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Equalization of the Measurements of the Altitude, the Azimuth and the Time from Observation of Passages of Celestial Bodies

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ABSTRACT: The article is describing the computational model serving equalization of the astronomical measurements accomplished to navigational and geodetic purposes. Series of measuring data: the altitude, the azimuth and the time from observation of passing of celestial bodies in the field of view of the observing device are input parameters to calculations. This data is burdened with random error of the measurement. The equation of the movement of celestial body in the horizontal system is the result of the equalization. It is possible to calculate the azimuth and the altitude for the chosen moment or to fix the time of the given azimuth or the altitude from this equation.

1 INTRODUCTION

Assuming, that the movement of celestial bodies on celestial sphere results only from rotary motion of the earth, then these bodies are moving along circles, which center is in the vicinity of closer pole and their radius is equal to the complement of the declination to the right angle. This assumption is correct during navigational or geodetic measurements due to short time of their duration. When the low accuracy of measurement is allowed (for example for the purposes of celestial navigation accuracy of altitude of 0.1' and accuracy of time of 1 second is required), then measuring series compound of several measurements of the altitude or the azimuth and the time can be equalized with straight line. The correction for the curve of celestial latitude is taken into account in such series in methods of the astronomical geodesy, and thanks to this it is possible to treat these series as linear in relation to the center thread. Both mentioned methods of the processing of measuring data results from the tendency of reduction of the amount of calculations connected with their processing. In the case, when measuring data is processed automatically, for the equalization one can accept the path of celestial body along circle and derive equation of the movement of the body in horizontal coordinates system (approximating equation). And then choose any location of the body on the circle, which data will be put to the reduction father.

2 APPROXIMATING EQUATION IN THE FIELD OF VIEW

Celestial bodies in their daily movement should theoretically form the arcs of the small circles on the celestial sphere with radiuses equal to the complement of the declination δ to the right angle and with centers in the closer celestial pole. In the particular case, when the body lies on the celestial equator it is great circle and the path of the body form straight line. The real path is influenced additionally by: the change of refraction with the altitude of the body and oscillations of its image, and at the measurements random errors of the measurements. One uses series n of the measurements of the position of celestial body: zenith distance z_i, the azimuth a_i and the time of registration t_i appropriate for point P_i, for derivation of equation of the movement. Zenith distances z_i have to be corrected for the refraction $r(z_i^r)$ appropriate for z_i^r

$$z_i = z_i^r + r_i\left(z_i^r\right). \qquad (1)$$

The variable z_i^r is measured and burdened with refraction, and z_i already corrected for the value of refraction.

The approximating equation is described by horizontal coordinates z_P (zenith distance) and a_P (azimuth) of the center P of circle along which body moves, with its radius r equal in first approximation of its polar distance

$$r = \pi / 2 - \delta \qquad (2)$$

and by horizontal coordinates z_G and a_G and time t_G of indicated point P_G on this circle (Fig. 1). The point P should theoretically agree with the pole which is nearest to the celestial body. Additional parameters of which values are known these are a declination δ of celestial body and an angle speed of change of right ascension v_{max} equal 7,29212E-05 radian for a second result from rotation of the Earth.

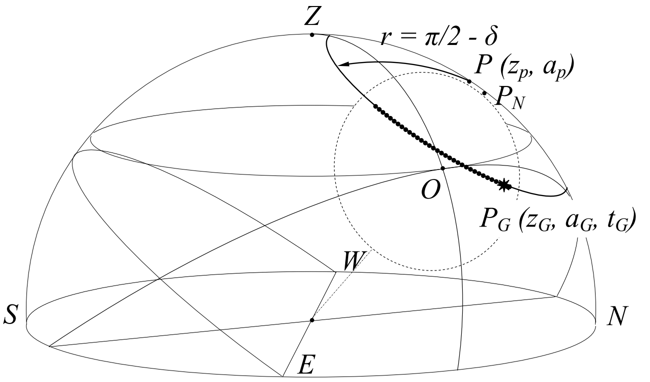

Figure 1 Parameters of the equation of the movement of celestial body in the horizontal system: circle with radius r and with centre in the point P as well as point P_G on this circle and time t_G appropriate to this point.

2.1 *Determination of the centre of the circle $P(z_P, a_P)$*

By the means of least square roots method, one seek such point P on the spherical surface, so that the sum Δ of square roots of the shortest great circle distances δ_i between respective point $P_i(z_i, a_i)$ and circle with the centre in the point P is minimal

$$\Delta = \sum_{i=1}^{n} \delta_i^2 = \min . \qquad (3)$$

The function of distance δ_i is difference of radius r and distance r_i of given point P_i from centre of circle P (Fig. 2)

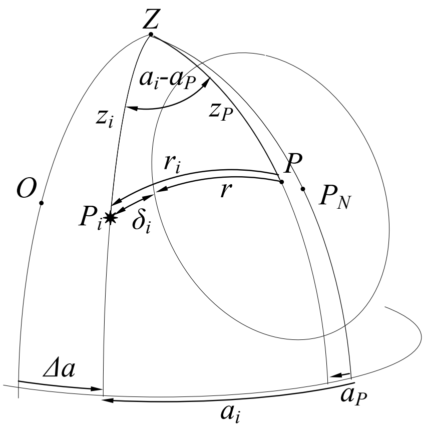

Figure 2 The shortest great circle distances δ_i between the point P_i and the circle with centre in the point P.

$$\begin{aligned} \delta_i &= r - r_i \\ \delta_i &= r - \arccos\left(\cos z_i \cos z_P + \sin z_i \sin z_P \cos(a_i - a_P)\right) \end{aligned} \qquad (4)$$

The condition (3) is met if derivatives of variables z_P and a_P are equal 0

$$\sum_{i=1}^{n} \frac{\partial \delta_i^2}{\partial z_P} = 0, \quad \sum_{i=1}^{n} \frac{\partial \delta_i^2}{\partial a_P} = 0 . \qquad (5)$$

Differentiating (4) through variables z_P and a_P, substituting to (5) and summing up for all points we receive a pair of non-linear equations with two unknown quantities z_P and a_P. It is possible to solve the pair of equations with iteration method taking horizontal coordinates of the pole nearest to celestial body as first approximation of the centre. We receive values in demand z_P and a_P in the result of the solution of the pair of equations.

2.2 *Reducing measurements to time t_G*

Each point P_i is reduced to any chosen time t_G. This reduction is made by the rotation of the point around determined centre of the circle for the angle $\Delta\alpha_i$ of the change of the right ascension for the difference of time Δt_i between times t_G and t_i (Fig. 3)

$$\begin{aligned} \Delta t_i &= t_G - t_i \\ \Delta \alpha_i &= v_{\max} \cdot \Delta t_i \end{aligned} . \qquad (6)$$

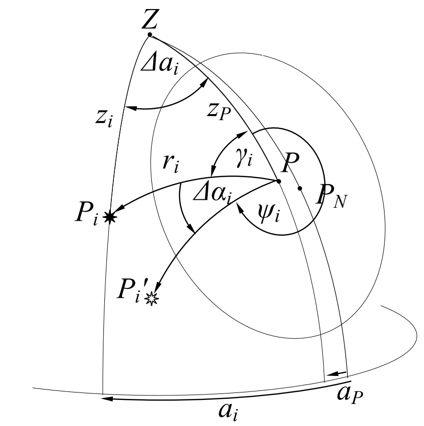

Figure 3 Reduction of points P_i to time t_G by the rotation of points around P for the angle of the change of right ascension $\Delta\alpha_i$.

It is necessary to calculate angle γ_i and distance r_i shown on figure 3 to determine coordinates of reduced point $P_i'(z_i', a_i')$. Defining the angle Δa_i as

$$\Delta a_i = a_i - a_P \qquad (7)$$

and keeping its value in the range $(0, 2\pi)$, then γ_i and r_i are calculated from formulae

$$\begin{aligned} \cos r_i &= \cos z_i \cos z_P + \sin z_i \sin z_P \cos(\Delta a_i) \\ \cos \gamma_i &= \frac{\cos z_i - \cos z_P \cos r_i}{\sin z_P \sin r_i} \end{aligned} , \qquad (8)$$

in addition for $\Delta a_i < \pi$

$$\gamma_i = 2\pi - \gamma_i . \qquad (9)$$

Defining the angle ψ as

$$\psi_i = \gamma_i + b \cdot \Delta\alpha_i \tag{10}$$

where b is equal 1 for P lying near north pole and -1 for south pole, and then keeping its value in the range $(0, 2\pi)$, z_i' and a_i' of reduced point P_i' are calculated from formulae

$$\cos z_i' = \cos z_P \cos r_i + \sin z_P \sin r_i \cos(\psi_i)$$
$$\cos \Delta a_i' = \frac{\cos r_i - \cos z_P \cos z_i'}{\sin z_P \sin z_i'} , \tag{11}$$

in addition for $\psi_i < \pi$

$$\Delta a_i' = 2\pi - \Delta a_i' \tag{12}$$

and then

$$a_i' = a_P + \Delta a_i' . \tag{13}$$

2.3 *Determination of the point $P_G(z_G, a_G)$ on the circle for the time t_G*

Point P_G is made by averaging coordinates of reduced points $P_i'(z_i', a_i')$ (Fig. 4).

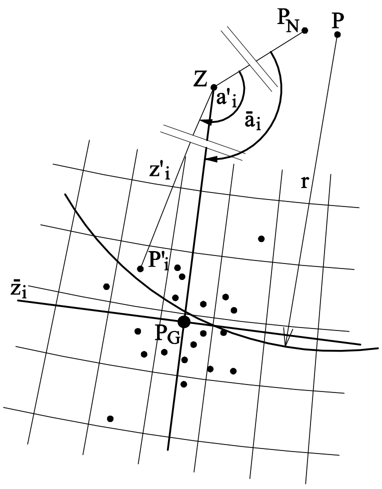

Figure 4 Point P_G calculated as the average of reduced points $P_i'(z_i', a_i')$.

Assuming that P_G is in the considerable distance from the Zenith and from the Nadir compared with the error of position of P_i', then the mean zenith distance and mean azimuth are calculated from formulae

$$\bar{z} = \sum_{i=1}^{n} z_i' \quad \bar{a} = \sum_{i=1}^{n} a_i' . \tag{14}$$

The standard deviation of position of the measurement point along the vertical circle σ_z, along almucantar σ_l and on the plane m_i are calculated from

$$\Delta z_i = \bar{z} - z_i' \quad \Delta a_i = \bar{a} - a_i'$$
$$l_i = \Delta a_i \cdot \sin \bar{z}$$
$$\sigma_z = \sqrt{\frac{\sum_{i=1}^{n} \Delta z_i^2}{n-1}} \quad \sigma_l = \sqrt{\frac{\sum_{i=1}^{n} \Delta l_i^2}{n-1}} \tag{15}$$
$$m_i = \sqrt{\sigma_z^2 + \sigma_l^2}$$

and the standard deviation of position of P_G on the plane

$$m = \frac{m_i}{\sqrt{n}} . \tag{16}$$

Function (3) of determining of the centre P of the circle is sensitive in the square roots of the distance between the point P_i and the arc of the circle but the point P_G is calculated as the average (14), it is proportionally to the distance from mean point, so the point P_G doesn't lie on the circle (Fig. 4). One can move this point onto the circle, by projection along radius r. Or one can determine new radius r' and new angle γ' from (17), assuming that the arithmetic mean is a better estimator for the measurement of passage of celestial bodies then the square roots average.

$$\Delta a = a_G - a_P$$
$$\cos r' = \cos z_G \cos z_P + \sin z_G \sin z_P \cos(\Delta a) \tag{17}$$
$$\cos \gamma' = \frac{\cos z_G - \cos z_P \cos r}{\sin z_P \sin r}$$

For $\Delta a < \pi$ from (17) (value Δa kept in the range $(0, 2\pi)$)

$$\gamma' = 2\pi - \gamma' . \tag{18}$$

3 DETERMINATION OF TIME AND COORDINATES FROM THE EQUATION OF THE MOVEMENT

3.1 *Calculation of coordinates on the circle for the given time t_i*

Having the point P_G with coordinates z_G and a_G and its time t_G on the circle with the centre in the point $P(z_P$ i $a_P)$ and with radius r', it is possible to determine coordinates z_i and a_i of the other point P_i on this circle for any given time t_i, the same way as measurement points were reduced to time t_G – formulae (6), (10)-(13). Appropriate formulae have the form

$$\Delta t_i = t_i - t_G$$
$$\Delta\alpha_i = v_{\max} \cdot \Delta t_i , \tag{19}$$

$$\psi_i = \gamma' + b \cdot \Delta\alpha_i , \tag{20}$$

$$\cos z_i = \cos z_P \cos r' + \sin z_P \sin r' \cos(\psi_i)$$

$$\cos \Delta a_i = \frac{\cos r' - \cos z_P \cos z_i}{\sin z_P \sin z_i} , \quad (21)$$

in addition for $\psi_i < \pi$

$$\Delta a_i = 2\pi - \Delta a_i , \quad (22)$$

$$a_i = a_P + \Delta a_i . \quad (23)$$

3.2 *Calculation of the time t_i of reaching the zenith distance z_i*

Converting (21) with taking into consideration (20) and (19), it is possible to calculate the appropriate time t_i for any given zenith distance z_i. ψ value from formula

$$\cos\psi = \frac{\cos z_i - \cos z_P \cos r'}{\sin z_P \sin r'} \quad (24)$$

corresponds with two values of the angle ψ_i from the formula (20)

$$\psi_{i1} = \psi , \quad \psi_{i2} = 2\pi - \psi \quad (25)$$

so substituting each of them to

$$t_i = \frac{\psi - \gamma'}{b \cdot v_{\max}} + t_G \quad (26)$$

we receive two values t_i. There is no solution of the equation (24) for $z_i < |z_P - r'|$ and $z_i > |z_P + r'|$. Exchanging inequalities for equalities above formulae describe conditions, by which (24) has only one solution.

3.3 *Calculation of the time t_i of reaching the azimuth a_i*

In order to calculate t_i appropriate to any given value a_i, it is necessary to determine from formula

$$\cos r' = \cos z_P \cos z_i + \sin z_P \sin z_i \cos \Delta a_i \quad (27)$$

involved value z_i, where Δa_i is calculated as (7). There is one (for the equality) or two solutions (for the inequality) of z_i in the case, when

$$\sin r' \geq= \sin z_P \sin \Delta a_i \quad (28)$$

or there is no solution in remaining cases. The farther proceedings comes down to the calculation of the time of reaching the obtained zenith distance z_i, which was descried higher. To the formula (26) one should substitute only one value calculated from the equation (25), that is ψ_{i2} for $\Delta a_i < \pi$, and ψ_{i1} in the opposite case.

Time t_i and coordinates z_i, a_i serve as data corrected for refraction for the reduction in various methods of making astronomical fix.

4 CONCLUSIONS

The described method of the equalization of measurements assumes that celestial bodies rotate on circles with constant speed and only random errors of measurement are found in measurement data. It is possible to determine data for the reduction for the any given point on the circle. If this point is in vicinity of the arc containing measuring data, then the accuracy of this point results directly from the accuracy of the measurement and the number of measurements.

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