INTRODUCTION

The determination of the height in the vertical reference frame in force in Poland is based both on national regulations (Council of Ministers Regulation, 2012) and on the resolution adopted by the EUREF subcommittee in Tromsø (Resolution No. 5, EUREF Symposium, 2000). Currently, the PL-KRON86-NH vertical reference frame is in use in Poland – a normal height system referred to a quasi-geoid of the average level of the Baltic Sea determined by the zero level of the mareograph in Kronstadt. According to the Regulation, by the end of 2019, Poland will adopt the PL-EVRF2007-NH vertical reference frame, i.e. a normal height system referred to the zero level of the mareograph in Amsterdam. The authors present a method of determining normal heights of seabed referred to the zero level of the mareograph in Amsterdam for coastal areas of the Baltic Sea. This method uses GNSS measurements, the EGM 2008 model and depth measuring methods typical for underwater mining.
2 DESCRIPTION OF THE METHOD

In the description of the method applied the authors presented models and formulas necessary to implement the method, as well as definitions of height systems used.

2.1 Geodetic height systems

The fundamental concept in the height systems applied is the so-called geopotential number $C$. It is equal to the difference between the potential of the geoid $W_0$ and of the point on the surface of the Earth $W_P$. The following relationship can be shown between the geopotential number $C$, gravitational acceleration $g$ and the height $h$.

$$ C = W_0 - W_P = \int g \, dh $$

Depending on the gravitational acceleration used to determine the height system, we may receive:

$H_{\text{ort}}$ – orthometric height system

$$ H_{\text{ort}} = \frac{C}{\bar{g}} $$

where:

$\bar{g}$ – average value of gravitational acceleration between the geoid and the physical surface of the Earth.

$H_{\text{nor}}$ – normal height system

$$ H_{\text{nor}} = \frac{C}{\bar{\gamma}} $$

where:

$\bar{\gamma}$ – the average value of normal gravitational acceleration along the vertical line of the normal gravity field.

$H_{\text{dyn}}$ – dynamic height system

$$ H_{\text{dyn}} = \frac{C}{\gamma_{45}^{\circ}} $$

where:

$\gamma_{45}^{\circ}$ – the normal gravitational acceleration for latitude 45°.

The normal acceleration of gravity is related to the concept of equipotential ellipsoid that meets the following conditions:

- The mass of the ellipsoid is equal to the mass of the Earth.
- The angular velocity of the ellipsoid spin $\omega$ corresponds to the velocity in the rotation of the Earth.
- The surface of an ellipsoid is by definition an equipotential surface with potential $U_0$. This potential is equal to the potential on the geoid $W_0$.

This condition is as follows:

$$ U_0 = W_0 = \text{const} $$

Normal acceleration is equal to:

$$ \gamma = \text{grad}U $$

The applicable global GRS 80 reference system can be represented by the following relationship:

$$ \gamma = 9.780327 \times (1 + 0.0053024 \cdot \sin^2 \varphi - 0.0000058 \cdot \sin 2 \varphi \ldots) \text{ms}^{-2} $$

where:

$\varphi$ – latitude.

The vertical gradient of gravitational acceleration can be calculated using the formula:

$$ \frac{d\gamma}{dh} = -\frac{\gamma}{a} \left(1 + f + q - 2f \sin^2 B\right) $$

where:

$a$ – semi-major axis of ellipsoid,

$f$ – flattening of the ellipsoid,

$B$ – ellipsoidal width,

$$ q = \frac{\omega^2 a^2 b}{GM} $$

$b$ – semi-minor axis of ellipsoid,

$\omega$ – rotational velocity of the Earth,

$GM$ – geocentric gravitational constant.

According to the national regulation (Council of Ministers Regulation, 2012) and to the resolution adopted by the EUREF Subcommittee – Resolution No. 5 of the EUREF Symposium in Tromsø (EUREF Symposium, 2000). By the end of 2019 current vertical reference frame will be replaced with PL-EVRF2007-NH reference system. It is a normal height system in which the quasi-geoid refers to the average level of the North Sea defined by the zero level of the mareograph in Amsterdam.

For hydrotechnical works the system of dynamic heights is of key importance.

2.2 Determining the normal height of a point at sea level

The principle of determining the normal height of a point at sea level, e.g. a drilling platform, research vessel, etc. is shown in Figure 1.
The ellipsoidal height of a point on the sea surface, for example on a drilling platform, can be determined by GNSS observations with the use of RTK network technology implemented on the ASG-EUPOS network. The evaluation of the potential use of this technique in the offshore area was the subject of prior research (Rogowski et al., 2015). The cited work involved a number of measurement experiments conducted to evaluate the usability of the ASG-EUPOS system with the use of RTK/VRS technology. Data for correction was obtained via the Internet from Orange mobile network. In the offshore area, discrepancies in the horizontal component reach up to 3 cm because the RTK/VRS correction is extrapolated. Since the height component is determined with an error twice as large, one can estimate that it will be determined with an error of up to 5 centimetres. Another problem is the range of mobile network which covers almost the whole Gulf of Gdańsk but in the coastal zone does not exceed 10 Mm. These are only estimates because, unfortunately, network administrators do not want to share any data. On the area beyond 10 Mm it will be necessary to apply the method which in turn will result in the deterioration of accuracy to approx. 25 cm (Przestrzelski, 2017), (Rogowski et al., 2015).

As follows from the principle presented in Fig. 1, the knowledge of the geoid model as well as the difference between the ellipsoidal height of the quasi-geoid ζ and the height of the geoid N are necessary in order to establish the normal height. Values of this correction in numerical form are available at the GUGiK’s website. We propose that the EGM 2008 model is used to calculate the value N. It is an official Earth Gravitational Model 2008 that has been publicly released by the U.S. National Geospatial-Intelligence Agency (NGA) EGM Development Team. This gravitational model consists of spherical harmonic coefficients complete to degree and order 2159, and contains additional coefficients extending it to degree 2190 and order 2159. For this model, there are online calculators available on the following websites:


These calculators allow you to determine the height of the geoid with a relatively high accuracy (3 – 5 cm) and grid resolution (1’ x 1’). For the area we are interested in, we can create a map that can further be used to determine the value N corresponding to the object position. An example of such a map for Gulf of Gdańsk is shown in Fig. 2 (Pałczyńska, 2017). The fact that all of the above calculators use the WGS 84 ellipsoid proves to be problematic. A slight difference in the parameters of the WGS 84 and GRS 80 ellipsoids (see Table 1) can be found in the data obtained from the abovementioned calculators. For accuracies we are interested in, this difference can be omitted.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>WGS 84</th>
<th>GRS 80</th>
</tr>
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<tbody>
<tr>
<td>a</td>
<td>6378137</td>
<td>6378137</td>
</tr>
<tr>
<td>1/f</td>
<td>298.257 223 563</td>
<td>298.257 222 101</td>
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</table>

The distances between the geoid and the quasi-geoid can be calculated with the following relationship (Barlik, 2000):

\[ N - \zeta = - \frac{H^2 \partial A_g}{2\gamma} + \frac{H}{\gamma} A_g B, \]

where the second element represents the influence of Bouguer gravimetric anomaly. This anomaly is defined in the following formula:

\[ A_g B = g - 2\pi k \sigma H + \frac{\partial g}{\partial h} H - \gamma_0, \]

where \( k \) is the Newton’s constant, and \( s \) is the density of subsurface material. The formula includes also the vertical gradient of gravitational acceleration \( \frac{\partial g}{\partial h} \).

Normal heights for the PL-EVRF2007-NH system can be determined using \( \Delta \zeta \), height anomaly correction for data for the closest points from the official GUGiK model. The transition from the normal heights referred to the EGM 2008 quasi-geoid to the
quasi-geoid passing through the zero level of the mareograph in Amsterdam is shown in Figure 3.

where:
\[ \Delta \zeta \] – height anomaly correction referred to the Amsterdam system

Figure 3 Normal height correction referred to the EGM 2008 quasi-geoid to the Amsterdam system.

Table 2 presents differences between the GUGiK geoid and the EGM 2008 geoid for selected points on the Baltic shore.

<table>
<thead>
<tr>
<th>B</th>
<th>L</th>
<th>EGM2008</th>
<th>PL-geoid-2011</th>
<th>Differences</th>
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<tr>
<td>54.72</td>
<td>18.46</td>
<td>29,574</td>
<td>29,136</td>
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<td>18.46</td>
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<td>54.69</td>
<td>18.47</td>
<td>29,586</td>
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<td>54.66</td>
<td>18.46</td>
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<tr>
<td>54.63</td>
<td>18.48</td>
<td>29,632</td>
<td>29,206</td>
<td>0.426</td>
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<td>54.5</td>
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</table>

Calculating the height of the sea floor requires the knowledge of the height difference between the position of the GNSS antenna and the seabed. It can be determined with an echo sounder or with other methods used in underwater mining.

3 SUMMARY AND CONCLUSIONS

The data necessary for the transition between reference systems is available on the website of the Head Office of Geodesy and Cartography at the following address: http://www.gugik.gov.pl/bip/prawo/model-danych. The data to determine the correction to the EGM 2008 geoid in order to calculate heights in the PL-EVRF2007-NH system can be extrapolated from the GUGiK model available at http://www.gugik.gov.pl/__data/assets/text_file/0016/1843/gugik-evrf2007.txt. The authors estimate that the seabed height in the area of Polish territorial waters in the PL-EVRF2007-NH frame can be determined with the accuracy of approximately 10 cm.

LITERATURE

Barlik M., (2000), On the contribution of vertical gravity gradient anomalies to the separation between the geoid and Molodensky’s quasigeoid, Reports on Geodesy, no. 2 (50).


Przestrzeński P., (2017), Siościowe przyczynianie różnicowe z wykorzystaniem obserwacji GPS i GLONASS. (Doctoral dissertation), University of Warmia and Mazury.


Internet sources

http://earth-info.nga.mil/GandG/wgs84/gravimod/egm2008/.
http://icgem.gfz-potsdam.de/ICGEM/.