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Inherent Properties of Ship Manoeuvring Linear Models in View of the Full-mission Model Adjustment

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ABSTRACT: The paper presents new results on the inherent properties of ship linear dynamics. The focus is made on the second-order formulation for the uncoupled equations of sway and yaw, and on their unique, unknown performance within the zigzag test. From the standpoint of application to full-mission model tuning, a very important loop in the drift-yaw domain of the zigzag behaviour, as governed by the rudder rate dependent time constants (of T3-type), is brought to the light. This and some other dependent effects, like overshoot angle performance, are likely to be lost, if the well-known, rather ambiguous, first-order approximations are deployed.

1 INTRODUCTION

The linear dynamic models of first- or higher-order, not only in the field of ship steering or manoeuvring, have been inspiring researchers for decades, and still draw our attention nowadays. Despite some drawbacks, they are simple, can often provide an efficient analytical solution that can be easily studied for exact and direct inherent relationships within the investigated dynamics, mostly constituting a more or less nonlinear problem. The dynamic models of ship manoeuvring can be of hydrodynamic type (with parameters as hydrodynamic derivatives) or the equivalent input-output (transfer function) type. The parameters of the latter type cover various time constants and amplification ratios.

With regard to the coupled ship sway (drift) and yaw motions in the linear formulation, they can be well either described by a single two-dimensional linear model of first-order (as set of two coupled linear ODEs of first-order) or by two uncoupled one-dimensional models of second-order for each motion. We cautiously omit here a discussion on the validity range of this linearity.

Over the years, various identification techniques (including system identification) for parameters of the two hydrodynamic and input-output types of models, especially in their linear form and for the combined sway-yaw motions, as of concern in the present paper, were developed and are still under improvement efforts – e.g. [Kallstrom, 1979], [Holzhuter, 1990], [Terada, 2015]. The ship motion phenomenon and measurement experiments are actually complicated. The last word has not been said yet. Although the conversion of hydrodynamic description to transfer function description, and analysis of dynamic systems in the latter, convenient form, is firmly established in literature, e.g. [Nomoto et al., 1957], [Lisowski, 1981], [Dudziak, 2008], the inverse transformation is practically missing.

Within the full-mission ship handling simulator mathematical models, very sophisticated and nonlinear, the so-called four-quadrant operation and lookup-table data storage is standard requirement for modelling the hull, propeller, and rudder hydrodynamics – e.g. [Lebedeva et al., 2006], [Artyszuk, 2013] , [Sutulo, Guedes Soares, 2014]. With regard to hull and rudder forces in particular, we focus on arbitrary combinations of drift angle and dimensionless yaw velocity, as their arguments, and consider appropriate plots/curves in the drift-yaw plane. Based on recorded motions, an attempt is made to efficiently fix the values of hydrodynamic coefficients – the nodes of lookup-tables. In this context, a special interest is being placed on designing high quality manoeuvring trials, such that bring a lot of information for comprehensive and unique calibration of the math model. In this process, we are also looking for analytical techniques of those trials, similar in type to that of [Nomoto, 1960], to effortlessly and quickly arrive at some parameters, that can be next transformed to the background 'full-mission' hydrodynamic model.

The existing zigzag test also seems to provide necessary data. However, the most frequently used first-order Nomoto approximation for uncoupled motions, though originally introduced and discussed for yaw motion [Nomoto et al., 1957], proves to be inadequate when we want to revert back to the basic hydrodynamics. In the latter aspect, at least for zigzag test, we are thus forced to fully maintain the original second-order formulation of uncoupled motions. Very crucial parameters of this representation are the so-called T_3-time constants, derived from and responsible for the essential interaction between sway and yaw. These constants surprisingly lack a proper appreciation in the past research. A tribute shall here be passed to [Norrbin, 1996], who as one of not many tried to consider some aspects of T_3 problem in respect of ship hydrodynamics.

Of course, a big challenge is here to develop a deterministic, curve fitting method of zigzag data for this dual (sway & yaw) second-order model, but it is out of the scope of the present study. Instead of, some new facts on sensitivity effects of T_3 are revealed, which shall be helpful in designing and implementing such an identification method.

This conceptual, theoretic paper, though supported by a numerical analysis, is subdivided into several chapters. We start from recalling and discussing the basic linear system of differential equations for sway and yaw manoeuvring motions, its hydrodynamic structure and the second-order uncoupled version. The most innovative yet very important and meaningful results, though simple in methodology, are presented in the next three chapters. Therein starting from deriving the inverse formulas for the second-order models, by which the transfer function parameters are converted to hydrodynamic coefficients. Based on them, some investigations are next conducted on the great role of the mentioned so-called T_3 time constants in transfer function description. Finally, a rational proposal follows on how to fix the detailed hydrodynamic coefficients, if the aggregate hydrodynamic coefficients, as obtained from the mentioned inverse formulas, are known.

2 2D LINEAR MODEL OF SHIP MANOEUVRING

The coupled linear ordinary differential equations of the first-order with constant coefficients for sway and yaw velocities of a ship are worldwide known in the field of ship manoeuvring and ship control engineering. They constitute a basis for deriving the very famous direct (uncoupled, or independent of sway) the 2nd order linear differential equation of yaw motion, traditionally referred to as the 2nd order Nomoto model. This can be next approximated to the first-order linear equation of yaw, the so-called 1st order Nomoto model [Nomoto et al., 1957], [Dudziak, 2008].

The stated above models can be formulated in dimensional, i.e. absolute units of velocities and time, or just made dimensionless. The dimensionless time, by rule, is expressing the advancing time counted in units of time that a ship requires to cover its own length and is fully equivalent to dimensionless distance, i.e. the distance travelled by a ship as rated in her own length units.

The dimensionless quantities are much better in analysis, since they provide universal steering characteristics, as independent of the ship's size/length and forward (surge) velocity. One of essential assumptions underlying the linear model, much stronger in the fully dimensionless case, is the constant surge velocity.

The adopted notation of coefficients in the coupled equations for sway and yaw varies from author to author, where we may generally distinguish two styles - the western (international) and the eastern (Russian) one. For the purpose of the present study, however, the following is applied:

$$\begin{cases} \dfrac{d\beta}{ds'} = a_1\beta + b_1\omega'_z + c_1\delta \\ \dfrac{d\omega'_z}{ds'} = a_2\beta + b_2\omega'_z + c_2\delta \end{cases} \tag{1}$$

where:
β - drift angle [rad] at ship's origin, positive when laterally moving to port, $\beta = \arctan(-v_y / v_x)$, v_x and v_y - ship's surge and sway velocities,
ω'_z - relative yaw velocity [-], positive for turning to starboard, $\omega'_z = \omega_z L / v$, where: ω_z - yaw velocity, L - ship's length, v - total linear velocity (as resulted from v_x and v_y),
s' - dimensionless time/distance [-], $s' = s/L = t \cdot v/L$, in which: t - time or s - distance,
δ - rudder angle in [rad] as input control, positive when to port.

Additionally, one can write $\omega_z = d\psi/dt$, where ψ is the heading angle.

3 STRUCTURING THE MODEL

Except for c_1 and c_2, as solely connected with the rudder hydrodynamic force, all other coefficients in (1) combine the effects from both a ship's hull ('*H*') and rudder ('*R*'). In addition, the coefficient b_1 has a very important contribution from the centrifugal force

('C') involved in the development of drift angle. All the coefficients can easily be derived (or approximated) from a detailed description of hydrodynamic forces laid down at a core of the mentioned full mission models. The core mainly consists in storing relevant relationships in the form of lookup-tables. Around a certain point, those can make up the usual analytical form, known from other simpler models, and even be reduced to linear model. A practical example of such relationships, which can be suited to any existing approach was presented in [Artyszuk, 2013]. Those are quoted, rearranged, and simplified to meet the definition of the six coefficients in (1) - a_i, b_i, c_i, where $i = 1, 2$ - as follows:

$$a_1 = a_{1H} + a_{1R}\,,\;\; b_1 = b_{1H} + b_{1R} + b_{1C} \tag{2a}$$

$$a_2 = a_{2H} + a_{2R}\,,\;\; b_2 = b_{2H} + b_{2R} \tag{2b}$$

$$a_{1H} = -\frac{0.5\dfrac{L}{B}\dfrac{1}{c_B}}{1+k_{22}} Y'_b \cdot \frac{180}{\pi}\,, \quad \text{where } Y'_b > 0 \tag{3}$$

$$a_{1R} = -\frac{R}{1+k_{22}} \frac{c_{Ry}}{(1-w)\sqrt{1+c_{Th}}}\,, \text{ where } c_{Ry} > 0 \text{ and} \tag{4}$$

$$R = 0.5\frac{L}{B}\frac{1}{c_B} A'_R (1-w)^2 (1+c_{Th}) \left.\frac{\partial c_L(\alpha, c_{Th})}{\partial \alpha}\right|_{\alpha=0°} \cdot \frac{180}{\pi}(1+a_H)\,,$$
where $a_H > 0$ (5)

$$b_{1H} = -\frac{0.5\dfrac{L}{B}\dfrac{1}{c_B}}{1+k_{22}} Y'_w\,, \text{ where } Y'_w > 0 \text{ (mostly for}$$
modern ships, of skeg-shaped stern), (6)

$$b_{1R} = -\frac{R}{1+k_{22}} \frac{c_{Ry} \cdot -x'_{Reff}}{(1-w)\sqrt{1+c_{Th}}} = -a_{1R} x'_{Reff}\,, \text{ where } x'_{Reff} < 0 \tag{7}$$

$$b_{1C} = \frac{1+k_{11}}{1+k_{22}} \tag{8}$$

$$c_1 = -\frac{R}{1+k_{22}} \tag{9}$$

$$a_{2H} = \frac{0.5\dfrac{L}{B}\dfrac{1}{c_B}}{r'^2_z + r'^2_{66}} N'_b \cdot \frac{180}{\pi}\,, \quad \text{where } N'_b > 0 \tag{10}$$

$$a_{2R} = \frac{R}{r'^2_z + r'^2_{66}} \frac{c_{Ry}}{(1-w)\sqrt{1+c_{Th}}} \cdot x'_{Reff} = -\frac{1+k_{22}}{r'^2_z + r'^2_{66}} a_{1R} \cdot x'_{Reff} \tag{11}$$

$$b_{2H} = \frac{0.5\dfrac{L}{B}\dfrac{1}{c_B}}{r'^2_z + r'^2_{66}} N'_w\,, \quad \text{where } N'_w < 0 \tag{12}$$

$$b_{2R} = \frac{R}{r'^2_z + r'^2_{66}} \frac{c_{Ry} \cdot -x'_{Reff}}{(1-w)\sqrt{1+c_{Th}}} x'_{Reff} = \frac{1+k_{22}}{r'^2_z + r'^2_{66}} a_{1R} \cdot x'^2_{Reff} \tag{13}$$

$$c_2 = \frac{R}{r'^2_z + r'^2_{66}} x'_{Reff} = -\frac{1+k_{22}}{r'^2_z + r'^2_{66}} c_1 x'_{Reff} \tag{14}$$

Particular dimensionless elements of the above expressions (3) to (14) can be explained as below:

hull-related items:
L/B - ship's hull length-to-beam ratio,
c_B - block coefficient,
k_{11} - surge added mass coefficient, $k_{11} = m_{11}/m$, where m_{11} - surge added mass, m - ship's displacement (mass),
k_{22} - sway added mass coefficient, $k_{22} = m_{22}/m$, where m_{22} - sway added mass,
r'_z - ship's gyration dimensionless radius, $r'_z = J_z/(mL^2)$, where J_z - ship's mass moment of inertia,
r'_{66} - added gyration dimensionless radius, that is $r'_{66} = m_{66}/(mL^2)$, where m_{66} - added moment of inertia,
Y'_b, Y'_w, N'_b, N'_w - hull hydrodynamic (dimensionless) derivatives; Y'_b and N'_b, in view of the conversion factor $180/\pi$ in (3) and (10), are computed with reference to β in [°]; N'_b is assumed to include/integrate the Munk moment contribution, as usually seen while reporting experimental results.

rudder-related items:
A'_R - rudder area ratio, $A'_R = A_R/(LT)$, where: A_R - rudder area, LT - ship's length-draft product,
w - propeller wake fraction,
c_{Th} - propeller thrust loading coefficient, $c_{Th} = \dfrac{8}{\pi}\dfrac{k_T(J)}{J^2}$, where J - advance ratio, k_T - thrust coefficient,
$\left.\dfrac{\partial c_L(\alpha, c_{Th})}{\partial \alpha}\right|_{\alpha=0°}$ - rudder lift coefficient derivative vs. flow incidence angle α[°] for a given c_{Th}, taken at α=0°; c_L is defined herein with regard to propeller race velocity – see (5),
a_H - empirical amplification factor of (effective) rudder force due to hull-rudder interaction,
c_{Ry} - empirical multiplier (≥1 or <1) to the rudder geometric local drift angle to arrive at its effective local drift angle; c_{Ry}=1 means equality of both; furthermore, it is indirectly assumed that a ship's drift and yaw have equal effects on this effective local drift,
x'_{Reff} - effective rudder longitudinal position (the effective location of the rudder force), dimensionless in ship's length units; for $x'_{Reff} = -0.5$ we get the

nominal/physical position of the rudder force at aft perpendicular; as supplementing the 'action' of a_H, this coefficient also arises from hull-rudder interaction but in terms of the effective rudder force arm, $x'_{Reff} \geq -0.5$ or even $x'_{Reff} < -0.5$ are allowed.

All the terms in (3) to (14), except for the seven mostly uncertain and empirically determined coefficients - 4 related to hull (Y'_b, Y'_w, N'_b, N'_w) and 3 associated with rudder (a_H, c_{Ry}, x'_{Reff}) - can be referred to as the formal (reference, nominal) quantities. Their values are to be established by means of usually available geometric or hydrodynamic prediction methods. Any uncertainty/bias within them is allowed since the 'final' accurateness of forces and moments is to be reached through tuning of the aforementioned 7 dimensionless empirical coefficients. At this stage of research, the three rudder parameters are considered constants, however, according to this author's past investigations, a certain functional relationship with motion and control variables seems quite likely.

4 DECOUPLED CLASSICAL DRIFT AND YAW EQUATIONS

The basic hydrodynamic equations (1) impose problems when someone wants to relate a ship's kinematic response for a given control input (in terms of rudder angle) to their coefficients. The improvement goals using such efforts may be multiple – from ship design, through ship steering control, to full mission simulator performance in nautical studies, like in our case. In this context, and in view of transformations proposed in the next section, it seems necessary to recall and briefly discuss the well-known classical relationships relevant to the uncoupled equations.

The set of linear equations of the first-order (1) can easily be transformed to fully equivalent time responses of drift and yaw, being the second-order linear equations of particular motions:

$$T_1T_2\frac{d^2\beta}{ds'^2}+(T_1+T_2)\frac{d\beta}{ds'}+\beta=K_b\left(\delta+T_{3b}\frac{d\delta}{ds'}\right) \tag{15}$$

$$T_1T_2\frac{d^2\omega'_z}{ds'^2}+(T_1+T_2)\frac{d\omega'_z}{ds'}+\omega'_z=K_w\left(\delta+T_{3w}\frac{d\delta}{ds'}\right) \tag{16}$$

Although the drift equation (15) is rather of less interest and seldom challenged in literature, it is obviously very crucial for keeping uniqueness and identification of the basic set (1). Particular definitions of time constants (marked with 'T' symbols) and amplification constants ('K' notation), both of practical response interpretation, are summarised below:

$$T_1=-\frac{1}{0.5\left(a_1+b_2+\sqrt{(a_1+b_2)^2-4(a_1b_2-a_2b_1)}\right)} \tag{17}$$

$$T_2=-\frac{1}{0.5\left(a_1+b_2-\sqrt{(a_1+b_2)^2-4(a_1b_2-a_2b_1)}\right)} \tag{18}$$

$$T_{3w}=\frac{c_2}{-a_1c_2+a_2c_1} \tag{19}$$

$$T_{3b}=\frac{c_1}{-b_2c_1+b_1c_2} \tag{20}$$

$$K_w=\frac{-a_1c_2+a_2c_1}{a_1b_2-a_2b_1} \tag{21}$$

$$K_b=\frac{-b_2c_1+b_1c_2}{a_1b_2-a_2b_1} \tag{22}$$

The time constants T_1 and T_2, given above explicitly and appearing identically in both equations (15) and (16), are sometimes quoted in a more convenient, equivalent way, namely implicitly in the form of their product and sum:

$$T_1T_2=\frac{1}{a_1b_2-a_2b_1} \tag{23}$$

$$T_1+T_2=-\frac{a_1+b_2}{a_1b_2-a_2b_1}=-(a_1+b_2)T_1T_2 \tag{24}$$

All the expressions (17) to (22), particularly when applying (23) and (24), have a direct, practical meaning while studying the time response of a ship to certain rudder actions.

T_{3b} and T_{3w}, called hereafter as T_3-type constants, are connected with rudder (deflection) rate – its sign and magnitude. They can oppose or magnify the effect of rudder angle.

For a dynamically (directionally) stable ship there holds a practical dual condition (see e.g. [Dudziak, 2008]):

$$T_1T_2>0 \tag{25}$$

$$\frac{T_1+T_2}{T_1T_2}>0 \quad \text{or} \quad T_1+T_2>0 \quad \text{in view of (25)} \tag{26}$$

which lead to the basic stability criterion, in which T_1 and T_2 should be both positive. These inequalities are satisfied when:

$$a_1b_2-a_2b_1>0 \tag{27}$$

$$a_1+b_2<0 \tag{28}$$

but with regard to (3), (4) - ensuring $a_1<0$ - and (12), (13) - leading to $b2<0$ - the stability condition for marine vessels can be solely reduced to equation (27). However, the magnitudes for both a_1 and b_2 have their direct influence on the first term in (27) and thus on the stability.

The time-domain simulation of response to any rudder action is symmetrical versus T_1 and T_2 in that if we interchange their values in place of one another there will be no change in response. In addition, T_1 and T_2 calculated by the expression (17) and (18) accordingly always provide the case $T_1>T_2$ (even $T_1>>T_2$) for a stable ship.

In summary, we have a set of 6 new coefficients (of *T*-, and *K*-class) instead of the original set (of *a*-, *b*-, and *c*-class) in equations (1). Both sets are invertible to each other as being shown next. However, as mentioned before, the inverse problem of getting the original coefficients of (1) is seldom undertaken in research. Moreover, the identification procedures of *T*- and *K*-class constants based on ship motion response do not exist for the full second-order linear equations. This is even true in case of the single equation for yaw motion (16).

Such algorithms mostly deal with the reduced (of lower amount of information), first-order equations of two unknown parameters, and stable ships. The widely used here zigzag of 10°/10° type or of another type, but with finite yaw response, often seems to be excessive to establish a linear yaw model for an unstable ship. In that, the identification procedure itself (of a certain integral approximation/fitting towards a linear model), as redefined in [Nomoto, 1960], and the used actually 'overlinearized' zigzag response due to the assumed relatively large variation of nominal rudder and heading (even of only 10° magnitude), nearly always leads to response models of more or less but stable ships.

5 DERIVATION OF INVERSE FORMULAS

The mentioned inverse conversion of the six *T*-, and *K*-class constants, if such are known for both drift and yaw, to the basic six hydrodynamic coefficients (of *a*-, *b*-, and *c*-class) in (1) is presented below in condense, natural order:

$$a_1 = \frac{T_1T_2 - T_{3b}\left(T_1 + T_2 - T_{3w}\right)}{\left(T_{3b} - T_{3w}\right)T_1T_2} \tag{29}$$

$$b_1 = \frac{K_b}{K_w} \cdot \frac{-T_1T_2 + T_{3b}\left(T_1 + T_2 - T_{3b}\right)}{\left(T_{3b} - T_{3w}\right)T_1T_2} \tag{30}$$

$$a_2 = \frac{K_w}{K_b} \cdot \frac{T_1T_2 - T_{3w}\left(T_1 + T_2 - T_{3w}\right)}{\left(T_{3b} - T_{3w}\right)T_1T_2} \tag{31}$$

$$b_2 = \frac{-T_1T_2 + T_{3w}\left(T_1 + T_2 - T_{3b}\right)}{\left(T_{3b} - T_{3w}\right)T_1T_2} \tag{32}$$

$$c_1 = \frac{T_{3b}K_b}{T_1T_2} \tag{33}$$

$$c_2 = \frac{T_{3w}K_w}{T_1T_2} \tag{34}$$

where a_1 and b_2 are solely based on the time constants.

The details of those derivations are as follows:

Step 1

After combining the four equations (19) to (22) with the relationship (23) we have:

$$T_{3w} = \frac{c_2 \cdot T_1T_2}{K_w} \tag{35}$$

$$T_{3b} = \frac{c_1 \cdot T_1T_2}{K_b} \tag{36}$$

which lead straight to (33) and (34).

Step 2

Substituting the just received definitions of c_1 and c_2, stored in (27) and (28), to equations (21) and (22), and again deploying (23), we arrive at:

$$-T_{3w}K_wa_1 + T_{3b}K_ba_2 = K_w \tag{37}$$

$$T_{3w}K_wb_1 - T_{3b}K_bb_2 = K_b \tag{38}$$

which shall be next coupled with (23) and (24), as uniquely representing (17) and (18), but written in such a form:

$$a_1b_2 - a_2b_1 = \frac{1}{T_1T_2} \tag{39}$$

$$-\left(a_1 + b_2\right) = \frac{T_1 + T_2}{T_1T_2} \tag{40}$$

Hence a set of four, apparently nonlinear algebraic equations – (37) to (40) – is being received, that shall be solved against the missing unknowns: a_1, a_2, b_1, b_2. Speaking precisely, equation (39) is the only 'nonlinear' within this set, but this 'nonlinearity' can be resolved into elementary, linear relationships after taking advantage of the other three equations. At first glance, however, the required transformations for this task are not so clear.

The solution of the set (37) to (40) can be obtained analytically. For example, let's determine a_2 from (37) and b_2 from (38) and then substitute both to (39) and (40). The latter two equations shall now be solved for the unknowns a_1 and b_1. Using these values, the final values of a_2 and b_2 are provided after returning back to (37) and (38).

Not only the final relationships (29) to (34) are useful, but such are also the intermediate equations (37) to (40), especially while seeking for mutual relationships between the hydrodynamic coefficients in arbitrary groups, when some of them have already been fixed.

Of major importance also appears a sensitivity of the results for *a*- and *b*-class coefficients – see (29) to (32) – to the accuracy of estimating the time constants related to rudder rate: T_{3b} and T_{3w}, particularly to their difference. The value close to zero in the denominator of these expressions implies very high values of the parameters: a_1, a_2, b_1, b_2.

6 ROLE OF T_{3B} AND T_{3W} IN SHIP HYDRODYNAMICS – NUMERICAL EXAMPLE

Exemplary values of particular data in formulas (3) to (14) for a hypothetical ship, without any claim to be exact, are presented in Tab. 1, though to some extent they originate in the author's previous full-scale identification studies and lookup-table modelling on a small chemical tanker [Artyszuk, 2013]. The dimensional values of ship's length (constant) and her forward speed (variable as specific to a given manoeuvre) are necessary to convert the rudder rate from the absolute $d\delta/dt$ in [°/s] to $d\delta/ds'$[°/-]. Except as explicitly stated, the rudder rate of 2.5°/s has been chosen that is slightly above the minimum international requirement for steering gear (2.3°/s).

The individual contributions to the coefficients in (1), classified according to the source of forces, see (2a) and (2b), are collected in Tab. 2. Herein, the rudder has up to 30% significant contribution in all terms, which is sometimes forgotten, when using a simple rudder effect as coupled only with the helm angle δ. Practically, the sign of the rudder contribution, as compared to hull, is only opposite for the drift-related yaw moment – refer to a_{2H} and a_{2R} in Tab. 2.

The final values of the direct (*a*-, *b*-, and *c*-) constants in (1), in parallel with *T*- and *K*-constants, are demonstrated in the upper part of Tab. 3. This condition of the ship is referred to as the reference case. The lower part of Tab. 3 contains the influence of the variation of constants T_{3b} and T_{3w} (rather small in magnitude, at least, as compared to T_1) on the computation of *a*- and *b*-, c- constants while keeping the values of the other *T*- and *K*-type parameters. The huge sensitivity of the steering dynamic model to the considered T_3-type constants is here evident. They almost affect all basic parameters of the model in (1), sometimes even changing the sign.

Table 1. Elementary input data (dimensionless by default, except as given explicitly)

hull		rudder	
L[m]	97.4	A'_R	0.0177
v [m/s]	7.272	w	0.326
L/ v [s]	13.39	c_{Th}	2.127
		$\partial c_L/\partial \alpha$ @ c_{Th}	0.0385
L/B	5.867		
c_B	0.761	a_H	0.6
r'_z	0.247	c_{Ry}	1.0
k_{11}	0.056	x'_{Reff}	-0.5
k_{22}	1.004		
r'_{66}	0.225		
Y'_b	0.0043		
Y'_w	0.0260		
N'_b	0.0024		
N'_w	-0.0630		

Table 2. Final contributions to constants of equations by their nature (the reference case)

a_{1H} a_{1R}	b_{1H} b_{1R} b_{1C}	a_{2H} a_{2R}	b_{2H} b_{2R}
-0.479 -0.144	-0.050 -0.072 0.527	4.843 -1.291	-2.182 -0.646

Table 3. Constants of equations

CASE - reference	T_1 T_2	T_{3b} T_{3w}	K_b K_w
	10.491 0.298	0.154 0.983	-3.464 -4.896
	a_1 a_2	b_1 b_2	c_1 c_2
	-0.622 3.552	0.405 -2.827	-0.171 -1.539
	Below given the a- , b- & c-constants in same order as above		
CASE - T_{3b} variation T_{3b} =0.309	-0.046 4.366	-0.037 -3.403	-0.342 -1.539
CASE - T_{3w} variation T_{3w} =0.492	-1.458 2.594	0.996 -1.992	-0.171 -0.770

The simulation of standard 10°/10° zigzag manoeuvre is performed in the subsequent Figs. 1 to 9. The '*REF*' curves here correspond to the reference case, see Tab. 3. This rather simple test, as compared to others, brings comprehensive information on ship behaviour, especially if we consider both drift and yaw together of varying signs All the computations have been made by direct integration of (1), including the differential equation for heading angle, using the Euler method (still powerful for this specific problem) with dimensionless time step $\Delta s'$=0.05.

Figs. 1 and 2 present the heading variation, helm angle and the both kinematical variables as directly governed by our dynamic equations – drift angle and dimensionless rate of turn. The manoeuvre itself for our ship essentially lies within transient states because the range of kinematics shown in Fig. 2 is much lower than the steady-state values – β_0= 34.6° for the drift angle (= $K_b\delta_0$) and ω'_{z0}= 0.854 for the relative yaw velocity (= $K_w\delta_0$).

One of the most noticeable features of the second-order linear formulation of uncoupled steering dynamics with regard to drift and yaw, see (15) and (16), as equivalent to the full set of (1), is a partly independent change of drift angle and dimensionless yaw velocity. Moreover, the increase/decrease in yaw is much higher ('of low inertia') than of the drift angle – Fig. 2 and 5. For zigzag manoeuvre, such a behaviour produces – Fig. 3 and 4 – a certain, closed loop of the mutual relationship $\omega'_z=\omega'_z(\beta)$ in the plane of the domain of the dimensionless hull hydrodynamic forces, being functions of just drift angle and dimensionless yaw velocity. The wider the loop, the better for the fitting or validation of the hull force response surface as a 3D representation of two-variable relationship [Artyszuk, 2013]. The stated herein performances are not exhibited at all by the so-called first-order uncoupled Nomoto models, for certain reasons, much more frequently used than the former, original ones. These first-order approximations are based on the criterion proposed in [Nomoto et al., 1957] and quoted below:

$$T_b \frac{d\beta}{ds'} + \beta = K_b \delta \text{, where } T_b = T_1 + T_2 - T_{3b} \qquad (41)$$

$$T_w \frac{d\omega'_z}{ds'} + \omega'_z = K_w \delta \text{, where } T_w = T_1 + T_2 - T_{3w} \qquad (42)$$

Since T_{3b} and T_{3w} are rather small, T_b of (41) is quite close in magnitude to T_w in (42).

For detailed comparison, the output of the above 1st order models is also included in our analysis and marked by '1st ORD' in Figs. 3 to 6, and 8. However, in the case of these 1st order models, the derivatives of drift angle and yaw velocity in Figs. 5 , 6, and 8, and the resulting direct values of these two variables in Figs. 3 and 4, are considered only for the most representative, initial period of the zigzag manoeuvre with the first rudder execute. The rudder is then simplistically kept in this position (the counter-rudder is no longer applied), that enables a very efficient analytical solution of (41) and (42), which is adopted. Two versions of rudder control are studied for the 1st order models - the infinitely rapid (step) movement, as the limiting case, denoted by 'δ=const' and cyan-coloured, and the trapezoidal steering ('δ=var', brown color), with the same rudder rate as used in computing the corresponding 'second-order' response.

The corresponding β-ω'_z curve for the first-order models is practically an open, straight line inclined an angle arising from the ratio of steady state values of drift angle and dimensionless yaw velocity, or just directly from K_w/K_b. Figs. 3 and 4 present the 1st-quadrant section of this curve, which is quite independent of the model version used – with infinite or with finite rudder rate – and of the rudder alternate control strategy like in zigzag test. Combining both models (41) and (42), this curve is defined by:

$$\omega'_z = \omega'_{z0}\left[1-\left(1-\frac{\beta}{\beta_0}\right)^{T_b/T_w}\right] \qquad (43)$$

It thus means that the $\omega'_z=\omega'_z(\beta)$ relationship of the 1st order uncoupled models, (41) and (42), loses a lot of essential information from the original background hydrodynamics expressed by (15) and (16), or just by (1). Moreover, in the latter case, the derivative of yaw velocity in Fig. 5 experiences a significant peak that is damped for the 1st order approximation, which in consequence leads to quite different overshoot angles and oscillation periods in the heading diagram. However, this heading performance for the 1st order uncoupled equations, has not been shown in the paper.

When reducing the rudder rate from the reference 2.5°/s to the abstract value of 0.5°/s, one can achieve an efficient convergence of the second-order uncoupled dynamics to the first-order one because of the relatively low influence of the T_{3b} and T_{3w} constants, which shall be rather obvious – see Figs. 6 and 7. However, the derivative of yaw in the second-order response still displays the initial jump that is responsible for the occurrence of a loop around the straight line section of the first-order model in the β-ω'_z domain.

Furthermore, besides the case of simultaneously very low T_{3b} and T_{3w}, the second-order uncoupled dynamics also converges to first-order one for T_{3b} close to T_{3w}, independent of their absolute values. Of course, in view of (29) to (34), this gives exaggerated, almost enormous values for *a*-, *b*-, *c*-constants.

T_{3b} and T_{3w} have great impact (being much higher for T_{3w}) on flattening or spreading of the β-ω'_z plot in Figs. 3 and 4. The corresponding coefficients of (1) for the tested variations in those time constants were quoted in Tab. 3. The constant T_{3b} does not affect the yaw behaviour at all. The same should obviously happen with regard to the T_{3w} variation as expected to completely preserve the drift angle image. This is demonstrated in Fig. 8 and can even be proved analytically. The heading accompanying the reduction of T_{3w} has already been incorporated in the initial Fig. 1. However, since the rudder control in the zigzag manoeuvre is essentially heading- or yaw-based, the preservation of the drift angle for the varying T_{3w} is being held only within the initial period of the test, i.e. up to the first counter-rudder. Thereafter, the drift angle differential equation is being solved with the relative yaw velocities as 'not corresponding' to the actual drift angles and helm angles. Fig. 9 shows this situation. In general, T_{3b} implies a horizontal expansion/contraction in the loop of the β-ω'_z, while T_{3w} acts more universally, namely changing the loop in both direction - see again Figs. 3 and 4. Increasing T_{3w}, which is however not shown in the chart of Fig. 4, leads to the inverse scaling of the $\omega'_z(\beta)$ loop, such that we have a significant contraction along β-axis and a large expansion in the direction of ω'_z-axis.

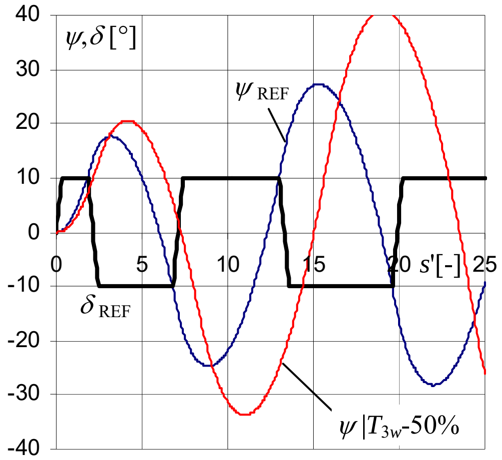

Figure 1. Heading and helm (incl. T3w variation)

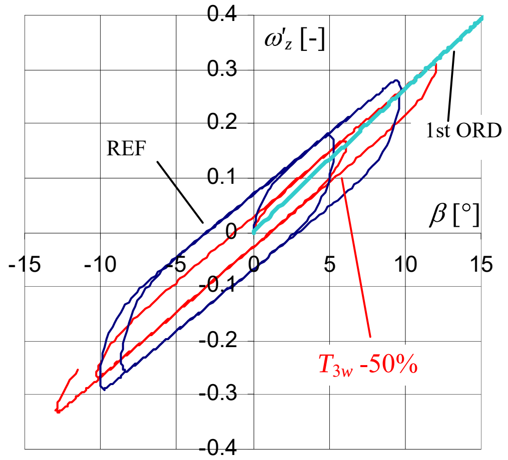

Figure 4. Dimensionless drift-yaw domain - T3w effect

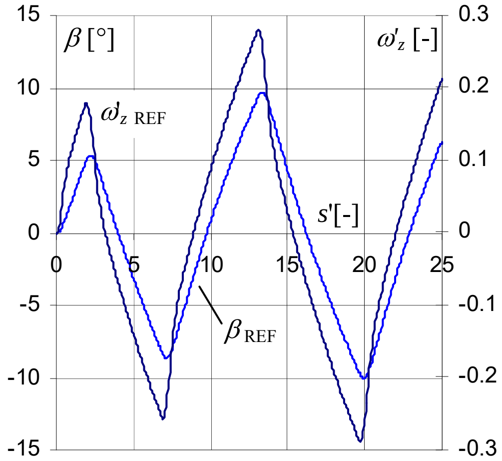

Fig. 2. Drift angle and relative yaw velocity

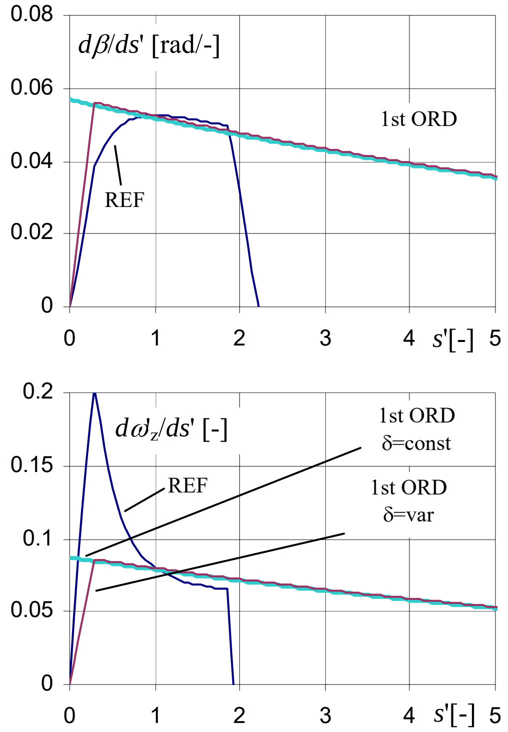

Figure 5. Derivatives vs. 1st order Nomoto model

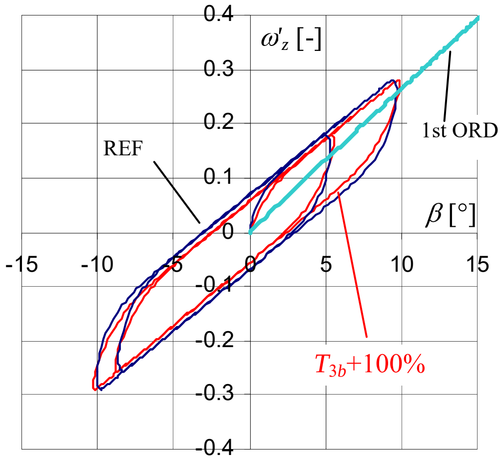

Figure 3. Dimensionless drift-yaw domain - T3b effect

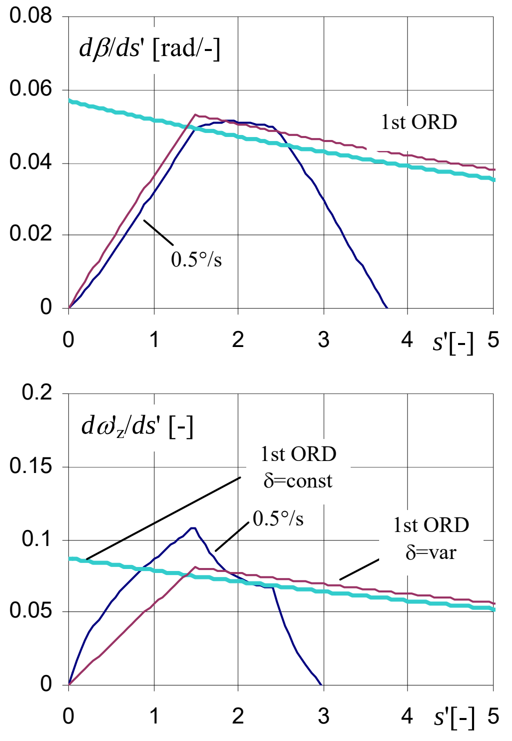

Figure 6. Derivatives for reduced rudder rate

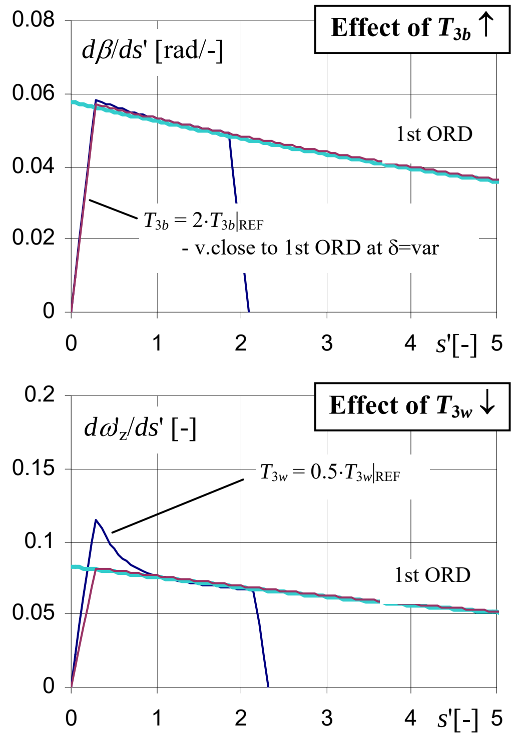

Figure 8. Effects of T3b and T3w on derivatives

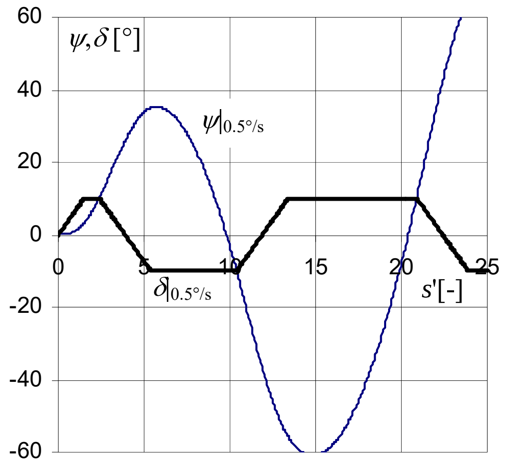

Figure 7. Helm and heading for reduced rudder rate

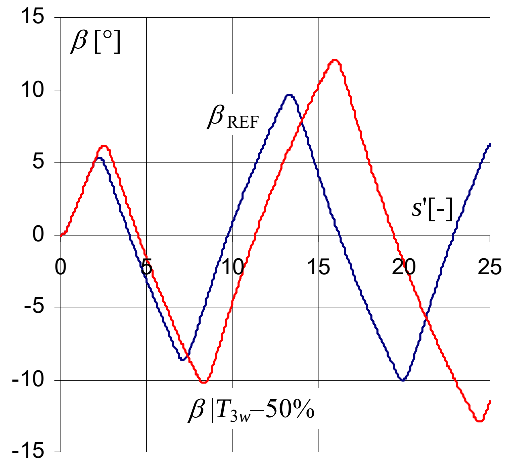

Figure 9. Drift angle response to increasing T3w

7 PROPOSED DETERMINATION OF DETAILED HYDRODYNAMIC PARAMETERS

If we move from the *T*- and *K*-class constants, six in total, determined through the analysis of kinematic response to a certain rudder control, to the six *a*- to *c*-class constants, by means of the inverse formulas (29) to (34), we can attempt to obtain the detailed parameters underlying the latter constants. In the 'Structuring the model' section seven natural unknowns – $\{Y'_b, Y'_w, N'_b, N'_w\}$ and $\{a_H, c_{Ry}, x'_{Reff}\}$ – were specified in this context. However, we now arrive at an indeterminate (overparameterised) set of algebraic equations because of too many unknowns in relation to the number of equations. One of the parameters should be thus fixed. Based on available model test data and/or other methods, any of those 7 coefficients could be selected for this purpose. In view of the potential estimation uncertainty, such selection will have an impact on the model validity while simulating specific manoeuvres.

In view of our derivations, the terms c_1 and c_2 responsible for the effect of helm angle δ, see (5), (9) and (14), can be resolved into:

$$c_1 = c_1(a_H) = (1 + a_H) \cdot f_1 \tag{44}$$

$$c_2 = c_2(a_H, x'_{Reff}) = (1 + a_H) x'_{Reff} \cdot f_2 \tag{45}$$

where f_1 and f_2 have been introduced to reflect the rest part of the corresponding expressions.

Hence, at the first stage we directly receive a_H from (44), and next x'_{Reff} from (45), and come to the following 4 equations with 5 unknowns:

$$\begin{cases} a_1 = Y'_b \cdot f_{3a} + c_{Ry} \cdot f_{3b}(a_H) \\ b_1 = Y'_w \cdot f_{4a} + c_{Ry} \cdot f_{4b}(a_H, x'_{Reff}) + b_{1C} \\ a_2 = N'_b \cdot f_{5a} + c_{Ry} \cdot f_{5b}(a_H, x'_{Reff}) \\ b_2 = N'_w \cdot f_{6a} + c_{Ry} \cdot f_{6b}(a_H, x'_{Reff}) \end{cases} \tag{46}$$

where similarly the f-symbols represent the appropriate relationships.

In the light of the state-of-the-art in ship manoeuvring hydrodynamics, it is suggested to fix one of the drift-dependent hull terms in (46) - Y'_b or N'_b - as relatively well worked out and published in the literature.

Of course, any uncertainty in estimating the rudder parameters will be 'corrected' by relevant recalibration of the hull parameters (and vice versa) due to the same physics or dependence involved in the background expressions. However, this will be paid for by errors in simulation when, for example, a ship is subject to manoeuvring without rudder action, like in wind, in which case the hull effects dominate.

8 CONCLUSIONS

It seems that existing ground-related ship's positioning (satellite) systems and collected data during full-scale sea manoeuvring trials are still insufficient in order to conduct a reliable identification of the uncoupled second-order ('full') linear dynamics, as aimed in the paper. This is also true, when we apply a certain amount of post-processing, as connected with filtering/smoothing the measurements and eliminating the environmental disturbances. In particular, the problem lies in a low adequacy/reliability of the indirectly acquired run of the drift angle, which is often subject to significant water current effects. If the very sensitive T_3-type constants are not accurately established, then we can lose the physical sense of the final hydrodynamic derivatives. Herein, a certain role herein is obviously played by a manoeuvring test (control input) selected.

To some extent, a remedy can be offered by free-running model tests. Nevertheless, this is really a big challenge for the future research to design special kind, high quality manoeuvres, conduct their measurements, and perform identification.

It is believed that the zigzag test, served in the present investigation as an example to demonstrate peculiarities of the second-order uncoupled linear dynamics, could also be used for that purpose. This manoeuvre has a very extensive record of published experimental and theoretical (simulation) data. However, some systematic parametric studies are required for zigzag performances being essentially 'generated' by the concerned dynamics. This would yield a benchmarking material in order to detect hidden nonlinearities in actual (of given ship) zigzag behaviour and then to prevent from fitting the considered linear part of the full-mission model.

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