TRANSNAV International Journal on Marine Navigation and Safety of Sea Transportation
Volume 5 Number 2 June 2011

Fuzzy Evidence in Terrestrial Navigation

W. Filipowicz
Gdynia Maritime University, Poland

ABSTRACT: Measurements taken in terrestrial navigation are random values. Mean errors are within certain ranges what means imprecision in their estimation. Measurements taken to different landmarks can be subjectively diversified. Measurements errors affect isolines deflections. The type of the relation: observation error – line of position deflection, depends on isolines gradients. All the mentioned factors contribute to an overall evidence to be considered once vessel's position is being fixed. Traditional approach is limited in its ability of considering mentioned factors while making a fix. In order to include evidence into a calculation scheme one has to engage new ideas and methods. Mathematical Theory of Evidence extended for fuzzy environment proved to be universal platform for wide variety of new solutions in navigation.

1 INTRODUCTION

In his recent papers the author presented application of Mathematical Theory of Evidence (MTE) in navigation. The Theory appeared to be flexible enough to be used for reasoning on the fix. Contrary to the traditional approach, it enables embracing knowledge into calculations. Knowledge regarding position fixing includes: characteristics of random distributions of measurements as well as ambiguity and imprecision in obtained parameters of the distributions. Relation between observations errors and lines of position deflection is also important. Uncertainty can be additionally expressed by subjectively evaluated masses of confidence attributed to each of observations.

New scheme enabling inclusion of knowledge into the fixing process was presented by Filipowicz 2009c. Way of computation of belief and plausibility as well as location vectors grades can be found in other papers by Filipowicz 2009a, 2009b. Location vectors were constructed assuming normal distribution of measurement errors. The latest was rather a result of limitation imposed on the publications. In order to fill up the hiatus empirical distributions are discussed herein.

Those interested in computational complexity of the fixing algorithms and ways of detecting local maxima should refer to another paper Filipowicz 2010a.

This paper is devoted to a new idea in position fixing in terrestrial navigation. Therefore characteristics of measurements errors are discussed, relation between imprecision of the measured values and lines of position or isolines is also presented.

During computation process abnormally high inaccuracy should be detected. In proposed approach the condition results in large mass of inconsistency, which occurs when no zero mass is assigned to empty sets. High inconsistency mass leads to rejection of the fix or undertaking steps towards fix adjustment. Selected position can be evaluated based on the final inconsistency but also on plausibility and belief values. It should be noted that constant errors are of primary importance when quality of the fix is considered. Using methods that remove systematic deflection of a measurement is recommended. Exploiting horizontal angles instead of bearings makes the fixed position independent from constant errors. The latest is a reason that part of the paper is devoted to the horizontal angle isoline.

MTE exploits belief and plausibility measures, it operates on belief structures. Belief structures are subject to combination in order to increase their initial informative context. The structures can be crisp, interval and fuzzy valued. Mainly crisp valued structures were presented and discussed in the author's previous papers. The structures consist of sets of normal location vectors along with crisp masses of confidence attributed to them. Vectors normality can be achieved through transformation procedure called normalization. Approaches known as Dempster and Yager methods are widely used. Advantages and disadvantages of the two proposals are discussed from nautical usage point of view. Being stuck to the original proposals proved to be not adequate while position fixing. For this reason a modified normalization procedure is proposed in this paper.

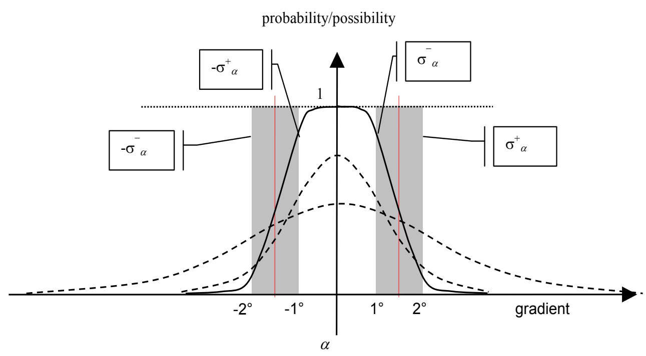

Figure 1. Graphic interpretation of the proposition "bearings mean error is between ±1°÷±2°"

2 FUZZY EVIDENCE

Crisp valued standard deviation of a measurement is inadequate. In recent navigation books mean error is described as imprecise interval value usually as: $[\pm\sigma^{-}_{d}, \pm\sigma^{+}_{d}]$. Mean error of a distance measured with radar variable range marker is within the interval of [±1% ÷ ±1.5%]. In the same condition mean error of a bearing taken with medium class radar is within [±1° ÷ ±2°] as presented by Jurdziński 2008 & Gucma 1995. Using fuzzy arithmetic notation it can be written as a quad (-2, -1, 1, 2). The latest means fuzzy value with core of [-1°, 1°] and support of [-2°, 2°], and reflects the statement that the error is within [±1° ÷ ±2°]. Graphic interpretation of the proposition is shown in Figure 1. The scheme engages probability and possibility theory. Observational errors are assumed to follow a normal distribution. Mean error estimates standard deviation (square root of a variance) of the distribution. The picture shows two confidence intervals related to two different distribution functions. A confidence interval is an interval in which a measurement falls within a range with selected probability. It is assumed that the confidence intervals are symmetrically placed around the mean. A confidence interval with probability equal to 0.683, for the Gauss probability density function is the interval $[\alpha - \sigma, \alpha + \sigma]$ where α is a mean and σ is a standard deviation.

Two confidence intervals introduce imprecision that is usually expressed by an interval or fuzzy value that is a synonym of fuzzy set.

Figure 1 shows trapezoid-like membership function that locates adjacent bearings within the defined set. The function returns possibility regarding given *x*, it attributes *x* degree of inclusion within the set. For example abscissa: $x = \alpha+0.5$ fully belongs to the given set, contrary to $x = \alpha+1.5$, its inclusion within the set is partial with degree of membership equal to 0.5. Different membership functions intended for nautical application were discussed by the author in his previous paper Filipowicz 2009a.

Empirical parameters are estimated based on observations. Empirical probability is widely used in practice. In terrestrial navigation it is exploited quite often. Theoretical probabilities are estimated by those calculated from experiments and observations. Empirical probability is the ratio of the number of those results that fall into a selected category to the total number of observations.

The empirical probability estimates statistical probability. Avoiding any assumptions regarding obtained data is the main advantage of estimating probabilities using empirical data. Histograms are widely used as graphical representation of empirical probabilities. Histogram is a diagram of the distribution of experimental data. Usually histogram consists of rectangles, placed over non-overlapping intervals also known as bins. The histogram is normalized and displays relative frequencies. It then shows the proportion of cases that fall into each of several bins. In normalized histogram total area of rectangles equals to one. The bins or intervals are usually chosen to be of the same size. There is no universal rule to calculate number of bins. In the presented application their quantity equals to the number of ranges established around measured value assumed as governed by normal distribution. Empirical distribution of observational errors with imprecise bin width and relative frequencies is shown in Figure 2.

Family of sets $\{\{l_k\}_i\}$ of measured values are given as a result of experiments. Therefore sets of mean values $\{\bar{l}_i\}$ and the bin width s can be obtained. Extreme deflection of means $\Delta\bar{l}^{-}$ and $\Delta\bar{l}^{+}$ can be also known. Modal value[1] $\bar{l}_m$ is calculated based on extreme means. Consequently empirical mean and bin widths are interval valued with above mentioned limits. Relative frequencies $\{p_j\}$ for each of consid-

[1] Modal value is defined for a fuzzy set. Usually it is calculated as a mean of the set's core, Piegat 2003. It should be noted that modal value is of secondary meaning in distribution characteristics.

ered bins are obtained as crisp or imprecise valued. Formulas from 1 to 4 define complete set of parameters for empirical distributions.

$$\left[\Delta\bar{l}^-,\ \Delta\bar{l}^+\right]=\left[\min_i(\{\bar{l}_i\})-\bar{l}_m,\ \max_i(\{\bar{l}_i\})-\bar{l}_m\right]$$

where: (1)

$$\bar{l}_m=\frac{\max_i(\{\bar{l}_i\})+\min_i(\{\bar{l}_i\})}{2}$$

$$s=\frac{\max_{k,i}(\{\{l_k\}_i\})-\min_{k,i}(\{\{l_k\}_i\})}{n} \tag{2}$$

$$\left[p_j^-,p_j^+\right]=\left[\min_{k,i}(\{\{p_{jk}\}_i\}),\max_{k,i}(\{\{p_{jk}\}_i\})\right] \tag{3}$$

$$\left[\Delta s^-,\ \Delta s^+\right]=\left[\Delta\bar{l}^-,\ \Delta\bar{l}^+\right] \tag{4}$$

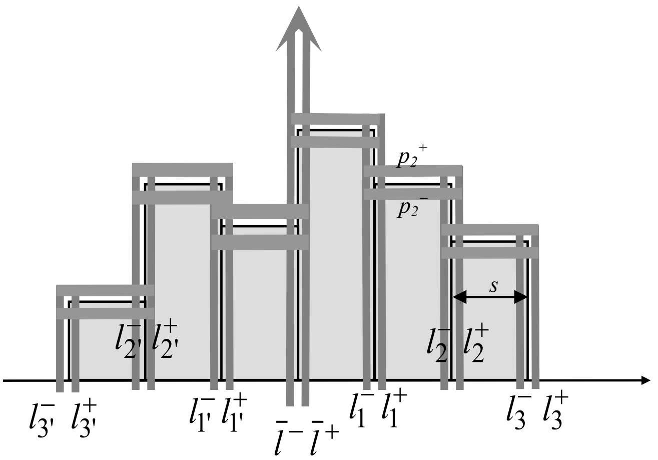

Figure 2. Empirical distribution with imprecise bin width and relative frequencies

3 ISOLINES AND THEIR GRADIENTS

Results of measurements plotted at a chart appear as lines of position. From the mathematic point of view the lines of position are isolines or in many cases lines tangent to them. An isoline for a function of two variables is a curve connecting points where the measurement has the same value. In terrestrial navigation, an isoline joins points of equal bearing, distance or horizontal angel. A bearing is the direction one object is from a vessel. Isoline of a bearing is a line, the same distance from an object produces circle. Isoline of the horizontal angle is also a circle since all inscribed angles that subtend the same arc are equal. The arc joins observed objects. Figure 3 presents isoline of a horizontal angle. A horizontal angle obtained as difference of two bearings is a valuable thing for navigator since it does not contain constant error.

The gradient of a function is a vector which components are the partial derivatives of the function. For function of two variables gradient is defined by Formula 5.

$$g(x,y)=\nabla f(x,y)=\left[\frac{\partial f}{\partial x},\frac{\partial f}{\partial y}\right] \tag{5}$$

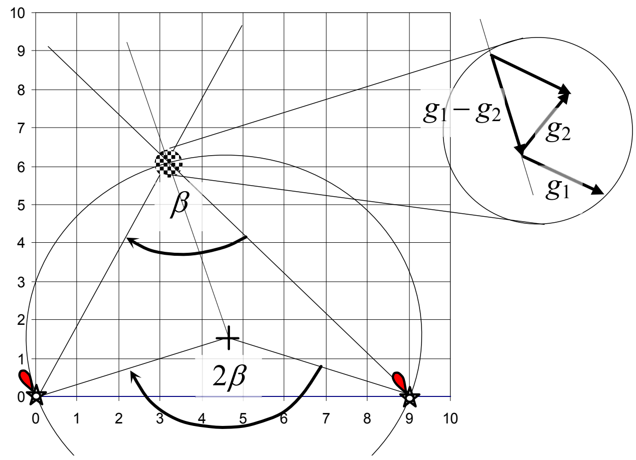

Figure 3. Isoline of a horizontal angle and its gradient in selected point (g_1 and g_2 refers to gradients of the first and second bearing, first bearing is taken to the left object)

Product of the gradient at given point with a vector gives the directional derivative of the function in the direction of the vector. The direction of gradient of the function is always perpendicular to the isoline. Gradients, measurements error and lines of position deflections are dependent values. Formula 6 shows the relation.

$$M(x,y)=\frac{\sigma}{|g(x,y)|} \tag{6}$$

Table 1. Parameters of the horizontal angle isoline

isoline parameter	formula
isoline radius	$r=\dfrac{d_{12}}{2\sin(\beta)}$
center coordinates	$(x_1+r\cos(90-\beta+\theta,\ y_1+r\sin(90-\beta+\theta))$
gradient module	$\lvert g\rvert=\dfrac{d_{12}}{d_1\,d_2}\quad\left[\dfrac{\text{rad}}{\text{Nm}}\right]$

d_{12} – distance between observed objects
θ – inclination, related to x axis, of the line passed through the objects ($\theta = 0$ in Figure 3)
x_1, y_1 – coordinates of the left object (see Figure 3)
β – horizontal angle calculated as difference of bearings ($\beta > 0$)
d_i – distance to *i-th* object ($d_i \neq 0$)

Error of the measurement divided by the module or length of the gradient in selected point gives deflection of the isoline at the point. In the proposed solution limits of introduced strips and possible isolines coverage are to be calculated accordingly.

Radius length, coordinates of the center and gradient module for horizontal angle isoline can be calculated with formulas presented in Table 1.

Table 2 contains data regarding isoline shown in Figure 3. The data embrace distances, gradients modules and isoline errors calculated for measurements standard deviation of ±1°. Appropriate values were obtained for selected points placed in the isoline.

Table 2. Selected points at the horizontal angle isoline, gradients and isoline errors

x	0	1	3	4	5	6	8	9
y	3.2	4.9	6.2	6.4	6.4	6.2	4.9	3.2
d_1 [Nm]	3.2	5.0	6.8	7.5	8.1	8.6	9.4	9.6
d_2 [Nm]	9.6	9.4	8.6	8.1	7.5	6.8	5.0	3.2
$\lvert g \rvert \left[\frac{\circ}{\mathrm{Nm}}\right]$	16.7	11.1	8.8	8.5	8.5	8.8	11.1	16.7
$\pm M$ [cables]	0.60	0.90	1.14	1.18	1.18	1.14	0.90	0.60

isoline error M was calculated for measurement mean error $\sigma = \pm 1°$

Isoline of a horizontal angle and its limits calculated for interval [β - 3σ, β + 3σ] is shown in Figure 4. Limits of an isoline shows its extreme shifts due to measurements errors. These limits can be of the same size along the line as for example for distances. For bearings and horizontal angles limits vary depending on the position of the observer. Within the limits strips related to confidence intervals are established. Levels of confidence and way of selecting stripes are discussed in the paper by Filipowicz 2010b.

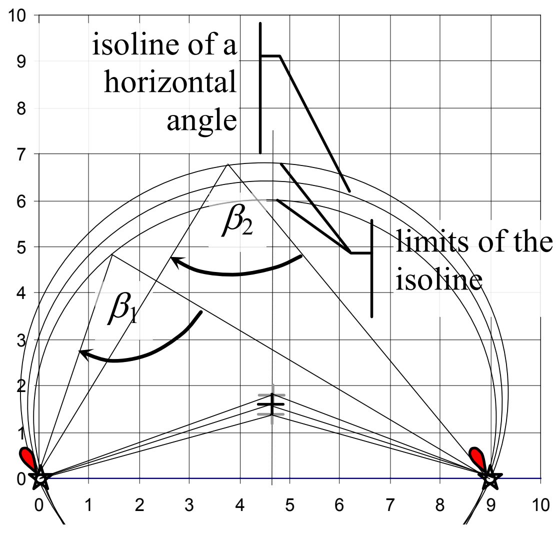

Figure 4. Isoline of the horizontal angle and its limits

4 SCHEME OF A POSITION FIXING

Let us consider three rectangular ranges related to three isolines as shown in Figure 5. Within ranges six strips were distinguished. Widths of the strips are calculated based on measurement errors and the isoline gradients. Each strip has fuzzy borders depending on imprecision in estimations of the isoline errors distribution. Theoretical or empirical probabilities of containing the true isoline within strips are given. Having particular point and all before mentioned evidence support on representing fixed position for given point should be found. This is quite different from traditional approach where single point should be found and available evidence hardly exploited.

The scheme of approach is as follows:

Given: available evidence obtained thanks to nautical knowledge

Question: what is a support that particular point can be considered as fixed position of the ship?

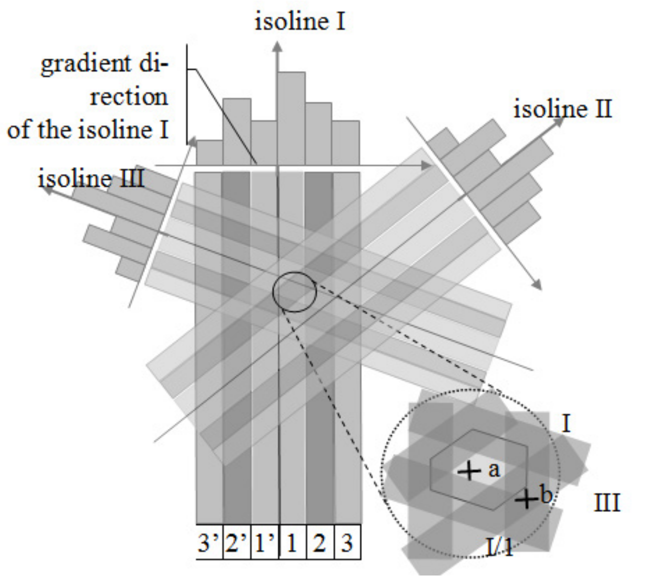

Figure 5. Three isolines with strips established around them

Figure 5 shows common area of intersection of three areas associated with three isolines. Six strips were selected around each isoline, the strips were numbered as shown in the figure. Number 3' refers to the far most section, number 3 indicates closest range according to gradient direction and regarding observed object(s). Assuming normal or empirical distribution probabilities attributed to each of the strips might be as shown in Table 3.

Table 3. Example probability values

strip	3'	2'	1'	1	2	3
normal distribution	0.021	0.136	0.342	0.342	0.136	0.021
empirical distribution	0.05	0.15	0.30	0.35	0.10	0.05

Figure 5 also shows magnified fragment of the area with two points situated within it. Points are marked with *a* and *b*. For both points hypothesis that they represent fixed position will be calculated. Support that point *a* can be considered as a fix is justified by the following probabilities related to (note that point *a* is entirely situated within crossing strips):

– membership within strip 1 regarding isoline I
– membership within strip 2' regarding isoline II
– location within strip 1' regarding isoline III

Position of point *b* is partial within strips related to isolines II and III. Its memberships are estimated as follows: II/2'→0.3, II/1'→0.7, III/1'→0.9, III/2'→0.1. Thus support that point *b* can be considered as a fix is justified by the following:

– full membership within strip 1 with reference to isoline I
– partial location within strip 2' regarding isoline II
– partial location within strip 1' regarding isoline II
– partial membership within strip 1' with reference to isoline III
– partial location within strip 2' with reference to isoline III

Evaluation of each of the measurements should also be included in calculation. Navigator knows which observation is good or bad, which are preferable to the others. Usually the opinion is subjective and can be expressed as linguistic term or a crisp value.

Table 4. Example probability values

strip	mass of evidence	sets	memberships ref. I		ref. II.		ref. III	
3'	$m_{3'} = 0.05$	$\mu_{i3'}$	0	0	0	0	0	0
2'	$m_{2'} = 0.15$	$\mu_{i2'}$	0	0	1	0.3	0	0.9
1'	$m_{1'} = 0.35$	$\mu_{i1'}$	0	0	0	0.7	1	0.1
1	$m_1 = 0.30$	μ_{i1}	1	0.5	0	0	0	0
2	$m_2 = 0.10$	μ_{i2}	0	0.5	0	0	0	0
3	$m_3 = 0.05$	μ_{i3}	0	0	0	0	0	0
uncertainty			0.3		0.2		0.1	

ref. stands for reference to:
index *i* indicates isolines (I, II or III)

Table 4 contains preliminary results of the example analysis. The table contains fuzzy points locations within selected strips, locations are given with reference to each of the isolines. Example empirical probabilities are included in column 2. Last row presents uncertainty, weights of doubtfulness, which is a complement of credibility, attributed to each measurement.

Belief structure is a mapping or an assignment of masses to normal location sets. Location vectors are to be normal it means that their highest grade must be one. Subnormal sets should be converted to their normal state using normalization procedure. Vectors are supplemented with all one set, which expresses uncertainty. It says that each location is equally possible. Mass attributed to this vector shows lack of confidence to a particular measurement. Thanks to this value all observations can be subjectively differentiated. All location vectors have assigned mass of confidence. Appropriate values are calculated as a product of empirical probability assigned to particular strip and complement of uncertainty related to given measurement. It should be noted that the sum of all masses within a single belief structure is to be equal to one. Table 5 presents three normalized belief structures constructed based on data from Table 4.

Belief structures are subject of combination in order to obtain knowledge base enabling reasoning on the position of the ship. It is known that combination of belief structures increase their initial informative context. By taking several distances and/or bearings a navigator is supposed to be confident on true location of the ship.

Plausibility and belief of the proposition represented by a fuzzy vector included in collection of result sets are calculated. In position fixing plausibility is of primary importance, for discussion on this topic see Filipowicz 2009a, 2010c. To calculate final plausibility and belief one has to use formulas presented by Denoeux 2000, the expressions were further simplified by the author Filipowicz 2010c. In presented example plausibility values that given points can be selected as a fixed position are: $pl_a = 0.62$, $pl_b = 0.60$. Obviously a dense mesh of points is to be considered in practical implementations.

Table 5. Final normalized belief structures

b.s. I		b.s.II		b.s. III	
{1 0.5}	0.21	{1 0.3}	0.12	{0 1}	0.08
{0 1}	0.07	{0 1}	0.28	{1 0.1}	0.31
{1 1}	0.72	{1 1}	0.60	{1 1}	0.61

b.s. stands for belief structure

5 NOTES ON NORMALIZATION OF PSEUDO BELIEF STRUCTURES

Two strips that do not embrace the common points are disjunctive and their intersection is empty. Result of combination of the disjunctive vectors is a null set. Therefore product of masses attributed to both combined disjunctive vectors is assigned to empty set what means occurrence of inconsistency. Inconsistency results in a pseudo belief structure that must be converted to its normal state. Two normalization procedures are used: one was proposed by Dempster another one by Yager. At first both of them considered crisp vectors. Further extensions for fuzzy environment were suggested by Yager 1995. Although it is quite often that many authors refer to them using original methods inventor names. Normalization procedures are quite different in two aspects, namely in allocation of inconsistency masses and modification of fuzzy sets contents called grades. Masses of inconsistency in Dempster approach increase weights attributed to not null sets. In Yager proposal the masses increase uncertainty. In case of subnormal sets Dempster suggested division by highest grade. It preserves allocation of points within selected strips. Yager proposed adding complement of the largest grade to all elements of the set. It corrupts allocation of points within selected strips. Therefore results of subnormal belief structures conversion to their normal state using the two methods are different, see Table 6 for case study. Fuzzy sets are location vectors containing fuzzy memberships of a search space points within selected strips. Thus Dempster transformation causes that points with not null locations increase their memberships, empty grades are not changed. In Yager normalization all considered points gain some degrees of membership. Unfortunately it may adversely affect computational process and ability of evaluation of the obtained fix. Therefore modified normalization method is proposed. In the approach inconsistency masses increase uncertainty very much like in Yager method. Conversion of subnormal sets remains in line with Dempster proposal. In order to obtain proper grades all of them are divided by the highest one. Modified method preserves location of search space points. The method also enables identification of all inconsistency cases as depicted by Filipowicz 2010b.

Table 6. Two example fuzzy sets, their normalizations and combinations

	Location vectors									$m(..)$
μ_1	{0	0.8	0	0	0	0	0	0.6	0}	0.41
μ_1^Y	{0.2	1	0.2	0.2	0.2	0.2	0.2	0.8	0.2}	0.41
μ_1^D	{0	1	0	0	0	0	0	0.75	0}	0.48*)
μ_1^M	{0	1	0	0	0	0	0	0.75	0}	0.33
μ_2	{0	0	0	0.67	0	1	0	0	0}	0.20
$\mu_{\mu1^Y \wedge \mu2}$	{0	0	0	0.2	0	0.2	0	0	0}	0.08
$\mu_{\mu1^D \wedge \mu2}$	{0	0	0	0	0	0	0	0	0}	0.10
$\mu_{\mu1^M \wedge \mu2}$	{0	0	0	0	0	0	0	0	0}	0.07

*) - according to Dempster proposal masses of non empty sets are modified during normalization

μ_1^Y - fuzzy set μ_1 normalized with Yager method

μ_1^D - fuzzy set μ_1 normalized with Dempster method

μ_1^M - fuzzy set μ_1 normalized with modified method

$\mu_{\mu1^Y \wedge \mu2}$ - result of combination of fuzzy sets μ_1^Y and μ_2

$\mu_{\mu1^D \wedge \mu2}$ - result of combination of fuzzy sets μ_1^D and μ_2

$\mu_{\mu1^M \wedge \mu2}$ - result of combination of fuzzy sets μ_1^M and μ_2

Table 7. Dempster versus Yager versus modified approaches

	Dempster normalization (Yager smooth normalization)*)	Yager normalization	modified normalization
way of modification of masses assigned to not null sets	increased by a factor calculated using inconsistency values	remain unchanged	reduced by complement of the highest grade
result uncertainty	solely depend on initial uncertainties	uncertainty is increased by total mass of inconsistency	increased by reduction of not null sets masses
modification of membership grades	general image of location vectors is preserved, null grades remain unchanged	null grades of location vectors gain some membership	general image of location vectors is preserved
ability to detect all inconsistency cases	possible	impossible	possible
recommendation	belief structures with fuzzy location vectors	belief structures with binary location vectors	belief structures with fuzzy location vectors
not recommended for	belief structures with binary vectors and high inconsistency	belief structures with fuzzy vectors and high inconsistency	belief structures with binary vectors and high inconsistency
computational complexity	rather high	rather low	rather low
final solution affected by high inconsistency	not observed	might adversely affect final solution	not observed

*) original method name suggested by Yager 1995

Table 6 embraces example of two fuzzy sets that are excerpted from belief structures. First of the sets is subnormal and needs to be converted. Their normal states obtained by three different methods are also presented. Results of combinations of the converted sets with the second one are included in last three rows of the table.

Combination is carried out using minimum operator and product of masses involved. Formula 7 delivers proper expressions.

$$\begin{aligned} &\mu_{\mu_1 \wedge \mu_2}(x_i) = \min(\mu_1(x_i), \mu_2(x_i)) \\ &m(\mu_{\mu_1 \wedge \mu_2}(x_i)) = m(\mu_1(x_i)) \cdot m(\mu_2(x_i)) \end{aligned} \tag{7}$$

Masses of credibility assigned to all vectors and to results of their combinations are shown in the last column of Table 6.

Table 7 contains comparison of Dempster, Yager and modified normalizations taking into account practical aspects presented in first column. It should be noted that position fixing engages fuzzy location vectors therefore modified normalization should be recommended. Most important feature of the Dempster and modified methods is ability to preserve general shape of location vectors, null grades remain unchanged. Consequently all inconsistency cases can be detected.

6 SUMMARY AND CONCLUSIONS

Bridge officer has to use different navigational aids in order to refine position of the vessel. To combine various sources he uses his common sense or relies on traditional way of data association. So far Kalman filter proved to be most famous method of data integration. Mathematical Theory of Evidence delivers new ability. It can be used for data combination that results in enrichment of their informative context. The Theory extension to a fuzzy platform proposed by Yen 1990 enables wider and more complex applications.

Based on the Theory concept new method of position fixing in terrestrial navigation is proposed. The method enables reasoning on position fixing based on measured distances and/or bearings. It was assumed that measured values are random ones with theoretical or empirical distribution. Knowledge on used aids and observed objects is included into combination scheme. Relation between measurement error and deflection of the isoline was also depicted. It was suggested that instead of bearings concept of horizontal angles should be used, obtained isoline is constant error free.

The true isoline of distance, bearing or horizontal angle is somewhere in the vicinity of the isoline linked to a measurement. To define true observation location probabilities six ranges were introduced. Probability levels assigned to each strip can be calculated based on features of normal distribution or they can be delivered from experiments. Standard deviation of the distribution is assumed to be within known range. Empirical data also varies within some range. In both cases imprecise interval valued limits of ranges are to be adopted. Sigmoid membership functions are used for establishing points of interest levels of locations within established ranges. Calculated locations are elements of fuzzy sets called location vectors. Vectors supplemented with the one expressing uncertainty compose one part of belief structure. Another part embraces masses of initial believes assigned to location vectors and uncertainty. Complete belief structure is related to each of measurements. Mass assigned to uncertainty expresses subjective assessment of measuring conditions. One has to take into account: radar echo signature, height of objects, visibility and so on to include measurement evaluation. Fuzzy values such as poor, medium or good can be used instead of crisp figures. Imprecise masses values engage different way of calculation and will be discussed in a future paper.

Belief structures are combined. During association process search space points within common intersection region are selected. Result of association is to be explored for reasoning on the fix. All associated items are to be taken into account in order to select final solution.

Mathematical Theory of Evidence requires that mass of evidence assigned to null set is to be zero and fuzzy sets are to be normal. Assignment for which above requirements are not observed is pseudo belief structure and is to be normalized. Pseudo belief structures can occur at the structures preparation stage as well as during association process. Usually null sets are results of combination of two ranges or areas without common search space points. The occurrences indicate abnormality in computation that might result from extraordinary erroneous measurements and/or wrongly adjusted search space. Therefore all null assignment cases are to be recorded and analyzed. Two normalization procedures proposed by Dempster and Yager are widely used. Converting procedures are quite different in two aspects. Masses of inconsistency in Dempster approach increase weights attributed to not null sets. In Yager proposal the masses increase uncertainty. In case of subnormal sets Dempster suggested division by highest grade, Yager proposed adding complement of the largest grade to all elements of the set. The latter causes that none of these approaches should be perceived as superior in case of position fixing. Therefore modified scheme was proposed. It takes best things from both proposals. Way of conversion of subnormal sets is taken from

Dempster method and managing of inconsistency comes from Yager approach.

REFERENCES

Denoeux, T. 2000. Modelling vague beliefs using fuzzy valued belief structures. *Fuzzy Sets and Systems* 116: 167-199.

Filipowicz, W. 2009a. Belief Structures and Their Application in Navigation. *Methods of Applied Informatics* 3: 53-83. Szczecin: Polska Akademia Nauk Komisja Informatyki.

Filipowicz, W. 2009b. Mathematical Theory of Evidence and its Application in Navigation. In Adam Grzech (eds.) *Knowledge Engineering and Expert Systems*: 599-614. Warszawa: Exit.

Filipowicz, W. 2009c. An Application of Mathematical Theory of Evidence in Navigation. In Adam Weintrit (ed.) *Marine Navigation and Safety of Sea Transportation*: 523-531. Rotterdam: Balkema.

Filipowicz, W. 2010a. Fuzzy Reasoning Algorithms for Position Fixing, Pomiary Automatyka Kontrola 56. Warszawa (in printing).

Filipowicz, W. 2010b. Belief Structures in Position Fixing. In Jerzy Mikulski (ed.) *Communications in Computer and Information Science* 104: 434-446. Berlin. Heidelberg: Springer.

Filipowicz, W. 2010c. New Approach towards Position Fixing. *Annual of Navigation* 16: 41-54

Gucma, S. 1995. Foundations of Line of Position Theory and Accuracy in Marine Navigation. Szczecin: WSM.

Jurdziński, M. 2005. Foundations of Marine Navigation. Gdynia: Gdynia Maritime University.

Piegat, A. 2003. Fuzzy Modelling and Control. Warszawa: EXIT.

Yager, R. 1996. On the Normalization of Fuzzy Belief Structure. *International Journal of Approximate Reasoning.* 14: 127-153.

Yen, J. 1990. Generalizing the Dempster-Shafer theory to fuzzy sets. *IEEE Transactions on Systems, Man and Cybernetics.* 20(3): 559-570.