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Cooperative and Non-Cooperative Game Control Strategies of the Ship in Collision Situation

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ABSTRACT: The paper introduces the positional cooperative and non-cooperative game of a greater number of met ships for the description of the process considered as well as for the synthesis of optimal control strategies of the own ship in collision situation. The approximated mathematical model of differential game in the form of triple linear programming problem is used for the synthesis of safe ship trajectory as a multistage process decision. The considerations have been illustrated an example of program computer simulation to determine the safe ship trajectories in situation of passing a many of the ships encountered.

1 INTRODUCTION

A large part in increasing the safety of navigation is the use of ARPA anti-collision system, which enables to track automatically at least 20 encountered *j* ships, determination of their movement parameters: speed V_j, course ψ_j and elements of approach: D_j – distance, N_j – bearing, $D_{j,\min} = DCPA_j$ - Distance of the Closest Point of Approach, $T_{j,\min} = TCPA_j$ - Time to the Closest Point of Approach (Fig. 1).

The functional scope of a standard ARPA system ends with manoeuvre simulation to achieve the safe passing distance D_s by altering course $\pm\Delta\psi$ or speed $\pm\Delta V$ selected by the navigator (Bist 2000, Cockroft & Lameijer 2006, Cahill 2000).

The most general description of the own control object passing the *j* number of other encountered objects is the model of a differential game of a *j* number of objects (Basar & Olsder 1982, Engwerda 2005, Isaacs 1965, Mesterton-Gibbons 2001).

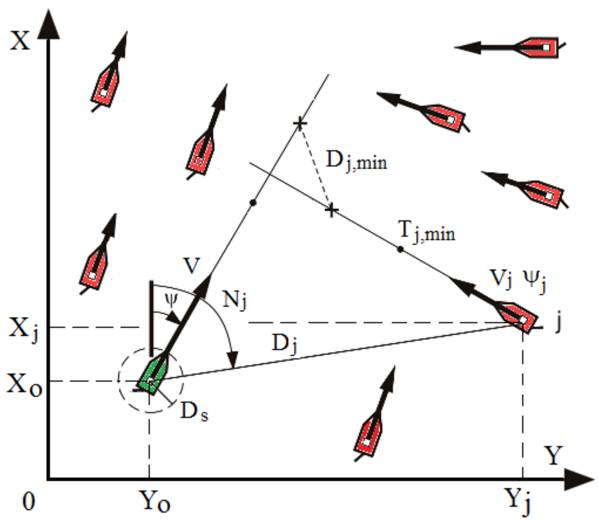

Figure 1. The own ship moving with speed V and course ψ during of passing *j* encountered ships.

This model consists both of the kinematics and the dynamics of the ship's movement, the disturbances, the strategies of the own ship and encountered ships and the quality control index (Clarke 2003, Kula 2015, Osborne 2004).

The diversity of possible models directly affects the synthesis of ship control algorithms which are afterwards affected by the ship control device, directly linked to the ARPA system and consequently determines effects of safe and optimal control (Fletcher 1987, Lisowski 2011).

2 MODEL OF GAME SHIP CONTROL PROCESS

2.1 *State and control variables*

The differential game described by state equation:

$$\dot{x} = f(x,u,t) \tag{1}$$

is reduced to a positional multistage game of a j number of participants (Fig. 2).

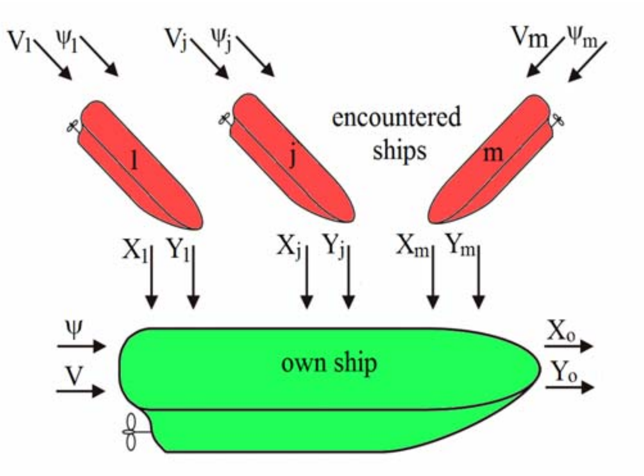

Figure 2. Block diagram of the positional game model of passing the own ship and encountered j ships.

The state x and control u variables are represented by (Lisowski & Lazarowska 2013):

$$\begin{aligned} &x_{0,1} = X_o,\ x_{0,2} = Y_o,\ x_{j,1} = X_j,\ x_{j,2} = Y_j \\ &u_{0,1} = \psi,\ u_{0,2} = V,\ u_{j,1} = \psi_j,\ u_{j,2} = V_j \\ &j = 1, 2, ..., m \end{aligned} \tag{2}$$

The making of a continuous positional game discrete and reducing it to a multistage positional game is determined by own ship and depends on: the maximum relative speed of the own ship under the current navigational situation, the range of the situation and the dynamic characteristics of the own ship (Gluwer & Olsen 1998, Isil & Koditschek 2001).

The essence of the positional game is to make the strategies of the own ship dependent on current positions $p(t_k)$ of the ships encountered at the current step k. In this way possible course and speed alterations of the objects encountered are considered in the process model during the steering performance (Lazarowska 2012, Luus 2000).

The current state of the process is determined by the co-ordinates for the position of the own ship and of the ships encountered:

$$\begin{aligned} &x_0 = (X_o, Y_o) \\ &x_j = (X_j, Y_j) \\ &j = 1, 2, ..., m \end{aligned} \tag{3}$$

The system generates its steering at the moment t_k on the basis of the data which are obtained from the ARPA anti-collision system concerning the current positions of the own and encountered ships:

$$\begin{aligned} &p(t_k) = \begin{bmatrix} x_0(t_k) \\ x_j(t_k) \end{bmatrix} \\ &j = 1, 2, ..., m \\ &k = 1, 2, ..., K \end{aligned} \tag{4}$$

It is assumed, according to the general concept of the multistage positional game, that at each discrete moment of the time t_k the position of the encountered ships $p(t_k)$ is known on the own ship (Gierusz & Lebkowski 2012, Lisowski 2012, Malecki 2013, Mohamed-Seghir 2016, Zak 2013).

The constraints of the state co-ordinates:

$$\{ x_0(t), x_j(t) \} \in P \tag{5}$$

constitute the navigational constraints, while the steering constraints:

$$\begin{aligned} &u_0 \in Uo \\ &u_j \in U_j \\ &j = 1, 2, ..., m \end{aligned} \tag{6}$$

take into consideration the kinematics of the ship movement, the recommendations of the COLREGS Rules and the condition to maintain the safe passing distance D_s:

$$D_{j,\min} = \min D_j(t) \geq D_s \tag{7}$$

2.2 *Sets of acceptable strategies*

The closed sets $U_{o,j}$ and $U_{j,o}$ defined as the sets of the acceptable strategies of the ships as players:

$$\begin{aligned} &U_{o,j}[p(t)] = S_{o1,j} \cup S_{o2,j} \\ &U_{j,o}[p(t)] = S_{j1,o} \cup S_{j2,o} \end{aligned} \tag{8}$$

are depended on the position $p(t)$, which means that the choice of the steering u_j by the j-th encountered ship alter the sets of the acceptable strategies of the other ships (Fig. 3).

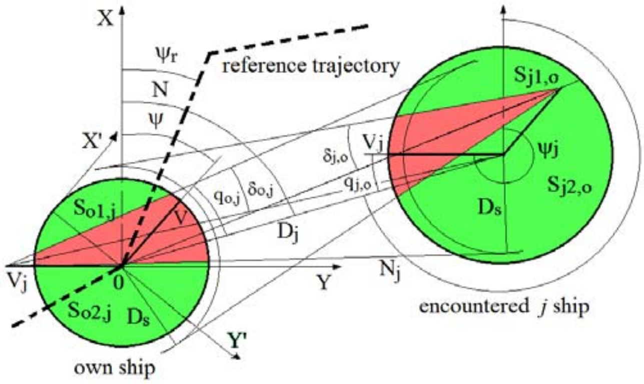

Figure 3. Determination of the acceptable safe strategies areas of the own ship $U_{o,j} = S_{o1,j} \cup S_{o2,j}$ and the encountered j ship $U_{j,o} = S_{j1,o} \cup S_{j2,o}$.

Let refer to the set of the acceptable strategies of the own ship while passing the *j*-th encountered ship at a safe distance D_s (Jaworski, Kuczkowski, Smierzchalski 2012; Lebkowski 2015, Nisan et al. 2007, Straffin 2001, Tomera 2015, Lisowski 2014a).

The area, when maintaining stability in time of the course and speed of the own ship and the ship encountered is static and is comprised within the semicircle of a radius equal to the set reference speed of the own ship *V* within the arrangement of the co-ordinates 0X'Y' with the axis X' directed to the direction of the reference course (Millington & Funge 2009, Modarres 2006, Szlapczynski 2012).

The set $U_{o,j}$ is determined with the inequalities:

$$
\begin{aligned}
& a_{o,j}\; u_{o,x'} + b_{o,j}\; u_{o,y'} \le c_{o,j} \\
& u_{o,x'}^2 + u_{o,y'}^2 \le V^2
\end{aligned}
\tag{9}
$$

where:

$$
\begin{aligned}
& \vec{V} = \vec{u}_o\,(u_{o,x'},\, u_{o,y'}) \\
& a_{o,j} = -\lambda_{o,j}\; \cos(q_{o,j} + \lambda_{o,j}\; \delta_{o,j}) \\
& b_{o,j} = \lambda_{o,j}\; \sin(q_{o,j} + \lambda_{o,j}\; \delta_{o,j}) \\
& c_{o,j} = -\lambda_{o,j} \begin{bmatrix} V_j\; \sin(q_{j,o} + \lambda_{o,j}\; \delta_{o,j}) + \\ V\; \cos(q_{o,j} + \lambda_{o,j}\; \delta_{o,j}) \end{bmatrix} \\
& \lambda_{o,j} = \begin{cases} -1 & for\;\; S_{o1,j} \quad (Port\;\; side) \\ \;\;1 & for\;\; S_{o2,j} \quad (Starboard\;\; side) \end{cases}
\end{aligned}
\tag{10}
$$

The value $\lambda_{o,j}$ is determined by using an appropriate logical function F_j characterising any particular recommendation referring to right of way contained in COLREGS Rules (Lisowski 2014b).

2.3 *Semantic interpretation of COLREGS Rules*

The form of function F_j depends of the interpretation of the above recommendations for the purpose to use them in the control algorithm, when:

$$
F_j = \begin{cases} 1 & then \;\;\; \lambda_{o,j} = \;\;1 \\ 0 & then \;\;\; \lambda_{o,j} = -1 \end{cases}
\tag{11}
$$

Interpretation of the COLREGS Rules in the form of appropriate manoeuvring diagrams enables to formulate a certain logical function F_j as a semantic interpretation of legal regulations for manoeuvring.

Each particular type of the situation involving the approach of the ships is assigned the logical variable value equal to one or zero (Lisowski 2015a, 2015b):

A – encounter of the ship from bow or from any other direction,
B – approaching or moving away of the ship,
C – passing the ship astern or ahead,
D – approaching of the ship from the bow or from the stern,
E – approaching of the ship from the starboard or port side (Lisowski 2015c, 2015d) .

By minimizing logical function F_j by using a method of the Karnaugh's Tables the following is obtained:

$$
F_j = A \cup \overline{A}\,(\overline{B}\,\overline{C} \cup \overline{D}\,\overline{E})
\tag{12}
$$

The resultant area of acceptable manoeuvres of the own ship in relation to the *m* encountered ships is:

$$
\begin{aligned}
& U_o = \bigcap_{j=1}^{m} U_{o,j} \\
& j = 1,\, 2,\, \ldots,\, m
\end{aligned}
\tag{13}
$$

is determined by an arrangement of inequalities (9).

On the other hand, however, the set of the acceptable strategies of the *j*-th object in relation to the own ship is determined by the following inequalities:

$$
\begin{aligned}
& a_{j,o}\; u_{j,x'} + b_{j,o}\; u_{j,y'} \le c_{j,o} \\
& u_{j,x'}^2 + u_{j,y'}^2 \le V_j^2
\end{aligned}
\tag{14}
$$

where:

$$
\begin{aligned}
& \vec{V}_j = \vec{u}_j\,(u_{j,x'},\, u_{j,y'}) \\
& a_{j,o} = -\lambda_{j,o}\; \cos(q_{j,o} + \lambda_{j,o}\; \delta_{j,o}) \\
& b_{j,o} = \lambda_{j,o}\; \sin(q_{j,o} + \lambda_{j,o}\; \delta_{j,o}) \\
& c_{j,o} = -\lambda_{j,o} V\; \sin(q_{o,j} + \lambda_{j,o}\; \delta_{j,o}) \\
& \lambda_{j,o} = \begin{cases} -1 & for\;\; S_{j1,o} \quad (Port\;\; side) \\ \;\;1 & for\;\; S_{j2,o} \quad (Starboard\;\; side) \end{cases}
\end{aligned}
\tag{15}
$$

The symbol $\lambda_{j,o}$ is determined by analogy to the determination of $\lambda_{o,j}$ with the use of the logical function F_j described by the equation (11).

Consideration of the navigational constraints, as shallow waters and coastline, generate additional constraints to the set of acceptable strategies:

$$a_{n,k}\ u_{o,x'} + b_{n,k}\ u_{o,x'} \le c_{n,k} \tag{16}$$

where:
k – is the nearest point of intersection of the straight lines approximating the coastline.

3 TYPES OF GAME AND SAFE SHIP CONTROL STRATEGIES

3.1 *Game optimal control rules*

The optimal control $u_o^*(t)$ of the own ship, equivalent for the current position p(t) to the optimal positional control $u_o^*(p)$, is determined in the following way:

- from the relationship (14) for the measured position $p(t_k)$, the control status at the moment t_k sets of the acceptable strategies $U_{j,o}[p(t_k)]$ are determined for the encountered ships in relation to the own ship, and from the relationship (9) the output sets $U_{o,j}[p(t_k)]$ of the acceptable strategies of the own ship in relation to each one of the encountered ships,
- a pair of vectors $u_{j,o}$ and $u_{o,j}$, are determined in relation to each j encountered ship and then the optimal positional strategy of the own ship $u_o^*(p)$ from the condition of optimum value I^* quality index control:
  - when the encountered ships non-cooperate:

$$I_{nc}^* = \min_{u_o^* \in U_o = \bigcap_{j=1}^{m} U_{o,j}} \left\{ \max_{u_{j,o} \in U_{j,o}} \ \min_{u_{o,j} \in U_{o,j}(u_j)} L[x_o(t_k), L_k] \right\} = I_{o,nc}^* \tag{17}$$

$$j = 1, 2, \ldots, m$$

  - when the encountered ships cooperate:

$$I_c^* = \min_{u_o^* \in U_o = \bigcap_{j=1}^{m} U_{o,j}} \left\{ \min_{u_{j,o} \in U_{j,o}} \ \min_{u_{o,j} \in U_{o,j}(u_j)} L[x_o(t_k), L_k] \right\} = I_{o,c}^* \tag{18}$$

$$j = 1, 2, \ldots, m$$

  - for the non-game optimal control:

$$I_{oc}^* = \min_{u_o^* \in U_o = \bigcap_{j=1}^{m} U_{o,j}} \left\{ L[x_o(t_k), L_k] \right\} = I_{o,oc}^* \tag{19}$$

$$j = 1, 2, \ldots, m$$

where:

$$L[x_o(t_k), L_k] = \int_{t_0}^{t_K} V(t)\,dt + r_o(t_K) + d(t_K) \tag{20}$$

refers to the goal control function of the own ship in the form of the payments – the integral payment and the final one (Lisowski 2016a).

The integral payment determines the distance of the own ship to the nearest turning point L_k on the assumed route of the voyage and the final one determines: $r_o(t_K)$ - the final risk of collision and $d(t_K)$ - final game trajectory deflection from reference trajectory.

The criteria for the selection of the optimal trajectory of the own ship is reduced to the determination of her course and speed, which ensure the smallest losses of way for the safe passing of the encountered ships at a distance not smaller than the assumed safe value D_s, having regard to the ship's dynamic in the form of the advance time t_m to the manoeuvre (Lisowski 2016b).

3.2 *Parameters of ship dynamics*

At the time advance maneuver t_m consists of element $t_m^{\Delta\psi}$ during course manoeuvre $\Delta\psi$ or element $t_m^{\Delta V}$ during speed manoeuvre ΔV.

The dynamic features of the ship during the course alteration by an angle $\Delta\psi$ is described in a simplified manner with the use of transfer function:

$$G_\psi(s) = \frac{\Delta\psi(s)}{\alpha(s)} = \frac{k_\psi(\alpha)}{s(1+T_\psi s)} \cong \frac{k_\psi(\alpha)\cdot e^{-T_{o\psi}s}}{s} \tag{21}$$

where:

$T_{o\psi} \cong T_\psi$ - manoeuvre delay time which is approximately equal to the time constant of the ship as a course control object,

$k_\psi(\alpha)$ - gain coefficient the value of which results from the non-linear static characteristics of the rudder steering.

The course manoeuvre delay time is as follows:

$$t_m^{\Delta\psi} \cong T_{o\psi} + \frac{\Delta\psi}{\dot{\psi}} \tag{22}$$

In practice, depending on the size and type of vessel advance time to the anti-collision manoeuvre through a change of course is: $t_m^{\Delta\psi} \cong 60 \div 720\ s$.

Differential equation of the second order describing the ship's behaviour during the change of the speed by ΔV is approximated with the use of the inertia of the first order with a time delay:

$$G_V(s) = \frac{\Delta V(s)}{\Delta n(s)} = \frac{k_v e^{-T_{ov}s}}{1+T_v s} \quad (23)$$

where:

T_{ov} - time of delay equal approximately to the time constant for the propulsion system: main engine-propeller shaft-screw propeller,

T_v - the time constant of the ship's hull and the mass of the accompanying water.

The speed manoeuvre delay time is as follows:

$$t_m^{\Delta V} \cong T_{ov} + 3\,T_v \quad (24)$$

In practice, depending on the size and type of vessel advance time to the anti-collision manoeuvre through a change of speed is: $t_m^{\Delta V} \cong 120 \div 900\, s$.

3.3 *Computer programs of positional multistage game ship control*

The smallest losses of way are achieved for the maximum projection of the speed vector of the own ship on the direction of the assumed course leading to the nearest turning L_k point.

The optimal control of the own ship is calculated at each discrete stage of the ship's movement by applying triple linear programming SIMPLEX method, assuming the relationship (20) as the goal function and the constraints are obtained by including the arrangement of the inequalities (8), (14) and (16).

The above problem is then reduced to the determination the function of control goal as the maximum of the projection of the own ship speed vector on reference direction of the movement:

$$\min L = \max\left[V\left(u_{o,x'}, u_{o,y'}\right) = u_{o,x'} \right] \quad (25)$$

with linear constraints approximating the joint set of the safe strategies of the own ship $U_{o,j}$:

$$\begin{aligned}
&a_{o,1}u_{o,x'} + b_{o,1}u_{o,y'} \le c_{o,1} \\
&a_{o,j}u_{o,x'} + b_{o,j}u_{o,y'} \le c_{o,j} \\
&a_{o,m}u_{o,x'} + b_{o,m}u_{o,y'} \le c_{o,m} \\
&a_{o,m+1}u_{o,x'} + b_{o,m+1}u_{o,y'} \le c_{o,m+1} \\
&a_{o,m+k+p}u_{o,x'} + b_{o,m+k+p}u_{o,y'} \le c_{o,m+k+p}
\end{aligned} \quad (26)$$

where:
m – number on encountered ships,
k – number of constraints approximating coastline,
p - number of segments approximating a semi-circle with a radius equal to own ship speed.

After the interval of time t_k the current fixing of the ship position is carried out and then comes the solving of the problem using the algorithm for the positional control.

Using the function of *lp* – linear programming from the Optimization Toolbox contained in the Matlab/Simulink the positional multistage game manoeuvring programs: *mpgame_nc* for criterion (17), *mpgame_c* for criterion (18) and *mpngame_oc* for criterion (19) has been designed for determination of the safe ship trajectory in a collision situation.

4 COMPUTER SIMULATION

4.1 *Navigational situation*

Computer simulation of mpgame_nc, mpgame_c and mpngame_oc algorithms was carried out in Matlab/Simulink software on an example of the real navigational situation of passing j=19 encountered ships in the Skagerrak Strait in good visibility Ds=0.5 1.0 nm and restricted visibility Ds=1.5 2.5 nm (nautical miles), (Fig. 4 and Tab. 1).

Figure 4. The place of identification of navigational situations in Skagerrak and Kattegat Straits.

The situation was registered on board r/v HORYZONT II, a research and training vessel of the Gdynia Maritime University, on the radar screen of the ARPA anti-collision system Raytheon (Fig. 5 and 6).

Table 1. Movement parameters of the own ship and encountered 19 ships.

j	Dj nm	Nj deg	Vj kn	ψj deg
0	-	-	20	0
1	9	320	14	90
2	15	10	16	180
3	8	10	15	200
4	12	35	17	275
5	7	270	14	50
6	8	100	8	6
7	11	315	10	90
8	13	325	7	45
9	7	45	19	10
10	15	23	6	275
11	15	23	7	270
12	4	175	4	130
13	13	40	0	0
14	7	60	16	20
15	8	120	12	30
16	9	150	10	25
17	8	310	12	135
18	10	330	10	140
19	9	340	8	150

Figure 5. The research-training ship of Gdynia Maritime University r/v HORYZONT II.

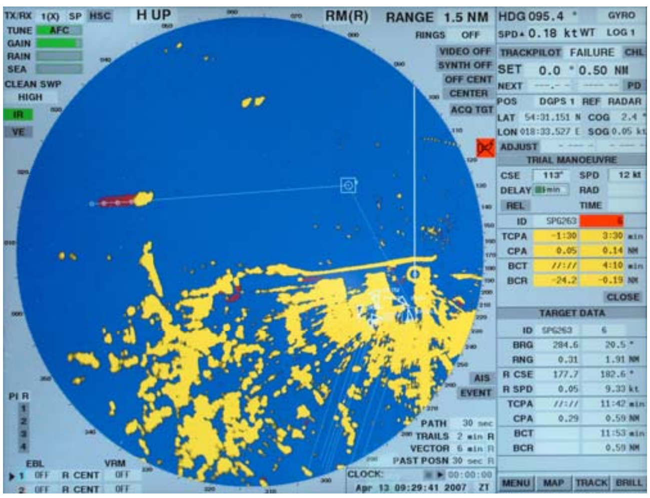

Figure 6. The screen of anti-collision system ARPA Raytheon, installed on the research-training ship of Gdynia Maritime University r/v HORYZONT II.

Examined the navigational situation, illustrated in the form of navigation velocity vectors of own ship and 19 met ships is shown in Figure 7.

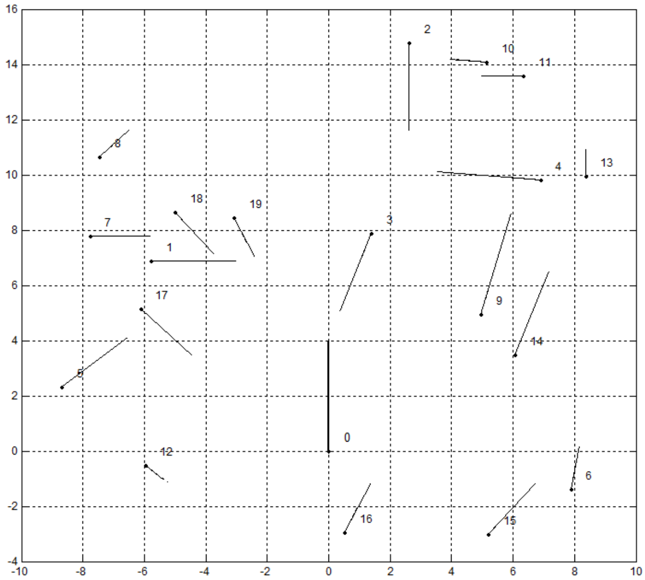

Figure 7. The 12 minute speed vectors of own ship 0 and j=19 encountered ships in navigational situation in Skagerrak Strait.

4.2 *Simulation of the multi-stage non-cooperative positional game*

Fig. 8 and 9 show the safe and optimal trajectory of the own ship in collision situation, which is determined using the algorithms of non-cooperative positional game in good and restricted visibility at sea.

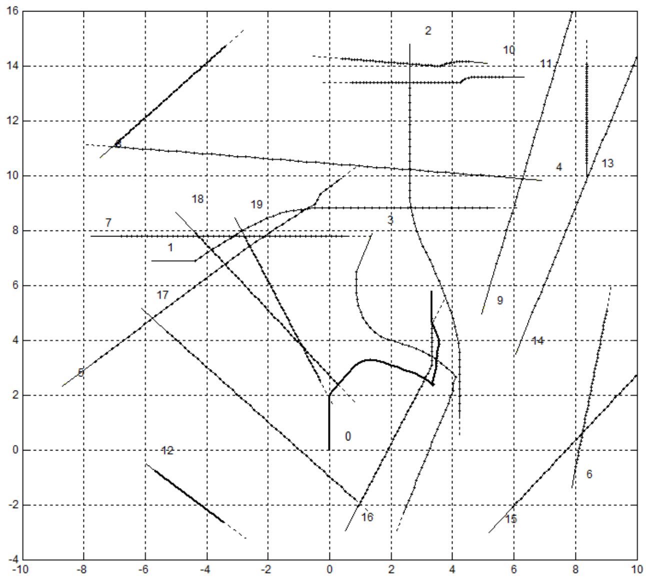

Figure 8. Computer simulation of multi-stage non-cooperative positional game algorithm *mpgame_nc* for safe own ship control in situation of passing 19 encountered ships in good visibility at sea, D_s=1.0 nm, $d(t_k)$=3.34 nm (nautical mile).

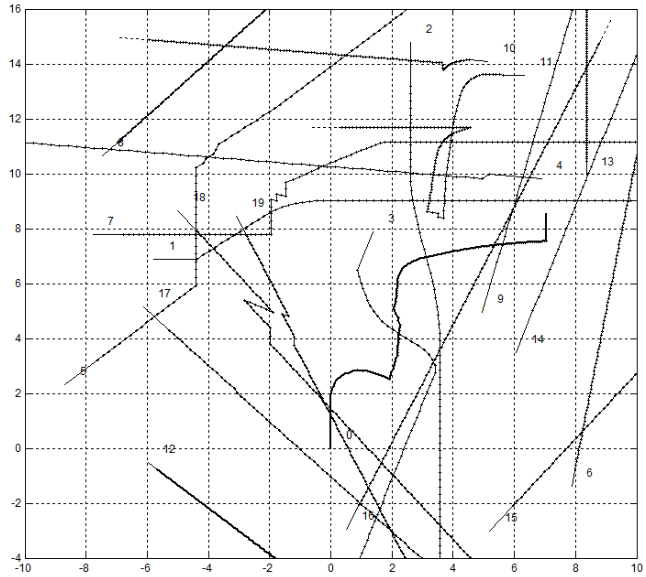

Figure 9. Computer simulation of multi-stage non-cooperative positional game algorithm *mpgame_nc* for safe own ship control in situation of passing 19 encountered ships in restricted visibility at sea, D_s=2.5 nm, $d(t_k)$=7.34 nm (nautical mile).

4.3 *Simulation of the multi-stage cooperative positional game*

Fig. 10 and 11 show the safe and optimal trajectory of the own ship in collision situation, which is determined using the algorithms of cooperative positional game in good and restricted visibility at sea.

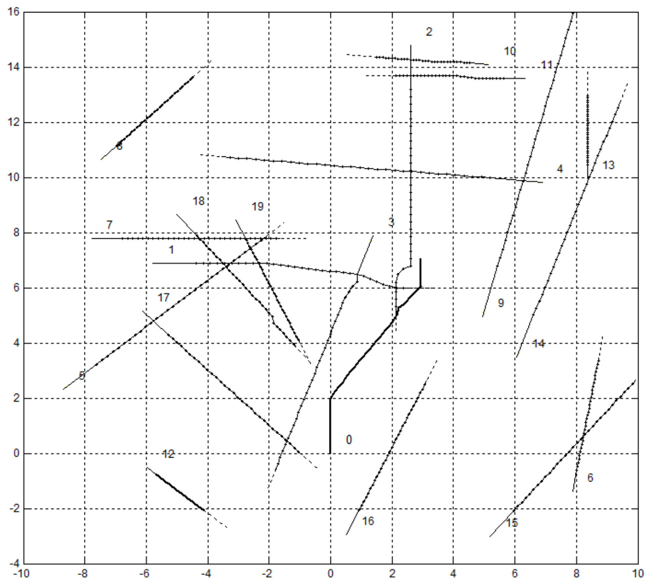

Figure 10. Computer simulation of multi-stage cooperative positional game algorithm *mpgame_c* for safe own ship control in situation of passing 19 encountered ships in good visibility at sea, D_s=1.0 nm, $d(t_k)$=2.94 nm (nautical mile).

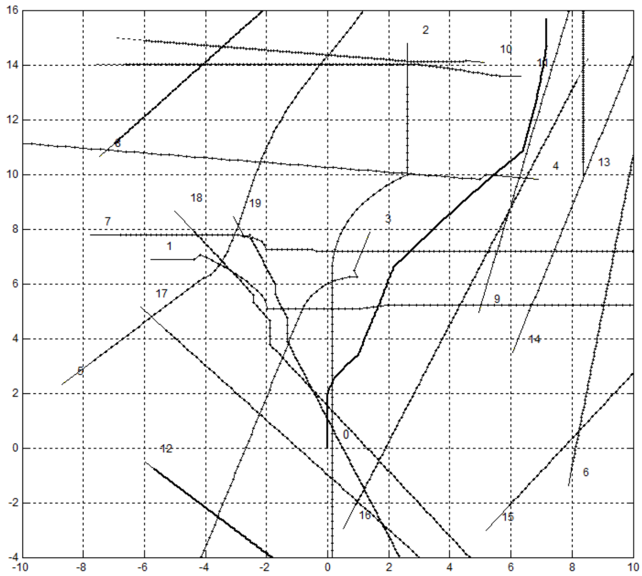

Figure 11. Computer simulation of multi-stage cooperative positional game algorithm *mpgame_c* for safe own ship control in situation of passing 19 encountered ships in restricted visibility at sea, D_s=2.5 nm, $d(t_k)$=7.06 nm (nautical mile).

4.4 *Simulation of the non-game optimal control*

Fig. 12 and 13 show the safe and optimal trajectory of the own ship in collision situation, which is determined using the algorithms of non-game optimal control in good and restricted visibility at sea.

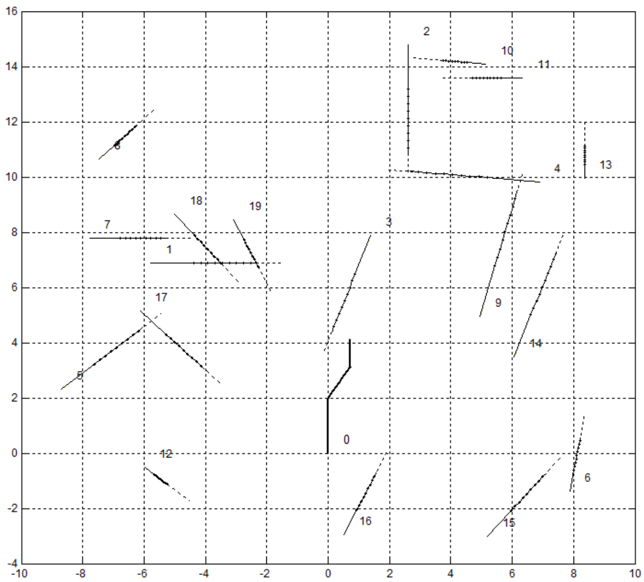

Figure 12. Computer simulation of non-game optimal control algorithm *mpngame_oc* for safe own ship control in situation of passing 19 encountered ships in good visibility at sea, D_s=1.0 nm, $d(t_k)$=0.72 nm (nautical mile).

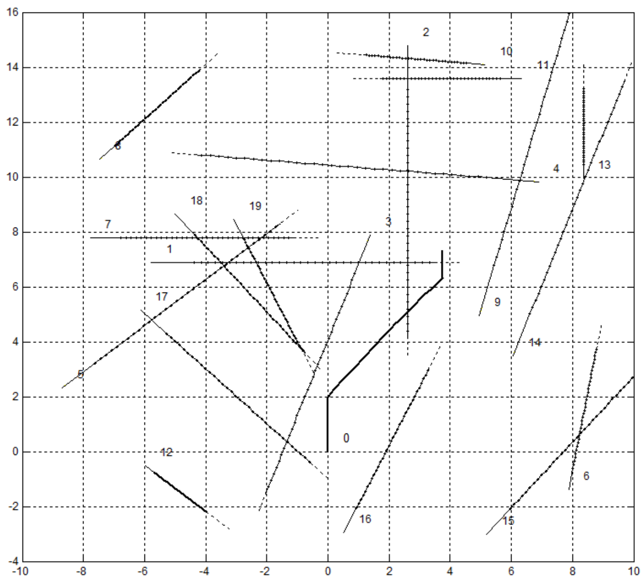

Figure 13. Computer simulation of non-game optimal control algorithm *mpngame_oc* for safe own ship control in situation of passing 19 encountered ships in restricted visibility at sea, D_s=2.5 nm, $d(t_k)$=3.76 nm (nautical mile).

5 CONCLUSIONS

The synthesis of an optimal on-line control on the model of a multi-stage positional game makes it possible to determine the safe game trajectory of the own ship in situations when she passes a greater j number of the encountered objects.

The trajectory has been described as a certain sequence of manoeuvres with the course and speed.

The computer programs designed in the Matlab also takes into consideration the following: regulations of the Convention on the International Regulations for Preventing Collisions at Sea, advance time for a manoeuvre calculated with regard to the ship's dynamic features and the assessment of the final deflection between the real trajectory and its assumed values.

The essential influence to form of safe and optimal trajectory and value of deflection between game and reference trajectories has a degree of cooperation between own and encountered ships.

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