# Combined Method Of Sight Reduction 

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#### Abstract

As ships and maritime transport have evolved, knowledge of navigation methods has also evolved, reaching today modern means that require less of the skills and time of navigators to determine the position of the ship on sees and oceans. However, the IMO resolutions maintain the obligation for seafarers to know the procedure for deter-mining the position of the ship based on the use of astronomical position lines, a process known simply as the "Intercept Method". As is well known, the classical "Intercept Method" involves a graphical stage aimed to determine the geographical coordinate of Fix position. This paper presents a combined method which eliminates the graphical construction which may involve plotting errors. The method introduces mathematical computation of fix geographical coordinates.


## 1 INTRODUCTION

From the time of French Admiral Marcq St. Hilaire sailors inherited and successfully used a method of determining the position of the ship, a method that bears her name and is known in the Anglo-Saxon language as the "Intercept Method".

The classical method, as it exists today, consists in calculating the elements of two astronomical lines of position (LOP) that can be determined on the basis of simultaneous observations, as a result the ship's position being a "fix" or on the basis of time-delayed observations, the ship's position being a "running fixed".

As it is known, the calculations are based on a DRP (Dead Reckoning Position). In the case of simultaneous observations, the calculations performed to determine the two LOPs are affected by
the error induced by DRP. This means that we introduce this error for twice.

The method proposed by this paper reduces the errors by half given that the second LOP is calculated starting from a point located on the first LOP which means that it can be assimilated to a fixed position. This point is the intersection point of the intercept with the azimuth.

The introduction of Intercept Point coordinates in the calculation of the second LOP leads to an increase in the accuracy of the final calculation of the ship's position coordinates.

Intercept Point coordinates are calculated using the Plane Navigational Triangle from the mathematics used by navigators and known as "Sailings".

The elements to be determined are difference of latitude and difference on longitude which will be applied to DRP coordinates.

With the obtained results the second Plane Navigational Triangle to find the coordinates of the ship's fix.

As can be seen, the graphic constructions are missing.

The concept of the problem can be the object of a computer program. [1-4]

## 2 PRINCIPLE OF THE METHOD

The classical "Intercept Method" involves the following steps:

- taking the sight of two celestial bodies and then sight reduction; as a result, the intercept and azimuth are obtained
- the two lines of position (LOP) are plotted using intercept and azimuth
- the fix position of the vessel is given by the intersection point of the two LOPs.
- the geographical coordinates of the fix are extracted using graphical procedure
The method proposed by this paper takes the first steps presented above and eliminates the graphical constructions. The geographical coordinates of the fix are determined by computation. In this way the graphical plotting errors are eliminated and accuracy of fix determination increases enough.

In many cases the sight reduction starts from estimated position (EP), from dead reckoning position (DRP) or from assumed position (AP). All of them include errors. The both LOPs are affected by the errors of EP, DRP or AP.

This method uses geographical coordinates of DRP only for the first LOP. To reduce the second sight the coordinates of the first intercept point (IP1) will be used. As a result, the origin of the second azimuth $(\mathrm{Zn} 2)$ will be placed just in the first intercept point.

The IP is plotted on the azimuth toward the celestial body (CB) if the intercept is positive or away the $C B$ if intercept is negative.

To compute the Fix coordinates simple trigonometric formulas are used. They are the same as those used by "Plane Sailing". A few words about the "Plane sailing".

The Earth surface is considered being a plane surface. In this way the navigational triangle (see picture bellow) is a plane right triangle. Its elements are:


A
Figure 1. The navigational triangle
$A$-departure position
$B$ - arrival position
C - course angle
Dist - distance to be travelled
Dep - departure = distance measured on the arrival point parallel between the meridians of departure and arrival positions
DLat - difference of latitude
Formulas:
DLat $=$ Dist $\cdot \cos (C)$
Dep $=$ Dist $\cdot \sin (C)$
DLon $=$ Dep $\cdot \sec ($ Lat $/$ med $)$
where:
DLon $=$ difference of longitude
Latm $=$ half the aritmetical sum of the departure position latitude (LatA) and the arrival position latitude (LatB).

The arrival point coordinates are computed as follows:
$L a t_{B}=L a t_{A}+D L a t$
$L^{\prime} n_{B}=L^{2} n_{A}+D L o n$

## 3 CLASSICAL METHOD

On November 20, 2020, at the chronometer time $\mathrm{CT}=19 \mathrm{~h} 20 \mathrm{~m} 16 \mathrm{~s}$ in DR position: Lat $=41^{\circ} 47.8^{\prime} \mathrm{N}$; Lon $=$ $029^{\circ} 34.6^{\prime} \mathrm{W}$ the following sights were taken and recorded: Altair star - observed altitude $\mathrm{Ho}=51^{\circ} 30.2^{\prime}$ and Vega star - observed altitude $\mathrm{Ho}=58^{\circ} 39.9^{\prime}$. Height of eye is 14 m and index correction IC $=+1.5^{\prime}$. The chronometer correction is $\mathrm{CC}=+00 \mathrm{~m} 00 \mathrm{~s}$. The geographical fix coordinates are required.

Table 1. Shorted computation

| Date | Nov. 20, 2020 | Nov. 20, 2020 |
| :--- | :--- | :--- |
| Body | ALTAIR | VEGA |
| Ho | $51^{\circ} 30^{\prime} 2$ | $58^{\circ} 39^{\prime} 9$ |
| Hc | $51^{\circ} 28^{\prime} 3$ | $58^{\circ} 37^{\prime} 7$ |
| Intercept(a) | $+1,8 \mathrm{Nm}^{\circ}$ | $+2,2 \mathrm{Nm}^{\circ}$ |
| Zn | $217^{\circ}, 9$ | $278^{\circ}, 5$ |

$D P R\left\{\begin{array}{c}\text { Lat }=41^{\circ} 477^{\prime}, 8 \mathrm{~N} \\ \text { Lon }=029^{\circ} 34^{\prime}, 6 \mathrm{~W}\end{array}\right.$
LOP ${ }_{1}$ ALTAIR $\left\{\begin{array}{l}a_{1}=+1,8 \mathrm{Nm} \\ Z n_{1}=217^{\circ} 9\end{array}\right.$
$L O P_{2}$ VEGA $\left\{\begin{array}{c}a_{2}=+2,2 \mathrm{Nm} \\ Z n_{2}=278^{\circ} 5\end{array}\right.$


Figure 2. Graphic representation

Diff $\left\{\begin{array}{l}\text { DLat }=-0 ', 5 \\ \text { DLon }=-3^{\prime}, 1\end{array}\right.$
Solution:

$$
\begin{aligned}
& \begin{array}{l}
D R L a t \\
+D L a t
\end{array}=+41^{\circ} 47^{\prime}, 8 \\
& \hline \text { Lat }=41^{\circ} 47^{\prime}, 5
\end{aligned}, 3 \mathrm{~N} .
$$

## 4 COMBINED METHOD

As we have stated above, the second azimuth will be plotted from the first intercept point (IP1). As a result, the graphical constructions will be changed.


Figure 3. Graphic representation
We can see the DRP, the first azimuth (Zn1), the first intercept (a1), the first intercept point (IP1) and the first LOP (LOP1).

The second azimuth $(\mathrm{Zn} 2)$ is plotted from the first intercept point (IP1) not from DRP. Being a point on the first LOP (LOP1) the IP1 is more accurate than DRP.
Algorithm:

1. reduce the first sight using the sin Hc
2. compute the geographical coordinates of the first intercept point (IP1)
3. reduce the second sight by means of the sin Hc formula using the IP1 coordinates instead of DRP coordinates
4. solve the right triangle formed by IP1, IP2 and Fix points to compute the size of Dist2
5. compute the geographical coordinate of Fix position.
6. Reduce the first sight

The computation was done separately and recorded in the Shorted Computation table: intersect ( $a_{1}$ ) $=1,8 \mathrm{Nm}$ and azimuth $\left(Z n_{1}\right)=217^{\circ} 9$.
2. Compute the geographical coordinates of the first intercept point (IP1)
We will consider DRP as departure point (DP) and IP1 as arrival point. Also, the first intercept (a1 = 1,8 Nm) will be considered as distance to be travelled (Dist1) and the direction of first azimuth ( $\mathrm{Zn} 1=217^{\circ}, 9$ ) as course angle (C1).

- DLat computation: DLat $=$ Dist $_{1} \cdot \cos C_{1}$

DLat $=1,8 \cdot \cos \left(217^{\circ}, 9\right)=1,8 \cdot(-0,789084)=-1^{\prime}, 4$

- Departure computation: DLat $=$ Dist $_{1} \cdot \sin C_{1}$

Dep $=1,8 \sin \left(217^{\circ}, 9\right)=1,8 \cdot(-0,6142852)=-1^{\prime}, 1$

- Latitude of $I P_{1}$ computation:

$$
\begin{aligned}
& L a t_{1}=\text { DRLat }+ \text { DLat } \\
& I P_{1} \text { Lat }=41^{\circ} 47^{\prime}, 8+\left(-1^{\prime}, 4\right)=41^{\circ} 46^{\prime}, 4
\end{aligned}
$$

- DLon computation:

DLon $=$ Dep $\cdot \sec \left(L a t_{m}\right)$
$L a t_{m}=\frac{\text { DRLat }+I P_{1} \text { Lat }}{2}=\frac{41^{\circ} 47^{\prime}, 8+41^{\circ} 46^{\prime}, 4}{2}=\frac{83^{\circ} 34^{\prime}, 2}{2}=41^{\circ} 47^{\prime}, 1$

DLon $=-1^{\prime}, 1 \cdot \sec \left(41^{\circ} 47{ }^{\prime}, 1\right)=-1^{\prime}, 1 \cdot 1,3411109=-1^{\prime}, 5$
$I P_{1}$ Lon $=-029^{\circ} 34^{\prime}, 6-1^{\prime}, 5=-029^{\circ} 36^{\prime}, 1$
$I P_{1}\left\{\begin{array}{l}\text { Lat }=41^{\circ} 46^{\prime}, 4 \mathrm{~N} \\ \text { Lon }=029^{\circ} 36^{\prime}, 1 \mathrm{~W}\end{array}\right.$
3. Reduce the second sight
(separate computation using sin Hc formula)
LOP $_{2}\left\{\begin{array}{c}a=+1,3 \mathrm{Nm} \\ Z n_{2}=278^{\circ}, 6\end{array}\right.$
4. Solving the right triangle formed by IP1, IP2 and Fix points
We can observe that this triangle is a right triangle.
We know:

- the side $I P_{1} / I P_{2}$ is equal to $a_{2}=+1,3 \mathrm{Nm}$
- the angle formed by $L O P_{1} / L O P_{2} /$ Fix is equal to difference in azimuths
$\Delta Z n=Z n_{2}-Z n_{1}=278^{\circ}, 6-217^{\circ}, 9=60^{\circ}, 7$
- the hypotenuse $L O P_{1} /$ Fix (labeled Dist $)_{\text {) }}$ can be computed as follows:

$$
\operatorname{Dist}_{2}=a_{2} \cdot \operatorname{cosec}(\Delta Z)=+1^{\prime}, 3 \cdot \operatorname{cosec} 60^{\circ}, 7=1,3 \cdot 1,1466978=+1,5 \mathrm{Nm}
$$

- orientation of $L O P_{1}$ can be considered the direction of course from $D R P$ to Fix ( $C_{2}$ )
$C_{2}=Z n_{1}+90^{\circ}=217^{\circ}, 9+90^{\circ}=307^{\circ}, 9$

5. Compute the geographical coordinate of Fix position
To compute the Fix geographical coordinates, we will consider the first intersect point (IP1) as a departure point (DP) and the Fix position as an arrival point (AP).

We have the following elements:

- departure point coordinates:

Lat $=41^{\circ} 46^{\prime}, 4 N ;$ Lon $=029^{\circ} 36^{\prime}, 1 \mathrm{~W}$

- distance to be traveled ( Dist $_{2}=1,5 \mathrm{Nm}$ )
- course angle ( $C_{2}=307^{\circ}$,9)

1. DLat computation:

DLat $=$ Dist $_{2} \cdot \cos \left(C_{2}\right)$
DLat $=1,5 \cdot \cos \left(307^{\circ}, 9\right)=1,5 \cdot 0,61422852=0^{\prime}, 9$
2. Dep 2 computation:

DLat $=$ Dist $_{2} \cdot \sin \left(C_{2}\right)$
$D e p_{2}=1,8 \cdot \sin \left(307^{\circ}, 9\right)=1,5 \cdot(-0,789084)=-1^{\prime}, 2$
3. Lat of Fix computation:
provides the fixed position of the ship based on measured inputs.
$L a t_{F i x}=I P_{1} L a t+D L a t$
$L a t_{\text {Fix }}=41^{\circ} 46^{\prime}, 4+0^{\prime}, 9=41^{\circ} 47{ }^{\prime}, 3 N$
d. DLon computation:

DLon $=$ Dep $_{2} \cdot \sec \left(\right.$ Lat $\left._{m}\right)$
$L a t_{m}=\frac{I P_{1} L a t+\text { Lat }_{F i x}}{2}=\frac{41^{\circ} 46^{\prime}, 4+41^{\circ} 47^{\prime}, 3}{2}=\frac{83^{\circ} 33^{\prime}, 7}{2}=41^{\circ} 46^{\prime}, 9$
DLon $=-1^{\prime}, 2 \cdot \sec \left(41^{\circ} 46^{\prime}, 9\right)=-1^{\prime}, 2 \cdot 1,3410412=-1^{\prime}, 6$
Lon $_{\text {Fix }}=-029^{\circ} 36^{\prime}, 1-1^{\prime}, 6=029^{\circ} 37^{\prime}, 7 \mathrm{~W}$
Fix $\left\{\begin{array}{c}\text { Lat }=41^{\circ} 47^{\prime}, 3 \mathrm{~N} \\ \text { Lon }=029^{\circ} 37^{\prime}, 7 \mathrm{~W}\end{array}\right.$

## 5 CONCLUSION

The results obtained by using the two methods are equal which demonstrates the accuracy of the combined method.

The proposed calculation method brings two major advantages:

- downsize of calculation errors of the two simultaneous LOPs
- elimination of errors created by graphic plotting.

The content of the method is in fact a step forward in the process of improving the use of astronomical navigation in our days by increasing the degree of accuracy in determining the astronomical position of the ship.

To facilitate the use of method, it is recommended to prepare by the time observation and computation sheets whose fields should be completed step by step in real time.

Mastering the method but especially its application does not mean a return to the past but rather an additional safety measure in keeping the navigation accurate.

## REFERENCES

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