however, it also brings the issue of potential overestimation. In this work, we introduced the static elliptical ship domain model as the new criteria.

Ship domain is firstly introduced by Fujii and Tanaka (Fujii and Tanaka, 1971) to represent an area around the ship that would like to keep clear of violation of other ships in the vicinity. If the violation occurs, it denotes that collision is likely to happen. Based on this fundamental concept various models and application have been proposed, e.g. (Szlapczynski et al., 2018; Wang, 2010), etc. In this paper, we replace the circular $ConfP$ with a static elliptical ship domain as the new criteria for collision candidate. The parameters of such area are semi-major and semi-minor axis, respectively. In this paper, they are set as 8 times and 4 times of own ship's length based on the research by (Szlapczynski et al., 2018). To integrate such domain into the TD-NLVO, a mathematical function needs to be proposed. To do so, two variables of own ships information needs to be integrated, which are length and course over ground, respectively. Such parameters can be obtained in historical AIS data. The general function of an ellipse can be written as Eq. (6):

$$A x^2 + B xy + C y^2 + D x + E y + F = 0 \tag{5}$$

Its identical form is shown in Eq. (7):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \tag{6}$$

Eq. (6) and Eq. (7) describes the ellipse whose foci are either on the major and minor axis. In practices, such domain needs to be described in the local coordinates of own ship whose major axis is along the course, instead of true north. Therefore, the ship domain needs to be rotated according to the course information. The rotation function is shown in Eq. (8):

$$x' = x\cos(\theta) - y\sin(\theta)$$
$$y' = x\sin(\theta) + y\cos(\theta) \tag{7}$$

Therefore, the rotated ship domain can be written as Eq. (9):

$$\frac{(x\cos(\theta) - y\sin(\theta))^2}{a^2} + \frac{(x\sin(\theta) + y\cos(\theta))^2}{b^2} = 1 \tag{8}$$

Rewriting Eq. (9) according to Eq. (1), the parameter function is shown in Eq. (10):

$$\left(a^2\sin^2(\theta)+b^2\cos^2(\theta)\right)\times x^2+\left(a^2\cos^2(\theta)+b^2\sin^2(\theta)\right)\times y^2 \quad (9)$$
$$+2\left(a^2-b^2\right)\sin(\theta)\cos(\theta)\,xy-a^2b^2=0$$

Then the corresponding parameters of the rotated ship domain can be obtained according to Eq. (11):

$$A=a^2\sin^2(\theta)+b^2\cos^2(\theta)$$
$$B=2\left(a^2-b^2\right)\sin(\theta)\cos(\theta)$$
$$C=a^2\cos^2(\theta)+b^2\sin^2(\theta) \quad (10)$$
$$F=-a^2b^2$$

3.3 Design of collision candidate detection model

With the integration of TD-NLVO and elliptical ship domain model, the new version of ship domain-based collision candidate detection model can be established. As aforementioned, collision candidate is detected according to the total process of encounter, instead of instance encounter information at a certain time interval. To do this, how to construct the trajectory data and process them are one of the important technical problems here. To implement the process perspective, the historical AIS data of ships navigating in the area are first reconstructed as chronological trajectory data according to their MMSI (Maritime Mobile Service Identifier). To accelerate the computing speed, such a trajectory is also divided into subsets using the same parameter settings in (Chen et al., 2018). The design of the new model is shown in Fig. (2):
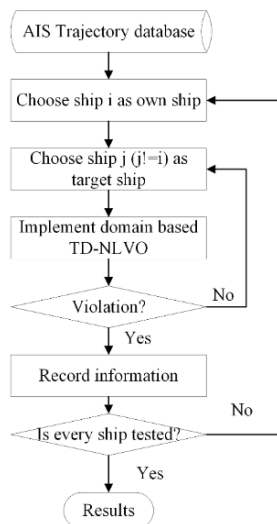

Figure 2. Flow chart of the ship domain-based collision candidate detection model

4 CASE STUDY

In this section, a case study on implementing the domain-based TD-NLVO is illustrated. The AIS data are obtained from the open access provided by the Danish Maritime Authority. Since the goal of this paper is to demonstrate the efficacy of the modified TD-NLVO and the comparison between the original and new method. The time span of the data is set to be 1 day. Here we introduced the AIS data on 1st Oct. 2018 in port Aarhus as the test datasets. The parameter settings are as follows: T_{Scan} is set to be 60 mins and $T_{theshold}$: 30s;

The encounter between tanker "219XXX000" and cargo ship "257XXX000" are shown with their trajectory and encounter situation in velocity space at a certain time step. Based on the AIS data and TD-NLVO, these two ships have an encounter that violates the domain of own ship. The trajectories are shown in Fig. 3:
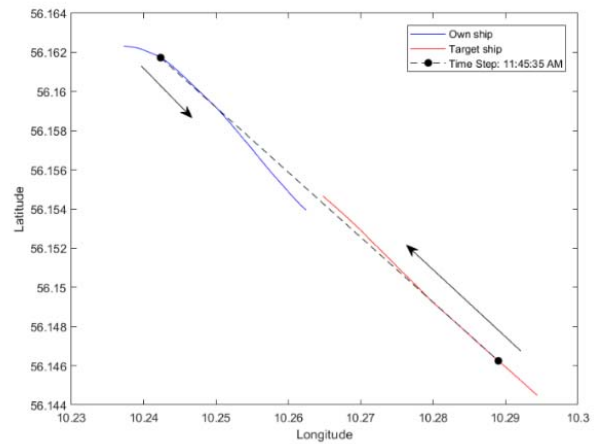

Figure 3. Trajectory between two ships

From the reconstruct of ship trajectories we can see that at the beginning of the encounter process, tanker "219XXX000" and cargo ship "257XXX000" were in "head on" situation. With both ships approaching each other, own ship (blue) detected that there might be danger of collision, therefore, she altered her course to her starboard to enlarge the distance between both ships, meanwhile the cargo ship also altered her course to her starboard side a bit to make sure both can pass each other on her starboard side, which is required by the COLREGs. Taking the encounter situation at 11:41:35 AM as the example, the spatiotemporal relationships between both ships in velocity space of own ship is shown in Fig. (4):
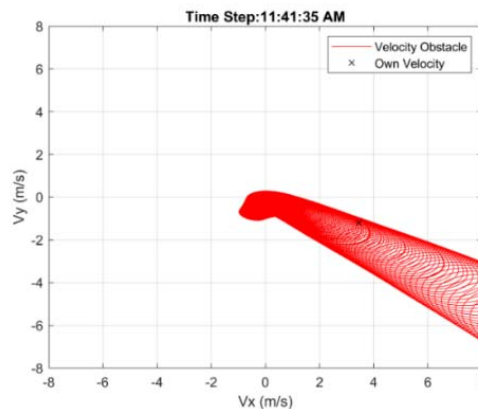

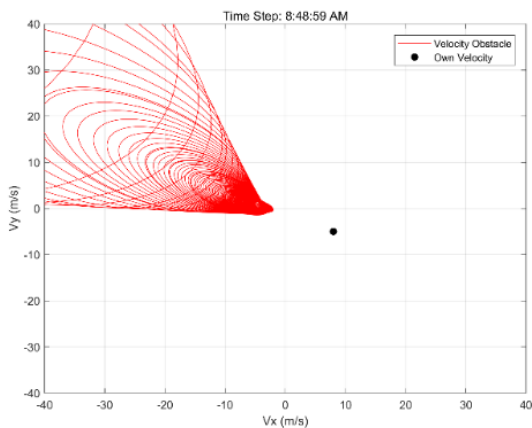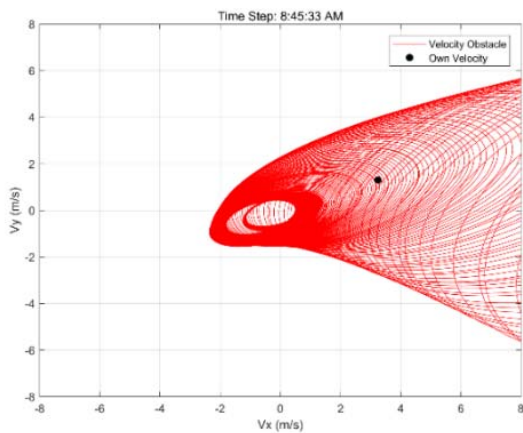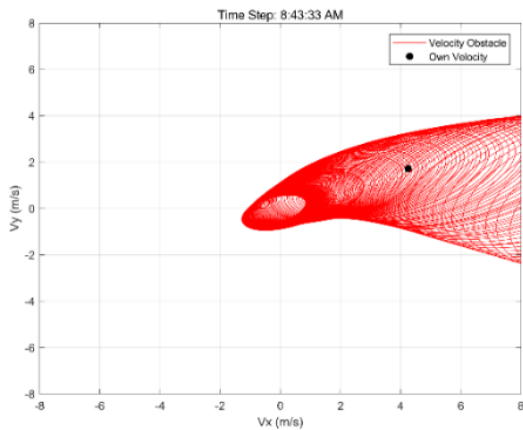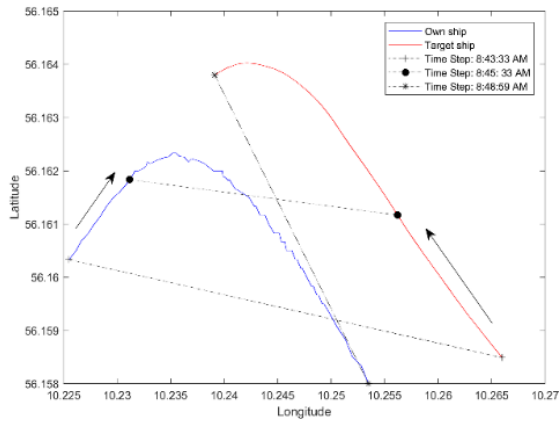Figure 4. Encounter situation in velocity space of own ship

Figure 5   Crossing encounter situation between "219XXX172" and "219XXX903"

As Fig. 4 indicates, the ship domain-based velocity obstacle algorithm successfully detect a violation of ship domain in the future of detection time 11:41:35, i.e. at that time, TD-NLVO detected that the ship domain of own ship would be violated in the future. Compared with trajectory information in Fig. 4, the alert from VO is earlier than the actual movement.

Fig. 5 illustrates another encounter situation between "219XXX172" and "219XXX903" at the entrance of port Aarhus. With the use of domain-based TD-NLVO, the encounter process can be easily demonstrated in velocity space of own ship. As we can see, at 8:43:33 AM when own ship went outbound, with trajectory information of target ship she can detect the violation of velocity obstacle at a certain point in the future at that time. With the development of time, own ship chose to make a turn to her starboard to follow the channel and avoid possible collision with target ship. Besides, since the modified TD-NLVO considered course information in domain modelling, the coverage of velocity obstacles are different during the encounter process as the course of own ship changes constantly.

5   DISCUSSIONS

In this section, a comparison between the original TD-NLVO (M1) and ship domain-based TD-NLVO (M2) is conducted. The comparison has two components: 1) comparison between results from M1 and M2 and 2) detail analysis of the common results from two different methods. The AIS data utilized is historical AIS data of port Arhus on 1st Oct 2018 from the open access of the Danish Maritime Authority. The parameter setting between the two methods are as follows:

Table 1. Parameter settings for M1 and M2

Parameter	M1	M2
$T_{theshold}$	30s	30s
T_{scan}	60min	60min
criteria	Circular safety region	Elliptical ship domain
radius	1000m	semi-major: 8*length(m) semi-minor: 4*length(m)

With the application of two methods on the same AIS data, the results are significantly different: for M1, the number of collision candidate is 19 meanwhile the number of collision candidate obtained with M2 is 7. The detail of the results can be found in Table 1, Appendix I. The difference can be explained by the difference in criteria choices. As proved in our previous work (Chen et al., 2018), difference choices on the criteria of collision candidate may reveal significant results. However, compared with circular shape region, ship domain, due to its capability on expressing the preference of coverage on different azimuth around ship according to various aspects, e.g. experience of the officer on watch, ship manoeuvrability, etc. is reasonable to be integrated into TD-NLVO. From the trajectory data of the collision candidate, we have found out that M1 have detected some encounters that do not have obvious collision avoidance behaviour, e.g. encounter between

ship "219XXX172" and "219XXX000" which is shown in Fig. 8 (a). For the 12 encounters that were identified by M1 but ignored by M2, 8 out of them falls into this type while rest of them shown certain avoidance behaviour which was ignored by M2, e.g. encounter between "219XXX000" and "248XXX000" (Fig.6 (b)).
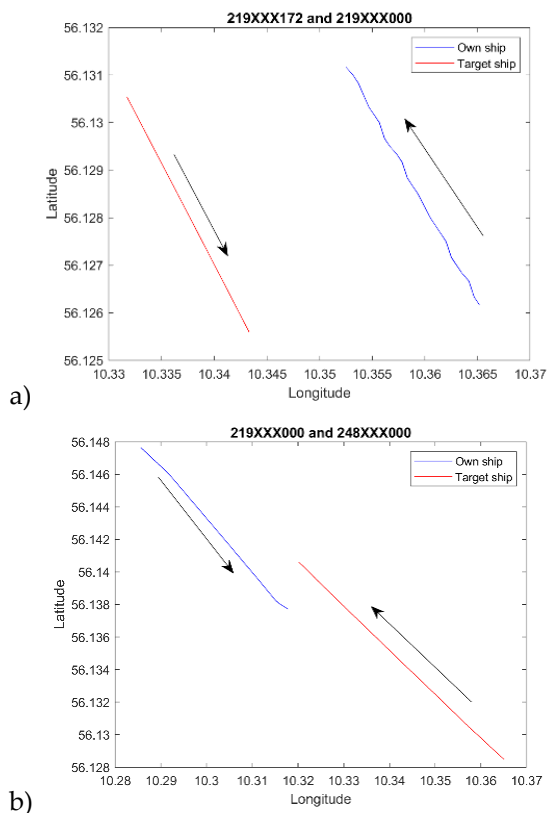

a)

b)

Figure 6. Illustration of encounter obtained with M1

From the data analysis, we can see that the determination of criteria for collision candidate have a strong influence on the absolute number. In practices, such criteria should be determined with caution and taking region traffic characteristics, ship characteristics, etc. into consideration.

As for the 7 common collision candidates from both methods, one can find that the start of detection and duration of violation for results obtained with M1 is in advance to the results obtained with M2 to some extent. The detailed information can be found in Table 2, Appendix I. Taking the encounter between "219XXX000" and "257XXX000" as an example. The trajectories of the two ships are shown in Fig. 3. The velocity obstacle utilized in M1 and M2 at time instance "11:41:35 AM" are shown in the figure below:
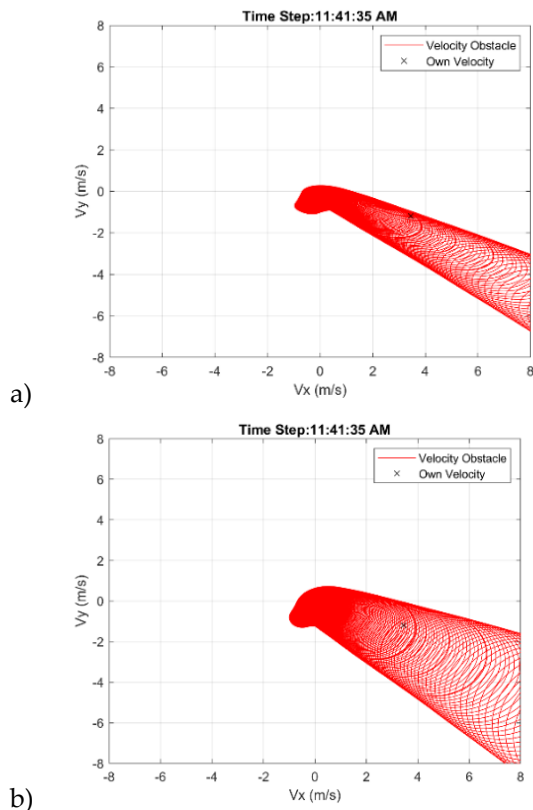

a)

b)

Figure 7. Illustration of Velocity obstacle obtained with M1 and M2 at 11:41:35 AM

From Fig. 7 we can see that at the time step the velocity obstacle obtained with the circular region is obviously larger than ship domain-based velocity obstacle. Two aspects can explain this: 1) the radius of the circular region is set to be 1,000 m while the ship domain of own ship "219XXX000"(85 meters in length) is smaller than such value; 2) due to the introduction of ship domain, the course information of own ship is also considered in the model as shown in Fig. 5. With the different course of own ship, the coverage of velocity obstacle is also different. Under the influence of these two factors, we can see that for Fig. 9(b) the velocity of own ship already falls into the VO while in Fig. 7 (a) it only falls at the boundary of the VO, which can be utilized to explain the time advance in the results from M1.

6 CONCLUSIONS

The maritime transport system is an important component of the global transportation system. In this paper, a modified non-linear velocity obstacle algorithm integrating elliptical ship domain is proposed. Elliptical ship domain was integrated as the new criteria of collision candidate. Based on the historical AIS data, a case study was implemented to demonstrate the efficacy and comparison between existing VO algorithm. The results indicate that: 1) with the integration of elliptical ship domain, the modified TD-NLVO algorithm can reflect the course changes in velocity space of own ship during the encounter; 2) compared to the original version of the algorithm, the modified version reduced the chances of false positive detection of the encounters that do not have obvious collision avoidance behaviours,

however, it also leads to a certain extent of false negative results; 3) compared to criteria of the safety region, the detection time and start of violation of domain-based model are later to some extent, which is caused by the reduced coverage of velocity obstacle.

Based on the results, one can see that with the integration of ship domain more information (course and length of ships) can be incorporated into the process of collision candidate detection. However, the choice of parameters, such as the parameters of the ship domain, have a strong influence on the absolute number of collision candidates detected by the methods. Therefore, to further improve the accuracy of the methods, further efforts can be devoted to determining the criteria considering characteristics of regional traffic, e.g. distribution of ship length, etc. Another aspect that needs further work is how to improve the quality of data to avoid potential underestimation of the results since in the data we introduced we have identified multiple cases where ship length information is missing. In future research, the domain of both ships will be taken into account to avoid the situation where two ships in encounter situation detects violation at different time.

ACKNOWLEDGEMENT

APPENDIX I COLLISION CANDIDATE INFORMATION OBTAINED FROM BOTH MODELS

Table 1. Results of collision candidate obtained from both models

Model	Own ship	Detection period	Target ship	Duration
M1	219 XXX 000	06:40:53 06:46:13	220 XXX 000	06:42:51 06:48:22
	219 XXX 313	07:31:27 07:34:26	219 XXX 172	07:31:30 07:34:48
	219 XXX 72	07:16:57 07:17:45	219 XXX 000	07:17:37 07:17:49
	219 XXX 172	08:43:23 08:47:39	219 XXX 903	08:46:19 08:47:42
	219 XXX 172	11:30:55 11:39:30	219 XXX 000	11:34:42 11:39:40
	219 XXX 172	11:26:57 11:27:43	257 XXX 000	11:27:03 11:28:00
	219 XXX 000	11:42:25 11:59:34	248 XXX 000	11:59:23 12:00:57
	219 XXX 000	11:39:55 11:47:15	257 XXX 000	11:45:54 11:47:39
	219 XXX 000	12:07:33 12:11:31	219 XXX 000	12:11:19 12:11:44
	219 XXX 000	12:04:37 12:06:49	248 XXX 000	12:06:28 12:06:58
	219 XXX 000	12:00:19 12:01:09	257 XXX 000	12:00:25 12:01:50
	219 XXX 000	12:00:04 12:00:38	248 XXX 000	12:00:12 12:00:57
	219 XXX 172	13:06:21 13:11:13	256 XXX 000	13:10:42 13:11:27
	219 XXX 903	14:00:02 14:01:52	246 XXX 000	14:00:42 14:02:08
	211 XXX 340	15:18:06 15:47:36	219 XXX 307	15:44:20 16:03:56
	219 XXX 172	15:42:39 15:43:01	219 XXX 000	15:42:45 15:43:05
	212 XXX 000	16:45:51 16:56:09	219 XXX 903	16:55:31 16:56:20
	219 XXX 172	17:00:18 17:03:45	219 XXX 903	17:02:44 17:03:48
	209 XXX 000	18:00:09 18:20:30	219 XXX 000	18:19:15 18:31:36
M2	219 XXX 000	06:42:55 06:46:13	220 XXX 000	06:44:56 06:48:22
	219 XXX 172	08:43:23 08:47:39	219 XXX 903	08:46:45 08:47:42
	219 XXX 172	11:33:07 11:39:30	219 XXX 000	11:38:20 11:39:40
	219 XXX 000	11:41:35 11:47:15	257 XXX 000	11:46:34 11:47:39
	219 XXX 903	14:01:18 14:01:52	246 XXX 000	14:01:26 14:02:08
	219 XXX 172	17:00:18 17:03:45	219 XXX 903	17:02:55 17:03:48
	209 XXX 000	18:00:09 18:20:30	219 XXX 000	18:19:21 18:31:36

Table 2. Details of common collision candidate from both models

Model	Own ship	Detection period	Target ship	Duration
M1	219 XXX 000	06:40:53 06:46:13	220 XXX 000	06:42:51 06:48:22
	219 XXX 172	08:43:23 08:47:39	219 XXX 903	08:46:19 08:47:42
	219 XXX 172	11:30:55 11:39:30	219 XXX 000	11:34:42 11:39:40
	219 XXX 000	11:39:55 11:47:15	257 XXX 000	11:45:54 11:47:39
	219 XXX 903	14:00:02 14:01:52	246 XXX 000	14:00:42 14:02:08
	219 XXX 172	17:00:18 17:03:45	219 XXX 903	17:02:44 17:03:48
	209 XXX 000	18:00:09 18:20:30	219 XXX 000	18:19:15 18:31:36
M2	219 XXX 000	06:42:55 06:46:13	220 XXX 000	06:44:56 06:48:22
	219 XXX 172	08:43:23 08:47:39	219 XXX 903	08:46:45 08:47:42
	219 XXX 172	11:33:07 11:39:30	219 XXX 000	11:38:20 11:39:40
	219 XXX 000	11:41:35 11:47:15	257 XXX 000	11:46:34 11:47:39
	219 XXX 903	14:01:18 14:01:52	246 XXX 000	14:01:26 14:02:08
	219 XXX 172	17:00:18 17:03:45	219 XXX 903	17:02:55 17:03:48
	209 XXX 000	18:00:09 18:20:30	219 XXX 000	18:19:21 18:31:36

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