# Fix Position Using Two Astronomical Line Of Position 

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#### Abstract

The Intercept Method (originally known as the Intercept Azimuth method) was created in 1875 by the French captain (latter admiral) Marq de Saint Hilaire. The method is still used today and is accepted by the International Maritime Organization as an component element of the Standards of Training, Certification and Watch-keeping for Seafarers. This paper aims to present the way of graphically determination of the vessel's fix position with two astronomical position lines computed using the intercept method.


## 1 INTRODUCTION

In the age of electronics, of constellations of satellites meant to determine the position of the ship anywhere on the Earth's surface, astronomical navigation continues to be the subject of study by future seafarers. We can consider astronomical navigation as a reserve means in keeping navigation that cannot miss from the necessary professional knowledges of the navigator just as neither the lifeboat nor the collective life raft can miss on board of any ship, regardless of its size or destination.
"Ars Navigation" - "the art of sailing" called the ancient Latins this wonderful human occupation, although they possessed, at that time, modest knowledge in the field of celestial mechanics and spherical trigonometry. And they were right, this is an art! What else can be reading an ECDIS device or a GPS display today? This ancient art, also known to the Arabs, has become routine!

But what can happen if these modern, ergonomic, smart systems fail? Art exists but we no longer have artists!

This paper intends to bring back to the stage of navigation the method of determining the fix of the ship using astronomical observations, a method that has proved particularly useful and accurate during ocean voyages both in peacetime and during major military events of the last century.

In the vastness of the Atlantic Ocean, the Sun and the stars were the guides of those who had taken on the noble mission of navigator.

But why in the era we were talking about at the beginning could astronomical navigation still be needed? Why use the formulas and tools we already see in museums, now that everything is computerized and is about to become robotic? Why right now when we start talking about artificial intelligence?

For the simple fact that under certain conditions the modern systems I mentioned may be "out of work" or may be the victims of a large-scale cyber attack.

Therefore, this paper presents a method for determining the fix of the ship with astronomical observations approved by the IMO as an international regulatory body in the field, based on the use of a
simple mathematical formula and the use of a pocket scientific computer to which are added the correction tables for the height measured with the sextant.

The determination of the coordinates of the astronomical fix of the ship is done through a graphical process that only needs the classic tools for chart work.

Incredible but true, the accuracy offered by the method is high enough, the error not being greater than 0.1 Nm . [1-4]

## 2 LOP'S COMPUTATION

On October 10, 2020, at the chronometer time $\mathrm{CT}=19 \mathrm{~h} 14 \mathrm{~m} 11 \mathrm{~s}$ in DR position: Lat $=34^{\circ} 13.4^{\prime} \mathrm{N}$; Lon $=$ $023^{\circ} 44.3^{\prime} \mathrm{W}$ the following sights were taken and recorded: Deneb star - sextant altitude Hs= $67^{\circ} 40.9^{\prime}$ and the Altair star - sextant altitude $\mathrm{Hs}=62^{\circ} 09.5^{\prime}$. Height of eye is 14 m and index correction IC $=+1.5^{\prime}$. The chronometer correction is $\mathrm{CC}=+1 \mathrm{~m} 12 \mathrm{~s}$. The astronomical fix coordinates are required.

Meridian angle (t) and declination (Dec) computation
Table 1. LOP 1 Deneb

| Date | 10 Oct. 2020 | 10 Oct. 2020 |
| :---: | :---: | :---: |
| Body | Deneb | Altair |
| CT | $19^{\text {h }} 14^{\text {m }} 11^{\text {s }}$ | $19^{\mathrm{h}} 14^{\mathrm{m}} 11^{5}$ |
| CC | $+1^{1 / 1} 12^{5}$ | $+1^{\text {m }} 12^{\text {s }}$ |
| UT | $19^{\text {h }} 15^{\text {m }} 23^{\text {s }}$ | $19^{\mathrm{h}} 15^{\mathrm{m}} 23^{\text {s }}$ |
| IC | +1,5' | +1,5' |
| Alt Corrn | $-6,9^{\prime}$ | -7,2' |
| Sum | -5,4' | -5,7' |
| Hs | $67^{\circ} 40,9^{\prime}$ | 62 ${ }^{\circ} 09,5^{\prime}$ |
| Ho | $67^{\circ} 35,5^{\prime}$ | $62^{\circ} 03,8^{\prime}$ |
| GHA $\gamma$ (hrs) | $304{ }^{\circ} 50,1^{\prime}$ | $304^{\circ} 50,1^{\prime}$ |
| Increments ( $\mathrm{m} / \mathrm{s}$ ) | $3^{\circ} 51,4^{\prime}$ | $3^{\circ} 51,4^{\prime}$ |
| Total GHA $\boldsymbol{r}$ | $308^{\circ} 41,5^{\prime}$ | $308^{\circ} 41,5^{\prime}$ |
| SHA | $49^{\circ} 27,9^{\prime}$ | $62^{\circ} 03,3^{\prime}$ |
| GHA* | $358^{\circ} 09,4^{\prime}$ | $10^{\circ} 44,8^{\prime}$ |
| Lon (E/W) | -23 $44,3^{\prime}$ | -23 ${ }^{\circ} 44,3^{\prime}$ |
| LHA* | $334^{\circ} 25,1^{\prime}$ | $347^{\circ} 00,5^{\prime}$ |
| t(E/W) | $25^{\circ} 34,9^{\prime}$ E | 120 ${ }^{\circ} 59,5^{\prime}$ E |
| Jab Dec. | N45 ${ }^{\circ} 21,5^{\prime}$ | N08 ${ }^{\circ} 55,6^{\prime}$ |

Computed altitude (hc) and intercept (a) computation

The following formula is used to determine the computed altitude ( Hc ):

$$
H c=\underbrace{\sin L a t \cdot \sin D e c}_{\text {part } a} \underbrace{+\cos L a t \cdot \cos D e c \cdot \cos t}_{\text {part } b}
$$

Table 2. Computed altitude (Hc)

| E Lat | $34^{\circ} 13.4^{\prime} \mathrm{N}$ | $\sin$ Lat | 0,562420154 | $\cos$ Lat | 0,826851601 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Dec | $\mathrm{N} 45^{\circ} 21,5^{\prime}$ | $\sin \operatorname{Dec}$ | 0,711515238 | $\cos \operatorname{Dec}$ | 0,702670667 |
| t | $25^{\circ} 34,9^{\prime} \mathrm{E}$ |  |  | $\cos \mathrm{t}$ | 0,901970738 |
| Ho | $67^{\circ} 35.5^{\prime}$ | part a | 0,400170510 | part $b$ | 0,524048937 |
| Hc | $67^{\circ} 33,1^{\prime}$ | part $b$ | 0,524048937 |  |  |
| Intercept (a) | $+2.4^{\prime}$ | $\sin \mathrm{Hc}$ | 0,924219447 |  |  |
|  |  | Hc | $67^{\circ} 33.1^{\prime}$ |  |  |

Zenith angle ( Z ) and azimuth ( Zn ) computation

Zenith angle ( Z ) is computed using the formula below:
$\sin Z=\sin t \times \sec H_{c} \times \cos D e c$
The zenith angle given by this formula has a quadrantal size.
Table 3. Quadrantal size of zenith angle

| ts | $25^{\circ} 34,9^{\prime} \mathrm{E}$ | $\sin t$ | 0,431797161 | Not necessary to compute Hpv because Dec > Lat |
| :---: | :---: | :---: | :---: | :---: |
| Hc | 67 $33.1{ }^{\prime}$ | sec Hc | 2,618748635 |  |
| Dec | N45 ${ }^{\circ} 21,5^{\prime}$ | cos Dec | 0,702670667 |  |
| Hfv | N45 ${ }^{\circ} 21,5^{\prime}$ | $\sin$ Z | 0,794557664 |  |
|  |  | Z | $52^{\circ} 36.8^{\prime}$ |  |
| Dec same name as Lat |  | Z | $52^{\circ} .6$ |  |
| Dec $>$ Lat |  | Z | NE52 ${ }^{\circ} 6$ |  |
|  |  | Zn | $052^{\circ} .6$ |  |

The following table helps to find the quadrant of horizon where the body is sighted.
Table 4. Quadrantal size of zenith angle

| Name of Dec | $\begin{gathered} \text { Magnitude } \\ \text { of Dec } \end{gathered}$ | Size of Hc | Origin (First letter) | Direction (Second letter) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | , | , 5 |
| Contrary to Lat | - | - | contrary to Lat | same name as meridian angle |
| $\begin{aligned} & \text { same name } \\ & \text { as Lat } \end{aligned}$ | Dec < Lat | $\mathrm{Hc}>\mathrm{Hpv}$ |  |  |
|  |  | $\mathrm{Hc}<\mathrm{Hpv}$ | same name |  |

## NOTES:

HPV represents the altitude of the celestial body in Prime Vertical. HPV is computed using the following formula:
$\sin H_{P V}=\sin D e c \cdot \operatorname{cosec}$ Lat
To convert the zenith angle $(\mathrm{Z})$ into azimuth ( Zn ) the following Conversion Table has to be used:
Table 5. Conversion table

| Conversion Table |
| :--- |
| Z |
| Zn  <br> $\mathrm{Z}=\mathrm{NE} \ldots .^{\circ}$ $\mathrm{Zn}=\mathrm{Z}$ <br> $\mathrm{Z}=\mathrm{SE} \ldots \ldots{ }^{\circ}$ $\mathrm{Zn}=180^{\circ}-\mathrm{Z}$ <br> $\mathrm{Z}=\mathrm{SW} \ldots \ldots$ $\mathrm{Zn}=180^{\circ}+\mathrm{Z}$ <br> $\mathrm{Z}=\mathrm{NW}_{\ldots}^{\circ}$ $\mathrm{Zn}=360^{\circ}-\mathrm{Z}$ |

LOP 2 Altair
Computed altitude (hc) and intercept (a) computation

Table 6. Computed altitude (Hc)

| E Lat | $34^{\circ} 13.4^{\prime} \mathrm{N}$ | $\sin$ Lat | 0,562420154 | $\cos$ Lat | 0,826851601 |
| :--- | :--- | ---: | :--- | ---: | ---: |
| Dec | ${\mathrm{N} 08^{\circ} 55,6^{\prime}}^{s i n}$ Dec | 0,155170187 | $\cos \operatorname{Dec}$ | 0,987887753 |  |
| $\mathrm{tt}_{5}$ | $12^{\circ} 59,5^{\prime}$ |  |  | $\cos \mathrm{t}$ | 0,974402772 |
| Ho | $62^{\circ} 03.8^{\prime}$ | part a | 0,087270840 | part $b$ | 0,795927819 |
| Hc | $62^{\circ} 01,8^{\prime}$ | part $b$ | 0,795927819 |  |  |
| Intercept (a) | $+2.0^{\prime}$ | $\sin \mathrm{Hc}$ | 0,883198659 |  |  |
|  |  | Hc | $62^{\circ} 01.8^{\prime}$ |  |  |

Zenith angle ( Z ) and azimuth ( Zn ) computation

Table 7. Quadrantal size of zenith angle

| E Lat | $34^{\circ} 13.4^{\prime} \mathrm{N}$ | $\sin$ Lat | 0,562420154 | $\cos$ Lat | 0,826851601 |
| :--- | :---: | ---: | :--- | ---: | ---: |
| Dec | $\mathrm{N} 08^{\circ} 55,6^{\prime}$ | $\sin \operatorname{Dec}$ | 0,155170187 | $\cos \operatorname{Dec}$ | 0,987887753 |
| $\mathrm{t}_{\mathrm{E}}$ | $12^{\circ} 59,5^{\prime}$ |  |  | $\cos \mathrm{t}$ | 0,974402772 |
| Ho | $62^{\circ} 03.8^{\prime}$ | part $a$ | 0,087270840 | part $b$ | 0,795927819 |
| Hc | $62^{\circ} 01,8^{\prime}$ | part $b$ | 0,795927819 |  |  |
| Intercept $(\mathrm{a})$ | $+2.0^{\prime}$ | $\operatorname{sin~} \mathrm{Hc}$ | 0,883198659 |  |  |
|  |  | Hc | $62^{\circ} 01.8^{\prime}$ |  |  |

Table 8. Quadrantal size of zenith angle

| Name of Dec | Magnitude <br> of Dec | Size of hc | Origin <br> (First letter) | Direction <br> (Second letter) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 3 | 4 | 5 |$|$|  | - | contrary to Lat |
| :---: | :---: | :---: |$⿻$| same name as |
| :---: |
| meridian angle |

Usually the method requires a few steps:

1. take the sights (the optimal situation is to be two observers on the bridge)
2. note the chronometer time (CT) to be able to compute UT for the moment of sight and the DR position of the vessel for the moment of sight
3. identify the two sighted bodies (if necessary)
4. compute the intercept (a) and azimuth $(\mathrm{Zn})$ for each sight

## 3 GRAPHICAL CONSTRUCTION

Algorithm

1. prepare a graphical scale of latitude and longitude on a blanc piece of paper (see the picture bellow)

- draw a vertical line in the middle of the white paper and mark its top with an arrow; label it as TN (True North). This vertical line represents for us the true meridian of vessel's DR position
- draw a horizontal line which will be used as longitude scale; mark the intersection point as origin (O) of a latitude-longitude graphical scale
- divide the horizontal line into equal segments from the origin to the edge and label the obtained points as minutes of longitude ( $1^{\prime}$, 2',...etc.)
- from the origin $(\mathrm{O})$ draw a line that makes with the longitude scale an angle equal to DR Lat
- from each point on the longitude scale draw perpendiculars till to the intersection with the line drawn before; mark the intersection points as minute of latitude. We have obtained a graphical scale of latitude

2. plot the two LOPs on the blanc paper:

- choose a point for DRP on the meridian (vertical line) and label it DRP
- draw a horizontal line through DRP which represent DRLat
- plot the two azimuths from DRP using a protractor and label them Zn 1 and Zn 2
- plot the two intercepts on each azimuth line (points I1 and I2) from DRP toward the celestial body (CB) if intercept is positive or away the CB if intercept is negative. Measure the size of intercepts on the latitude scale!
- draw the two line of position (LOP1 and LOP2) as perpendiculars through the intercept points I1 and I2. At the intersection of the two astronomical LOPs there is the astronomical Fix of the vessel

3. extract the fix coordinate as follows:

- draw a horizontal line through the Fix (the Fix parallel of latitude) and a vertical one (the Fix meridian)
- the difference of coordinates - difference of latitude and difference of longitude - will be sized as follows: the difference of latitude on the DRP meridian or between the parallel of Fix and parallel of DRP and the difference of longitude between the two vertical lines (meridians) or on the parallel of FIX position
- the difference of latitude will be measured on the graphical latitude scale
- the difference of longitude will be measured on the graphical longitude scale
- the geographical coordinates of the Fix position will be computed using following formulas:
$L a t=D R L a t+D L a t$
Lon $=$ DRLon + DLon
Inputs:
$D R P\left\{\begin{array}{l}\text { Lat }=34^{\circ} 13^{\prime}, 4 \mathrm{~N} \\ \text { Lon }=023^{\circ} 44^{\prime}, 3 \mathrm{~W}\end{array}\right.$
LOP1
DENEB $\left\{\begin{aligned} a & =+2,4 N m \\ Z n & =052^{\circ}, 6\end{aligned}\right.$
LOP2 $\left\{\begin{array}{l}a=+2,0 \mathrm{Nm}\end{array}\right.$
ALTAIR $\left\{\mathrm{Zn}=151^{\circ}, 7\right.$


Figure 1. Graphic representation

## Solution:

$$
\begin{gathered}
\text { DRLat }=34^{\circ} 13^{\prime}, 4 N \\
+ \text { DLat }=\quad-0^{\prime}, 5 \\
\hline \text { Lat }=34^{\circ} 12^{\prime}, 9 \mathrm{~N}
\end{gathered}
$$

DRLon $=023^{\circ} 44^{\prime}, 3 W$
$\frac{+ \text { DLon }=\quad+4^{\prime}, 1}{\text { Lon }=023^{\circ} 40^{\prime}, 2 W}$

$$
\text { Fix }\left\{\begin{array}{l}
\text { Lat }=34^{\circ} 12^{\prime}, 9 \mathrm{~N} \\
\text { Lon }=023^{\circ} 40^{\prime}, 2 \mathrm{~W}
\end{array}\right.
$$

## 4 CONCLUSION

The application of the described method is carried out in two steps. The first step is to perform the calculations needed to determine the elements of the astronomical position lines (LOP) that are azimuth and intercept. The second step is to create the graphical solution needed to determine the geographical coordinates of the ship's position for the moment when the observation were done.

The method is useful for verifying the accuracy of the indications of the existing navigation systems on
board the ship as well as for substituting their work in case of their failure.

The accuracy offered by the method is high enough given the size of the aquatic space at ocean crossings.

To facilitate the use of method, it is recommended to prepare by the time observation and computation sheets whose fields should be completed step by step in real time.

Mastering the method but especially its application does not mean a return to the past but rather an additional safety measure in keeping the navigation accurate.

## REFERENCES

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