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Construction and Analysis of New Mathematical Models of the Operation of Ship Propellers in Different Maneuvering Modes

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ABSTRACT: The influence of the curvilinear movement of the ship on the operation of the propeller was studied. It is shown that even at small values of the drift angle and the angular velocity of the vessel, the transverse component of the force on the propeller and the moment are non-zero and cannot be neglected. The existing and proposed new effective mathematical models of the longitudinal and transverse components of force and moment caused by the operation of the ship's propeller are analyzed. Simple expressions for the coefficients of the propeller thrust and the moment on the propeller shaft, the wake fraction, the thrust-deduction factor, and the flow straightening factor on the propeller at any drift angles and angular velocity are obtained. Numerical analysis of the obtained dimensionless components of forces and moments caused by the operation of the propeller is carried out, and their adequacy is shown. It is shown how the specified parameters change for all possible values of the drift angle and angular velocity. For a few commercial vessels of various types, technical characteristics and calculated dynamic parameters are given for the construction of mathematical models of propeller operation during curvilinear movement of the vessel.

1 INTRODUCTION

In the study of ship maneuvers, the development of ship autopilots and ship simulators, the availability of adequate mathematical models of the propulsive ship complex, which would describe the dynamics of the ship with high accuracy for the widest ranges of changes in kinematic parameters, plays a decisive role. The application of mathematical models of the ship's movement to the study of various maneuvers of the ship has recently become widely developed. The construction and active use of such models began in the middle of the twentieth century, these models are highlighted, in particular, in [1 - 5]. A significant contribution to the development and application of such models was also made by Japanese scientists, one of the first publications in this direction are works [6, 7]. This approach, which is also covered in [8] and is known as the MMG (Manoeuvring Modeling Group) method, was widely used, in particular, in [9 - 16]. A significant contribution to the development and application of mathematical models of ship dynamics was also made by the Portuguese school of researchers [17].

Mathematical models of the ship's propulsive complex consist of mathematical models of inertial and non-inertial forces and moments on the ship's hull. The latter include hydrodynamic and aerodynamic forces and moments on the ship's hull, forces and moments caused by the operation of propellers, auxiliary wind propulsors (sails), and ship's rudders. For the adequacy of the general mathematical model of ship dynamics, the adequacy of the mathematical models of all components of non-inertial forces and moments on the ship's hull is necessary.

Mathematical models of non-inertial forces and moments are usually empirical in nature and are built by processing data from experimental tests in ship model basins or during natural experiments. In particular, this applies to the forces and moments caused by the operation of the propeller. To describe the ship's maneuvers even in a horizontal plane (without taking into account roll and heel), linear mathematical models are mainly used. This is explained by the difficulty of using non-linear models and is argued by the small range of changes in the ship's kinematic parameters, such as the speed v, drift angle β, and angular speed ω_z during "weak maneuvers".

But the concept of "weak maneuver" is quite conditional, and even with small drift angles at the center of gravity of the vessel, the local drift angle at the stern increases significantly due to the angular velocity. Therefore, such important ship maneuvers as circulation, Kempf's zigzag, and, even more so, sharp evasion cannot be attributed to "weak" ship maneuvers.

In the mathematical modeling of the specified forces and moments, the rectilinear motion of the vessel, or motion with small values of the drift angles, is usually considered. At the same time, mainly, only the longitudinal action of the propeller is considered, and the transverse component of the force and moment on the propeller is not justifiably neglected. But experimental studies [18] show that even at small drift angles, the transverse component of the force on the ship's propeller takes on significant importance. In addition, known mathematical models usually contain coefficients and functions, which are defined rather complicatedly using tables and graphs, which is not convenient for numerical modeling. Among the shortcomings of the existing models of non-inertial forces should also be attributed their excessive, not always justified, simplification. Therefore, the construction of effective mathematical models of the ship's propulsive complex for a wide range of changes in kinematic parameters is not only an important scientific problem, but also an urgent practical task. In [19-24], new mathematical models of hydrodynamic forces on the hull were obtained for a wide range of changes in drift angle and angular velocity.

The development of the theory and calculation of the work of ship propellers is devoted, in particular, to works [2, 25 - 30]. These studies mainly concern a single propeller and the calculation of its parameters. The paper [28] provides a large array of experimental data for calculating the thrust coefficient of the propeller. In the works [2, 25], some aspects of the operation of the propeller during curvilinear movement of the ship were investigated, but the parameters of the models obtained there are presented in a form that is not convenient for use. In [3], the influence of auxiliary wind propulsors (sails) on the operation of the ship's propulsion complex was investigated.

There are known [18] experimental studies of the influence of the curvilinear movement of the ship on the operation of the propeller, which confirm the occurrence of significant transverse forces during such movement. Analysis of the latest research shows that, despite the recent significant development of mathematical models of the ship's propulsive complex, many problems require further resolution. In particular, the construction of adequate mathematical models that would take into account all components of the force and moment on the propeller for a wide range of changes in the kinematic parameters of the ship's movement and would be convenient for numerical modeling.

The goal of this work is the construction and numerical analysis of adequate mathematical models of forces and moments caused by the operation of ship propellers, which would, on the one hand, cover the entire range of changes in the kinematic parameters of the ship's motion, and on the other hand, would be convenient for use in solving various problems of the dynamics of the ship's propulsive complex.

The mathematical model of the operation of the ship's propeller is a multi-level model (as, after all, any complex empirical model), it consists of several dimensionless parameters, which are determined on the basis of experimental studies and each of which is important for the construction of the overall model. Therefore, we will first determine how the curvilinear movement affects the characteristics of the propeller, then we will determine and perform research for each parameter separately, and finally we will obtain a general mathematical model of the operation of the ship's propeller and conduct a numerical study of it as a whole.

2 THE INFLUENCE OF THE SHIP'S CURVILINEAR MOVEMENT ON THE CHARACTERISTICS OF THE SHIP'S PROPELLER

2.1 *Technical and kinematic parameters of the vessel and propeller*

The geometric and technical characteristics of the ship and propeller will be denoted as follows: L – length of the ship along the waterline; B – width of the vessel along the current waterline; T – draft of the ship on the midline, ρ – mass density of sea water, C_b – block coefficient; $W=C_bLBT$ – volume displacement of the vessel; $S = LT\sigma_D$ area of the underwater part of the centerplane of the ship; σ_D – reduced coefficient of the underwater centerplane of the ship; n_P, D_P – respectively, the rotation frequency and the diameter of the propeller; $\tilde{x}_P$ the relative position (in the stern, $\tilde{x}_P = x_P / L < 0$) of the propeller from the center of gravity of the vessel (for the main types of vessels, it is assumed that $\tilde{x}_P \in (-0.45; -0.5)$); $a = 4\pi^{-1} A_E D_P^{-2}$ – blade area ratio of the propeller, the value of which is within $0.4 \div 1.1$; $h = H \cdot D_P^{-1}$ – pitch ratio of the propeller, the values of which are, as a rule, within $0.5 \div 1.6$; κ – the number of propeller blades; v – the speed of the ship in the direction of movement; ω_Z –

the angular speed of rotation of the ship around the axis Z; $Fr = v / \sqrt{gL}$ – Froude number; v_0 – the value of the ship's speed at the time of the start of the maneuver (initial speed); β_P – local drift angle on the propeller; v_P – speed of flow on the propeller. The values of the main technical and kinematic parameters of some types of merchant ships are given in Table 1.

Mathematical models of forces and moments caused by the operation of the ship's propeller will be built relative to dimensionless kinematic phase coordinates: speed: $\tilde{v} = v \cdot v_0^{-1}$, drift angle β and angular velocity $\omega = \omega_Z L v^{-1} = v_0 \omega_Z L \tilde{v}^{-1}$.

Table 1. Technical and kinematic parameters of ships

Ship	DTC, contain. ship	KCS, contain. ship	KVLCC2 tanker	VLCC, tanker ESS0	VLGC, tanker	LPG, tanker
№	1	2	3	4	5	6
L [m]	355	230	320	325	226	147
B [m]	51	32.2	58	53	36.6	25.5
T [m]	14.5	10.8	20,8	21.79	11.8	8.8
C_B	0.661	0.651	0,809	0.831	0.72	0.740
D_P[m]	8.911	7.9	9.86	9.1	7.4	5.7
$\tilde{x}_P$	-0.45	-0.46	-0.48	-0.46	-0.47	-0.47
κ	5	5	4	5	4	4
a	0.8	0.8	0.431	0.682	0.42	0.601
h	0.959	0.997	0.721	0.715	0.905	0.784
ψ_{P0}	0.376	0.3365	0.4495	0.473	0.382	0.373
ζ_{P0}	0.281	0.2474	0.351	0.374	0.288	0.2796
μ_0	0.456	0.4749	0.292	0.319	0.3616	0.337
μ_1	-0.266	-0.264	-0.211	-0.251	-0.186	-0.248
μ_2	-0.265	-0.266	-0.218	-0.261	-0.218	-0.229
μ_3	0.085	0.0845	0.053	0.076	0.050	0.0723
μ_0^*	0.051	0.0530	0.028	0.031	0.0369	0.035
μ_1^*	-0.036	-0.037	-0.022	-0.027	-0.024	-0.029
μ_2^*	-0.033	-0.034	-0.009	-0.021	-0.01	-0.020
μ_3^*	-0.048	-0.055	-0.031	-0.022	-0.051	-0.024

2.2 *Influence of the curvilinear movement of the ship on the dynamic parameters of the propeller*

The operating parameters of a ship's propeller during curvilinear movement with a certain drift angle and angular velocity change significantly compared to straight-line movement. This is due to the following factors.

First, the local drift angle will change on the propeller, which will be determined by using the equality of the projections of the velocity vector at the center of gravity and on the propeller, respectively, on the X axis and the Y axis:

$$\begin{cases} v_P \cos\beta_P = v\cos\beta, \\ v_P \sin\beta_P = v\sin\beta - x_P \cdot \omega_Z. \end{cases} \tag{1}$$

On the right side of the last equality, the second term arose due to the rotation of the vessel around the Z axis. Further, after dividing the first equality by the second in relations (1), we represent the local drift angle as follows

$$\beta_P = \kappa_P \operatorname{arctg}(\operatorname{tg}\beta - \tilde{x}_P \frac{\omega}{\cos\beta}) \tag{2}$$

Here, by introducing a flow straightening factor κ_P, the change in flow due to the influence of the ship's hull during curvilinear motion is taken into account. The calculation of the coefficient will be discussed below.

Secondly, the speed of the flow on the propeller will change, which, using relation (1), will be given as follows:

$$v_P = (1-\psi_P) k_P v \tag{3}$$

Where k_P is a dimensionless coefficient that takes into account the influence of the ship's curvilinear movement on the velocity of flow on the propeller and can be calculated by the formula:

$$k_P = \sqrt{1 - 2\tilde{x}_P \omega \sin\beta + \tilde{x}_P^2 \omega^2}\,. \tag{4}$$

In representation (3), the influence of the ship's hull on the speed of flow on the propeller is determined by the wake fraction ψ_P during curvilinear movement, the calculation of which will be discussed below.

Thirdly, during curvilinear movement, the advance ratio of the propeller will also change, which in this case will be given as follows

$$J_P = \frac{v_P}{n_P D_P} = \frac{(1-\psi_P) k_P v_0 \tilde{v}}{n_P D_P}. \tag{5}$$

Fourthly, the longitudinal component of the force $X_P = (1-\zeta_P) T_P$, where T_P – the thrust of the propeller will change due to the change in the dimensionless thrust-deduction factor ζ_P on the propeller and the advance ratio J_P. In addition, as a result of a violation of the symmetry of the component forces acting in the plane of the propeller disk, a transverse force Y_P will be formed, which acts on the ship's propeller, as well as a moment $M_P = Y_P L_P$, where $L_P = |x_P|$ – the distance from the center of gravity of the ship to the propeller. Due to the change in the advance ratio J_P, the moment on the propeller shaft Q_P will also change. Dimensionless load coefficients [2, 24] correspond to the specified components of forces, in terms of the thrust of the propeller and in terms of the transverse force:

$$\sigma_P = \frac{8T_P}{\rho\pi v_P^2 D_P^2},\ \sigma_{yP} = \frac{8Y_p}{\rho\pi v_P^2 D_P^2}, \tag{6}$$

as well as dimensionless coefficients of propeller: thrust, transverse component of force and torque:

$$K_T = \frac{T_P}{\rho n_P^2 D_P^4},\ K_{Py} = \frac{8Y_P}{\rho n_P^2 D_P^4},\ K_Q = \frac{Q_P}{\rho n_P^2 D_P^5}. \tag{7}$$

3 MATHEMATICAL MODELS OF DYNAMIC AND KINEMATIC PROPELLER PARAMETERS AND THEIR NUMERICAL ANALYSIS

We will analyze existing and consider new mathematical models of dynamic and kinetic parameters of a ship's propeller. In particular, a number of studies [2], [25-29] show that the coefficients of propeller thrust and transverse force are functions of the advance ratio J_P, blade area ratio a, pitch ratio h and the number of propeller blades κ: $K_T = K_T(J_P, a, h, \kappa)$, $K_Q = K_Q(J_P, a, h, \kappa)$. Usually, these functions are approximated by polynomials:

$$K_T(J_P, a, h, \kappa_p) = \sum_{k=1}^{N} A_k \kappa^{\zeta_k} a^{\nu_k} h^{q_k} J_P^{g_k} \,, \tag{8}$$

$$K_Q(J_P, a, h, \kappa_p) = \sum_{k=1}^{N^*} B_k \kappa^{\zeta_k} a^{\nu_k} h^{q_k} J_P^{g_k} \,, \tag{9}$$

where the number of terms N, N^*, exponents ζ_k, ν_k, q_k, g_k and coefficients A_k, B_k, depend on the design features of the propellers and are determined by processing experimental data [27, 28] based on correlation analysis. In particular, for values of $\kappa = 2 \div 7$, it is not difficult to obtain the following representations:

$$K_T(J_P, a, h, \kappa) = \mu_0 + \mu_1 J_P + \mu_2 J_P^2 + \mu_3 J_P^3 \,, \tag{10}$$

$$K_Q(J_P, a, h, \kappa) = \mu_0^* + \mu_1^* J_P + \mu_2^* J_P^2 + \mu_3^* J_P^3 \,. \tag{11}$$

Table 1 shows the values of the coefficients of expressions (10) and (11), as well as the values of other technical and dynamic parameters necessary for the construction of mathematical models of forces and moment components for some types of commercial vessels.

The load coefficient on the thrust of the propeller can be easily expressed through the dimensionless coefficient of thrust of the propeller (10) and the advance ratio of the propeller (5):

$$\sigma_P = \frac{8K_T}{\pi J_P^2} \,. \tag{12}$$

For the transverse force load factor, we will use the presentation of work [2], and replacing there β_P with $\sin\beta_P$, we will generalize them to the entire range of changes in the local drift angle $0^\circ \le |\beta_P| \le 90^\circ$:

$$\sigma_{yP} = (\vartheta_1 + \vartheta_2 \sqrt{\sigma_{P0}}) \sin\beta_P \,, \tag{13}$$

$$\vartheta_1 = 0.177h + 0.087(a - 0.55);$$

$$\vartheta_2 = (0.275 + 0.233(a - 0.55))h^2 + (0.067 - 0.018(a - 0.55));$$

$\sigma_{P0} = \left.\dfrac{8K_T}{\pi J_P^2}\right|_{\beta=0;\omega=0}$ – the value of the thrust load factor of the propeller during rectilinear movement.

The calculation of the wake fraction for curvilinear motion is based on the wake fraction for rectilinear motion. The values of the latter are calculated by empirical formulas obtained on the basis of experimental data. The most acceptable representations here are [25]:

$$\psi_{P0} = 0.11 + 0.16(C_B)^{\varepsilon} \varepsilon^{-1} W^{1/6} D_P^{-1/2} - \Delta\psi'_{P0} \,, \tag{14}$$

$$\Delta\psi'_{P0} = \begin{cases} 0, & \text{if } Fr \le 0.2, \\ 0.3C_B(Fr - 0.2), & \text{if } Fr > 0.2, \end{cases}$$

where $\varepsilon = 1$, if the propeller is located in the diametrical plane and $\varepsilon = 2$ for the side propellers.

The following formula is usually used to calculate the wake fraction during curvilinear movement

$$\psi_P = \psi_{P0} \cdot q_\psi(\beta_P) \,, \tag{15}$$

where $q_\psi(\beta_P)$ – the function of the local drift angle on the propeller β_P, which is determined by processing the results of experiments, in particular, in [7] the following representation is given

$$q_\psi(\beta_P) = e^{-4\beta_P^2} \,. \tag{16}$$

At the same time, it should be noted that in representation (15) in [7], a simplified representation for the local drift angle is used: a linear dependence is used and the flow slope at the stern of the vessel is not taken into account. Representation (16), in addition to the indicated simplification, has the disadvantage that the values of the function decrease too slowly with an increase in the local drift angle and have a non-zero value in the region of the boundary angles. In work [2], a dependence is proposed

$$q_\psi(\beta_P) = \begin{cases} \left(1 - \dfrac{|\beta_P|}{\beta_0}\right)^n, & \beta_P \le \beta_0; \\ 0, & \beta_P > \beta_0, \end{cases} \tag{17}$$

where β_0 the limit value of the local drift angle on the propeller, at which the wake becomes zero, its value for commercial ships is: $\beta_0 \simeq 45^\circ$, the value of the indicator is selected from the range: $n = 5 \div 6$. Representation (17) contains a non-differentiable function with respect to the drift angle, which is inconsistent with the physical meaning of the coefficient ψ_P. Therefore, it is suggested to use the following improved representation:

$$q_{\psi}(\beta_P) = \begin{cases} \left(1 - \dfrac{\beta_P^2}{\beta_0^2}\right)^5, & \beta_P \le \beta_0; \\ 0, & \beta_P > \beta_0. \end{cases} \tag{18}$$

Figure 1 compares all three relationships for ship 1 (DTC, container ship) at initial speed and relative speed $v_0 = 7.7[\text{m/s}] = 14.97[\text{knots}]$. Graphs 1-3 correspond to the value of the relative angular velocity $\omega = 0$, graphs 4-6 correspond to the relative angular velocity $\omega = 0.9$.

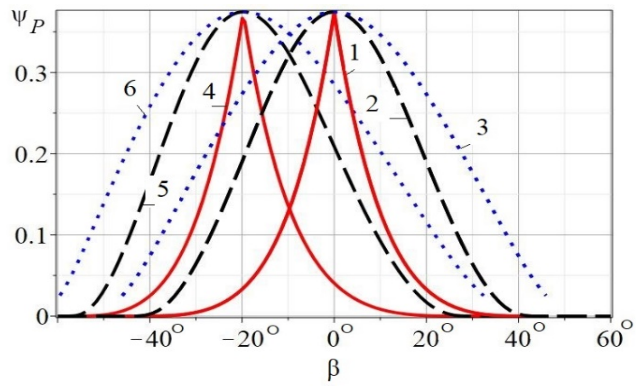

Figure 1. Comparison of the wake fractions

Solid red plots show representation (17), dashed black plots (18), and dashed blue curves (16). The calculation results show that dependence (18) is more accurate than dependence (16), approaches representation (17), while remaining a differentiated function.

To calculate the thrust-deduction factor ζ_p for propellers located in the diametrical plane, it is advisable to use the following generalized representations

$$\zeta_P = \begin{cases} 0.6\psi_P(1 + 0.67\psi_P), \text{ if } \varepsilon = 1, \\ 0.8\psi_P(1 + 0.25\psi_P), \text{ if } \varepsilon = 2, \end{cases} \tag{19}$$

Table 1 shows the values of the wake fraction ψ_{P0} and the thrust-deduction factor ζ_{p0} during straight-line movement for some types of vessels.

To calculate the slope of the flow coefficient κ_P on the propeller during curvilinear movement, it is advisable to use the representation:

$$\kappa_P = \begin{cases} \kappa_0 + (1 - \kappa_0)\left(\dfrac{\beta_P^*}{\beta_0}\right)^6, & \beta_P \le \beta_0; \\ 1, & \beta_P > \beta_0, \end{cases} \tag{20}$$

where κ_0 – the slope angle for small local drift angles, for commercial vessels, with a propeller in the diametrical plane, is usually chosen $\kappa_0 = 0.8$; to calculate β_P^*, you need to use formula (1), putting there $\kappa_P = 1$.

Figures 2 - 5 show three-dimensional graphs of the coefficients ψ_P and ζ_p, k_P, κ_P for all possible values of the drift angle β and angular velocity ω for ship 1 from table 1, at the initial speed $v_0 = 7.7[\text{m/s}] = 14.97[\text{knots}]$ and relative speed $\tilde{v} = 1.2$.

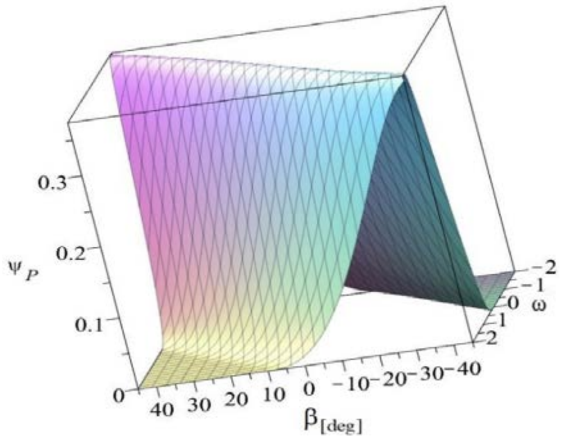

Figure 2. The wake fraction ψ_P

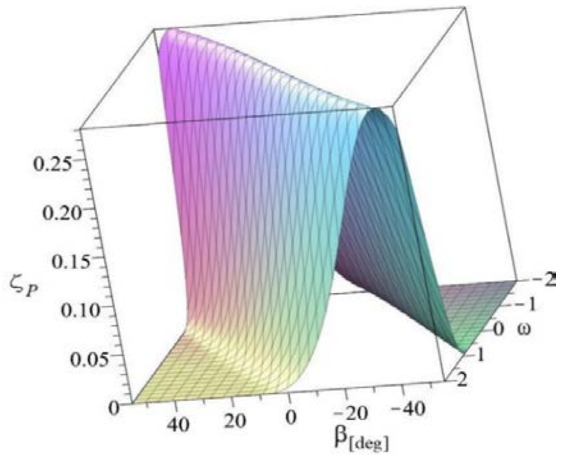

Figure 3. The thrust-deduction factor ζ_p

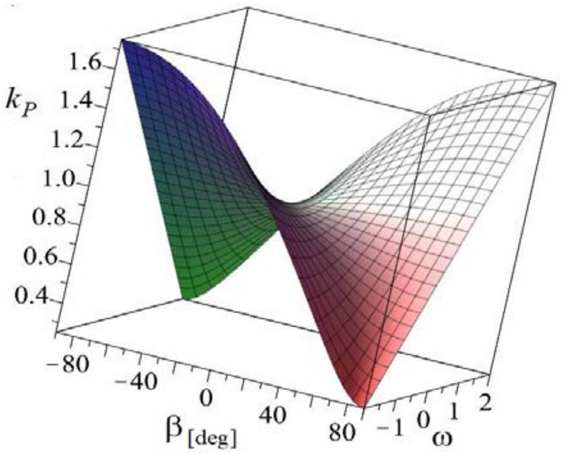

Figure 4. Dimensionless coefficient k_P

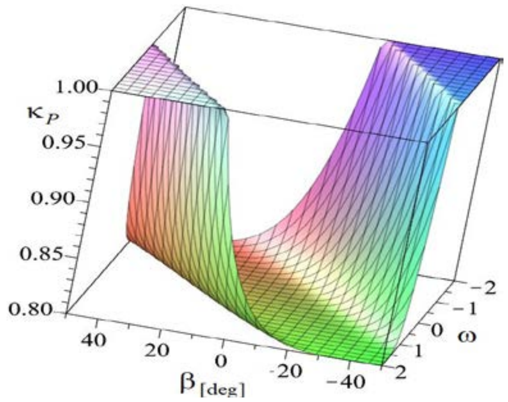

Figure 5. The slope of the flow coefficient κ_P

The results of the calculations show the adequacy of the given dependencies, their good consistency with the physical meaning of the coefficients for the entire possible range of changes in motion parameters, and prove the possibility of their application for solving any problems of ship maneuvering.

4 MATHEMATICAL MODELS OF FORCES AND MOMENTS CAUSED BY A PROPELLER DURING CURVILINEAR MOVEMENT AND THEIR NUMERICAL ANALYSIS

The expressions obtained above for dimensionless coefficients make it possible to determine the projections of forces and moments on the propeller during curvilinear movement. In particular, the longitudinal component of the force on the propeller and the moment on the propeller shaft are given through the dimensionless coefficients (10) and (11) as follows:

$$X_P = (1-\zeta_P)T_P = (1-\zeta_P)\rho n_P^2 \mathrm{D}_P^4 K_T\,,$$

$$Q_P = \rho n_P^2 \mathrm{D}_P^5 K_Q\,. \tag{21}$$

The transverse component of the force and moment that arise on the propeller and act on the hull of the vessel is given through the dimensionless load factor (13) as follows:

$$Y_p = \frac{1}{8}\rho\pi v_P^2 \mathrm{D}_P^2 \sigma_{yP}\,,\quad M_P = Y_P L_P\,. \tag{22}$$

When studying various maneuvers of the ship, in mathematical models of the dynamics of the propulsive complex usually pass to dimensionless phase coordinates. To do this, dividing the first two differential equations of ship motion [4, 5] by the expression $0.5\rho S v_0^2$, and the third by the expression $0.5\rho S L v_0^2$, we proceed to the dimensionless components of the inertial and non-inertial forces and moment acting on the ship. In particular, the longitudinal and transverse components of the dimensionless forces and the moment caused by the operation of the propeller during curvilinear motion can be represented as follows:

$$\tilde{X}_P = G_X K_T\,,\quad \tilde{Y}_p = G_Y \sigma_{yP}\,,\quad \tilde{M}_P = G_m \sigma_{yP}\,, \tag{23}$$

$$G_X = \frac{2n_P^2 D_P^4}{S v_0^2},\quad G_Y = \frac{2\pi\tilde{v}^2 D_P^2}{4S},\quad G_m = \frac{2\pi\tilde{v}^2 L_P D_P^2}{4SL}.$$

Figures 6 - 9 show three-dimensional graphs of the advance ratio of the propeller J_P, the longitudinal component of the force $\tilde{X}_P$, the transverse component of the force $\tilde{Y}_p$ and the relative moment $\tilde{M}_P$ for the widest range of changes in the drift angle and angular speed, respectively. Calculations were carried out for vessel 1 from table 1, at initial speed $v_0 = 7.7[\mathrm{m/s}] = 14.97[\mathrm{knots}]$ and relative speed $\tilde{v} = 1.2$. The calculated results confirm that even at small values of the drift angle and angular velocity, the transverse component of the force $\tilde{Y}_p$ and the moment on the ship's propeller $\tilde{M}_P$ are not zero, and as the drift angle increases, their values approach the longitudinal component of the force $\tilde{X}_P$, and near the drift angle $\beta \approx \pm 45^\circ$ have values of the same order (modulo). These results are in good agreement with the experimental studies of work [18].

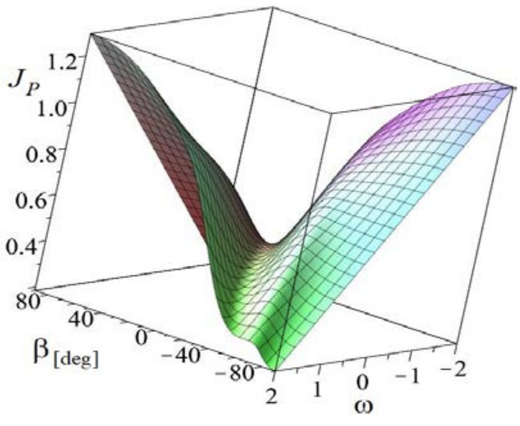

Figure 6. The advance ratio of the propeller J_P

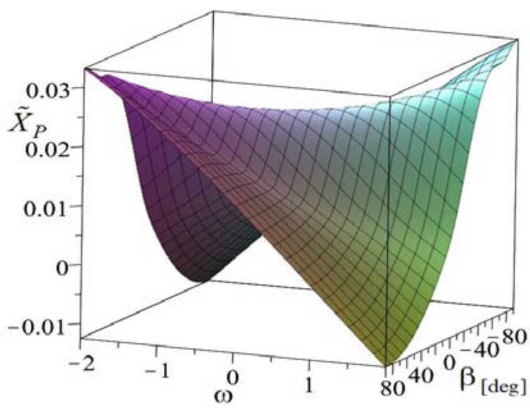

Figure 7. Longitudinal component of force $\tilde{X}_P$.

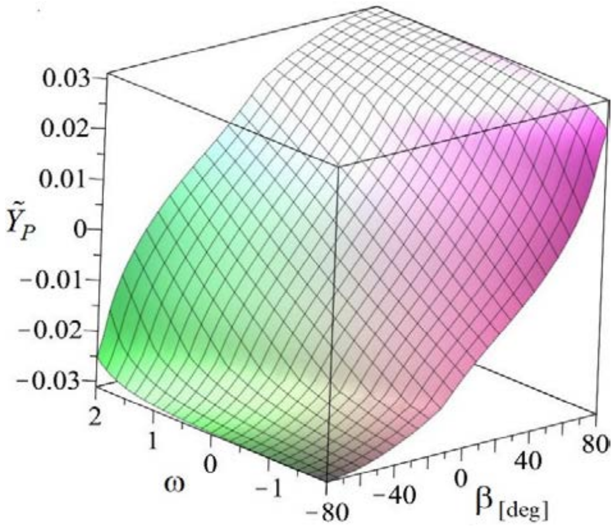

Figure 8. Transverse component of the force $\tilde{Y}_p$.

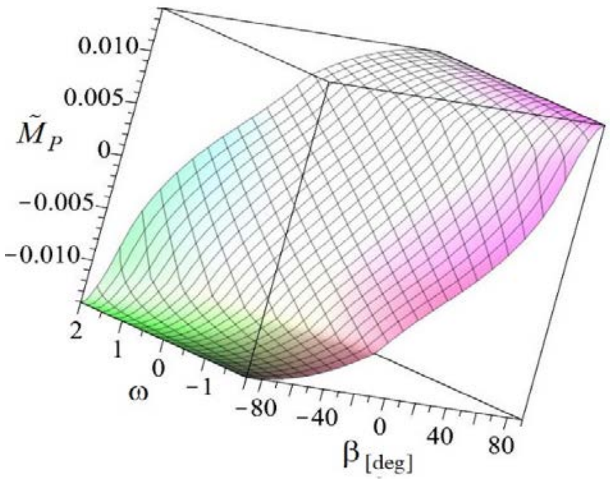

Figure 9. Relative moment $\tilde{M}_P$.

It should also be noted that the transverse component of force $\tilde{Y}_p$ and moment $\tilde{M}_P$ reach their largest values (modulo) at $\beta \approx \pm 80^{\circ}$, while the signs of the transverse component of force and moment coincide with the sign of the local drift angle. As for the longitudinal component of the force $\tilde{X}_P$, its greatest value is reached during rectilinear motion $(\beta = 0^{\circ})$, and when $\beta \to \pm 70^{\circ}$ and $\omega \to \pm 2$ approaches zero, when $\beta \approx \pm 80^{\circ}$ and $\omega \approx \pm 2$ takes a negative value, this is explained by the fact that at these values of the drift angle and angular velocity, the local drift angle β_P on the propeller takes the value: $|\beta_P| > \pm 90^{\circ}$, and therefore the direction of action of forces caused by the operation of the propeller and the direction of movement of the vessel form an obtuse angle.

5 CONCLUSIONS

Thus, the influence of the curvilinear movement of the ship on the operation of the propeller was investigated. It is shown that even at small values of the drift angle and the angular velocity of the vessel, the transverse component of the force on the propeller and the moment are non-zero and cannot be neglected. The existing and proposed new effective and convenient mathematical models of the longitudinal and transverse components of force and moment caused by the operation of the ship's propeller in dimensional and dimensionless forms are analyzed. In particular, simple representations are obtained for the dimensionless coefficients of the propeller thrust and the moment on the propeller shaft, the wake fraction, the thrust-deduction factor and the flow straightening factor on the propeller for any drift angles and angular velocity. Numerical analysis of the obtained dimensionless components of forces and moments caused by the operation of the propeller is carried out and their effectiveness is shown. The specified parameters for all possible values of the drift angle and angular velocity were studied and their adequacy and applicability were shown. For a number of commercial vessels of various types, technical characteristics and calculated dynamic parameters are given for the construction of mathematical models of propeller operation during curvilinear movement of the vessel.

The obtained results will make it possible to build general adequate mathematical models of the ship's propulsive complex.

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