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Evaluation of Navigation System Accuracy Indexes for Deviation Reading from Average Range

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ABSTRACT: The method for estimating the mean of square error, kurtosis and error correlation coefficient for deviations from the average range of three navigation parameter indications from the outputs of three information sensors is substantiated and developed.

1 INTRODUCTION

In modern navigation, there is a contradiction between the need for solving probabilistic problems, for example, the calculation of the navigation safety indications, and the lack of the necessary initial data. Such initial data are the parameters of navigation parameters accuracy (NP): the root-mean- square error (RMSE) *m*, the excess *E*, characterizing the form of the law of error distribution *ED*, and the correlation coefficient of latitude errors *r (Latitude)* and the longitude of the ship's position (*Longitude*) .

Therefore, the task of estimation the INS accuracy indicators in the process of operation in the specific operation conditions of the INS based on the internal signals of the IntNS is very relevant.

These internal signals are the readings of three or two INS that are part of the IntNS.

The estimation of indications during the voyage is conducted with a discreteness of Δ *t*=1 or 2 h diagrams and tables of deviations *u* between the samples y_{ji} NP from the outputs of three INS and their average value y^*_i, for example, for INS-1:

$$u_{1i}=y_{1i}-y^*_i; \qquad (1,a)$$

$$y^*_i=\frac{1}{3}(y_{1i}+y_{2i}+y_{3i}), \qquad (1,b)$$

And similar calculations are made for the testimony of other two INS. Here, the symbols denote: j -is the INS number, i- is the sequence number of the measurements or the time point.

Figure 1 shows the diagram of the variation in latitude production latencies, in conventional units, and deviations of the same NP for the day of navigation.

Contemplating the diagram, we find that the deviations as a whole reflect well the change in the errors in time. This allows us to assume that by evasion, it is possible to estimate correctly the accuracy indices of NP.

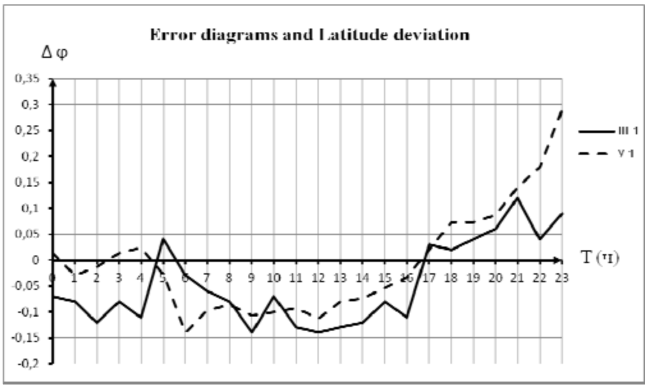

Figure 1 - Graphs of errors (solid line) and deviations (dashed line) latitude of INS, conventional units.

Method of estimation accuracy figure. When we estimate the accuracy indices by the deviations of u_{ji} from the average of three readings, $y^*_i = \frac{1}{3}(y_{1i}+y_{2i}+y_{3i})$, y3i), for example, for the first INS-1 at the i-th time of the deviation $u_{1i}=y_{1i}-y^*_i$.

The unknown error x_{1i} NP y_1 _of the first INS at i moment of time equals

$$x_{1i}=u_{1i}+y^*_i$$

This error cannot be determined, because Equation (2) contains two unknowns, but it is possible for n values u_{1i} to determine MSE $m_{x1}=m_{y1}$ of values Y_1[2]. In accordance with the theorem on the variance of the probability theory function [3], the RMSE m_{y1} of the sought-for value y_1 equals:

$$m^2_{y1}=m^2_{u1}+m^2_{y^*}+2r m_{u1} m_{y^*}; \quad (3,a)$$

$$m^2_{u1}=\frac{1}{n-1{,}45}\sum_1^n u_i^2 \ , \quad (3,b)$$

where n –is the number of deviations;
(n–1,45) - the number of deviations, is corrected in accordance with the Student distribution law;
$r=r_{u,y^*}$ - deviation correlation coefficient u average values of NP y^*, which can be found from the relation

$$r_{u,y^*}=m^2_{об}/m_u m_{y^*},$$

where $m_{об}$ - RMSE of general deviation for u and y^*, which, taking into account (1, b), can be taken equals to $m^2_{об} \approx \frac{1}{3} m_{y^*}$.

We take r = 0 at first-order estimate in (3, a), then $m^2_{y1}=m^2_{u1}+m^2_{y^*}$.

If we express $m^2_{y^*}$ in terms of (3, a) m^2_{y1} as $m^2_{y^*}=\frac{1}{3} m_{y1}$, then we arrive at :

$$m^2_{y1}- 0{,}3\, m^2_{y1}=0{,}67 m^2_{y1}=m^2_{u1},$$

and finally

$$m_{y1}=m_{u1}\sqrt{1/0{,}67}=1{,}23 m_{u1}. \quad (5,a).$$

Now it is possible to determine the correlation coefficient r plugged the obtained values in (4) and to make the second approximation for the unknown RMSE (root-mean-square error) m_{y1} with respect to (3, a)

$$my1=mu1\sqrt{1/0{,}57}=kmu1=1{,}32mu1 \quad (5,b)$$

Kurtosis of deviation is calculated according to the formula:

$$E=\frac{1}{n}\frac{\sum_1^n u_{in}^4}{\left(\frac{1}{n-1{,}45}\sum_1^n u_i^2\right)^2}-3 \quad (6)$$

where n – is the number of deviations and $\frac{1}{n-1{,}45}\sum_1^n u_i^2=m^2_u$ -is the root-mean-square value of deviation.

Covariance and correlation coefficient of errors by means of deviations can be computed from the formulas:

$$R_{\text{lat.long}l} \approx \frac{1}{n}\sum_1^n u_{Шi} u_{Дi} \ ; \quad (7,a)$$

$$R_{\text{lat.long}l} \approx R_{ШД}/m_Ш m_{Д.}, \quad (7,б)$$

where
$u_{\text{lat}.i}$ and $u_{\text{long}.i}$ - are INS reading deviation
$m_{\text{lat.}}$ и $m_{\text{long}.k}$ - the root-mean-square value of deviation computed from the formula (3,b);
n - the number of deviations in the cycle of determination of the correlation coefficient.

The validity checking of the method.

To check the validity of the method computed from formulas (3) - (5) and (6), the empirical ratio of the Root-Mean-Square Error $k^*=m_{y2}/m_{u2}$ was rated according to the data of the INS operating within 8 days, indications of which were recorded every hour. The results we can find in Table1 below, show the value of the Root-Mean-Square Error m_y computed from the formula (3,b) where $x_2=y_2-y_{эт2}$ is taken in place of u, and the value of the Root-Mean-Square Error m_{u2} of the second INS, which is the least precision device of the above three INS within twenty-four hours of observation.

Where $y_{эт2}$ is calibration value of Navigation Parameters.

In the same table, the values of kurtosis calculated for errors and deviations are also given.

Table 1. The results of the analysis of the possibility to estimate the INS accuracy values according to the NP deviations from the mean values, nominal units

Date	Value	Latitude Error	Latitude Deviation	Latitude Compar.	Longitude Error	Longitude Deviation	Longitude Compar.
25.10	RMSE, n.u.	0,08	0,06	1,32 +	0,17	0,12	1,42 +
	Kurtosis	2,9	2,6	<	1,6	3,1	>
26.10	RMSE, n.u.	0,21	0,16	1,31 +	0,25	0,18	1,39 +
	Kurtosis	-0,5	0	>	-0,7	0,7	>
27.10	RMSE, n.u.	0,03	0,02	1,5 -	0,08	0,09	0,89 -
	Kurtosis	-0,9	-1,3	<	-0,4	0,5	>
28.10	RMSE, n.u.	0,06	0,05	1,2 +	0,26	0,17	1,53 +
	Kurtosis	0,7	1,5	>	-0,1	-0,4	<
29.10	RMSE, n.u.	0,10	0,08	1,3 +	0,14	0,12	1,17 +
	Kurtosis	-1,0	-1,0	=	11,5	3,5	< (B)
30.10	RMSE, n.u.	0,15	0,11	1,36 -	0,26	0,18	1,44 +
	Kurtosis	-1,4	-1,6	<	-0,1	0,7	>
31.10	RMSE, n.u.	0,14	0,11	1,27 +	0,26	0,09	2,89 -
	Kurtosis	-1,5	-1,5	=	-0,1	-0,7	<
01.11	RMSE, n.u.	0,09	0,07	1,29 +	0,30	0,17	1,76 -
	Kurtosis	0,2	0,4	>	1,0	-0,4	<
Weighted average value		0,124	0,096	K*=1,29	0,226	0,151	k*=1,49

The analysis data of the validity checking of the method is based on the definition of limiting error $y^{\wedge}(P)$ of NP in form [2]

$$y^{\wedge}(P)=K_{P1E}(\underline{P})K_{hm}(P)m_y=K^{\wedge} m_y \qquad (8)$$

where
$K_{P1E}(\underline{P})$ – is the coefficient of conversion from the RMS to the limiting error, which depends on the specified probability P and error distribution law, determined by the upper confidence boundary for the kurtosis E;
$K_{hm}(P)$- is the Coefficient of the upper confidence limit for RMS;
m_y – is the value of RMS value of *y error*.

<u>Note:</u> The traditional definition of the limiting error in the form of $y^{\wedge}(P)=K_{P1}(\underline{P})m_y$, has two disadvantages. It contemplates that the errors of all Navigational Parameters are subjected to Gaussian Law and RMS is precisely known and is not random variable. These hypotheses lead to misidentification of the limiting errors. This attitude was proved 20 or more years ago, before the well-known Methodic appeared [5].

In the column "comparison" of Table 1 is given the following:

- The values of the ratio $k^*=m_y/m_u$ were obtained experimentally;
- Symbols «+» or «-» mean, for latitude channel $k^*\leq k=1{,}32$ and for longitude channel $k^*\leq k'=1{,}5$ and as a result of calculations the limiting error of the NP will not be greater than the proper value, or vice versa;
- Symbols «>», «<», «=», mean that the resulting ratio of kurtosis of errors and deviations will lead to the fact that the limit error of the NP will be greater than the proper value, or less, or equal the proper value. When there is a sign "<" kurtosis $E_{dev.}<E_{er.}$ coefficient of limiting conversion $K^{\wedge}_{dev.}<K^{\wedge}_{er.}$, and limiting error in the calculations based on the deviations will be determined with discrepancy.

Analyzing the values given in the Table we can come to the conclusion:

The coefficients k * related to the weighted mean values of the RMS m * (of errors) and m * (of deviations), have the following values:

$$k^*_{lat}=1{,}29<k=1.32;\ k^*_{long}=1{,}49>k=1{,}32 \qquad (9)$$

For the latitude channel, it is justified to use the theoretical coefficient k = 1.32 in determining the RMS by deviation. For the longitude channel, the use of the factor k = 1.32 can lead to serious errors in calculation of limiting errors. The upper confidence bound for the RMS ratio is assigned by Fisher distribution.

The Fisher coefficient K_{FD} for the confidence probability Pdv = 0.99, of the INS error correlation interval τ_K=4... 6 h and the equivalent digits of independent measurements N = 40 equals $K_{ФД}$ = 1.69. In this case, the upper confidence bound for the coefficient k equals $k_h=kK_{ФД}=2{,}23>k^*_{Д}=1{,}49$. Thus, the empirical value $k^*=1{,}49$ can be taken to compute from the formula (5, b) instead of the coefficient 1.32.

So, the calculating formulas for the RMS of the INS navigation parameters j are the following:

1 In latitude $m_j=1{,}32m_{uj}$, in longitude and the course $m_j=1{,}49m_{uj}$.
2 The limiting error of the NP may be fewer than it should be, due to out-of-true estimation of the kurtosis, in 44% of cases in the latitude channel and in 50% in the longitude channel. The maximum understatement of the limiting error may be 1% in the latitude channel and 48% in the longitude channel. The probability of such a significant understatement equals 1/16 = 0.06.

These results point us towards the following conclusion:

- In the process of IntNS operation it is advisable to monitor systematically the RMS values of coordinates and the course using the deviations of the NP from the average values and compare them with the priori values of the RMS. As a result, the larger value becomes a new priori one. In the absence of a priori values of the RMS, their definition by deviation becomes mandatory.

– It is expedient to determine kurtosis of coordinates and course errors on the long time intervals in cases when there are no priori values of kurtosis. As the upper confidence bounds to use the maximum values of the kurtosis's obtained within the intervals of twenty-four or seventy-two hours.

2 CONCLUSIONS.

1 The method to estimate the main accuracy figures of the INS based on the internal signals of the IntNS on the deviations of the INS indications from the mean values of three was substantiated and developed.
2 The validity of the method was verified by the experimental data.

BIBLIOGRAPHY

[1] Naval Defense. Catalogue.
[2] Mikhalskiy V. A., Katenin V. A. Metrology in navigation and solutions to navigation problems. Monography – St.Petr.: Elmor, 2009. -288 c.
[3] Ventsel E.C., OvcharovL.A. Theory of probability and its engineering applications. - M.: Science, 1988. - 480 c.
[4] Kondrashikhin V. T. The theory of errors and its application to navigational problems. -M.: Transport, 1969. -256 c.
[5] Mikhalskiy V. A., Ryabokon V. A. Methods of probability calculations to solve navigation problems on board ships and ships of Rus.Navy (MBP-96)./– St.Petr.: GUNiO Nav. RF, 1999. - 218 c.