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Availability of Traffic Control System Based on Servicing Model

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ABSTRACT: Traffic control is a component of transport system, on which safety and efficiency of means of transport movements substantially depend. It is not possible to achieve and maintain appropriate availability of traffic control system unless issues of appropriate maintenance servicing of traffic control equipment have been resolved. This will require applying a specific servicing policy, worked out on the basis of such system availability model. The servicing of a technical object is understood as any treatment, which results in restoring the object's state of availability. Servicing may consist in a repair of equipment or in its inspection. Classification of servicing optimisation models, in respect of using appropriate mathematical methods, such as Markov models, is presented.

The main goal of transport consists of movements of various means of transport, carrying people and cargo. This process shall be carried out maintaining a high level of safety. To this end movements of means of transport are based on ordered principles of traffic organisation, which from the point of view of technical functionality are fulfilled by systems of vehicles movements' control, so-called traffic control systems. These systems enable execution, through equipment localised in various places of the transport network, of appropriate control algorithms.

All components and systems of transport traffic control are required to show operation certainty. The operation certainty is understood as a probability of defect non-occurrence. A defect consist in – at two-state classification – a transition of a piece of equipment (in defined operating conditions and at defined time) from the state of availability (fitness) to non-availability.

Transport traffic control systems work in diversified, frequently most critical, operating conditions. The experience from such equipment operation confirms the dependence of proper systems' functioning on reliability of their components.

In transport traffic control equipment defects may cause only traffic disturbances (e.g. delays), but also occurrence of dangerous situations.

Traffic control system is a set of pieces of equipment, which change their states between the state of availability (i.e. the state of fitness to execute a task or function in the system) and states of non-availability at discrete moments in time, i.e. they are dynamic states. The set of such systems' states is a discrete set. Transitions between the following states are stochastic in nature and occur at random, in accordance with certain probability distribution.

The operation of traffic control systems to a large extent shall focus on achieving appropriate availability of traffic control equipment and on maintaining it through a required period. This results in a need to resolve problems of appropriate maintenance service of traffic control equipment (repairs and inspections)

Most important features of safe systems include: safety, availability, reliability, repairability.

Mutual links between main features of safe systems are presented in Figure 1.

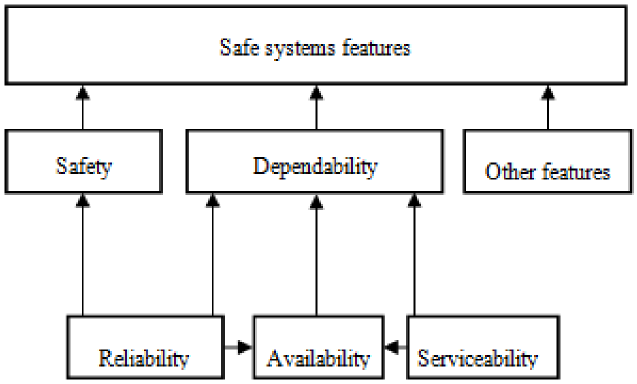

Figure 1. Links between features of safe systems

The servicing of a technical object (equipment) – Figure 2 – is understood as any treatment, which results in restoring the object's state of availability (operational). Servicing may consist in a repair of equipment or in its inspection, replacement of the entire equipment with a new one or in replacement of damaged components with new ones. Parameters characterising the equipment at servicing must ensure that it is operational, although they may differ from a new object (in particular this refers to defects intensity).

An inspection, maintenance and condition control are comprised (apart from a repair) by so-called technical service of equipment, which is opposite to its use.

To prevent adverse effects of unpredicted defects (failures), equipment which is still in the state of availability (operational) is subject to servicing. Such servicing is named preventive and distinguished from emergency servicing.

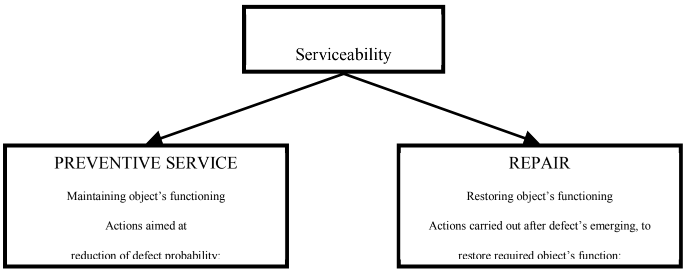

Figure 2. Servicing of a technical object

Properties of any system (in the case considered, of a transport traffic control system) indicate that such system, from the point of view of its servicing, may be presented (on certain level of generality) as a set of states of using, repair and inspection (Fig. 3). When analysing this diagram, the state of availability (initial) and the state of effective control (system transition between the basic state and the control state under influence of introduction of control command and after execution of the control task) may be distinguished within the state of using. But in servicing it is most important to distinguish the state of repair and inspection.

Classification of servicing optimisation models, both in respect of individual devices and systems, also because of using appropriate mathematical methods, such as inter alia linear and non-linear programming, dynamic programming, but first of all Markov models.

Game theory and stochastic processes theory, mainly of Markov processes, are used to model the process of technical objects operation (Koźniewska & Włodarczyk 1978). Mass servicing theory (referred to also as queuing theory) is strongly related to technology and its development resulted from practical demands.

In general form each queuing system may be presented using a block diagram (Fig. 4).

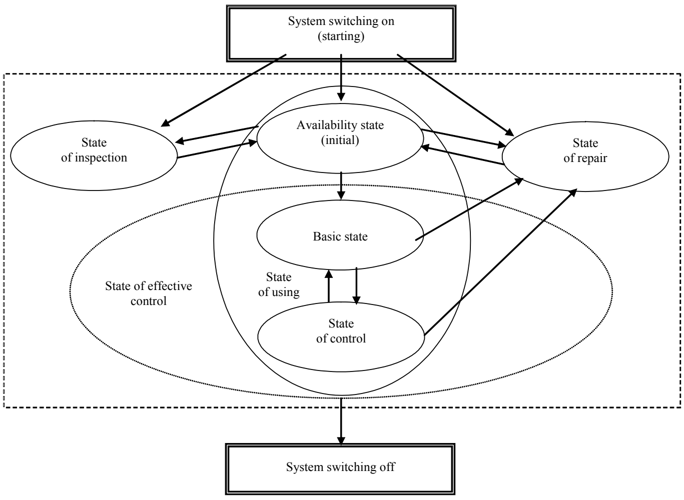

Figure 3. States of the transport traffic control system

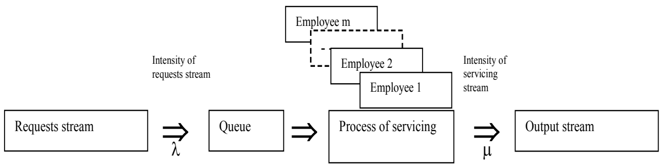

Figure 4. Block diagram of a queuing system

A queuing system may be described using three basic characteristics:

- Stream of requests – this is a statistical description of process of requests arriving at the system,
- Process of servicing – defines the process of requests servicing performance,
- Queue regulation (discipline) – defines the method of selecting the next request to be serviced, if there is a queue in the system.

The case when these variables are subject to exponential distribution is of great practical importance.

The stream of requests is a statistical description of the process of requests arrival at the servicing system. It is usually described using distribution functions for intervals between consecutive requests. If this stream does not show variability, these intervals are constant and the stream itself is of deterministic nature. But if requests are arriving at the system at random, then these intervals are a random variable and then the function of their distribution should be defined (Filipowicz 1997).

The following denotations are used:

$\overline{t_1}$ – average length of interval between two adjacent requests,

λ – average intensity of requests stream (requests intensity).

The relation between these values has the following form

$$\lambda = \frac{1}{\overline{t_1}}$$

Variable $\overline{t_1}$ stands for an important value in the reliability technique – it is so-called average time between failures. It is a measure of equipment reliability.

In practice it happens very often that the time of servicing is not constant and is subject to stochastic fluctuations. In such a case it must be described using appropriate distribution function. The time of servicing is an important value characterising the system of servicing. When considering the time of servicing as a random variable, its distribution function may be determined.

The following denotations are used:

$\overline{t_2}$ – average time of request servicing,

μ – average intensity of request servicing.

The relation between these values has the following form

$$\mu = \frac{1}{\overline{t_2}}$$

In the reliability technique the average time of request servicing may mean, apart from repairs, also servicing of inspections, and then inspections intensity is an inverse of average time between inspections.

A repair service of transport traffic control equipment (group of m service employees carrying out repairs of N pieces of equipment) is a typical queuing system. Each piece of equipment is a source of requests of intensity λ, while intensity of each employee servicing is equal μ. Overall requests intensity depends strictly on the number of damaged equipment, i.e. it is a function of system states. Such systems are named close queuing systems.

A visual diagram of traffic control equipment servicing, as a mass servicing position, is presented in Figure 5.

The simplest case of closed queuing system will be considered. A two-state configuration of element considered has been assumed:

– in state 0 the element fulfils its function (it is operational) – state of using,
– in state 1 the element is damaged (it is not operational) – state of servicing.

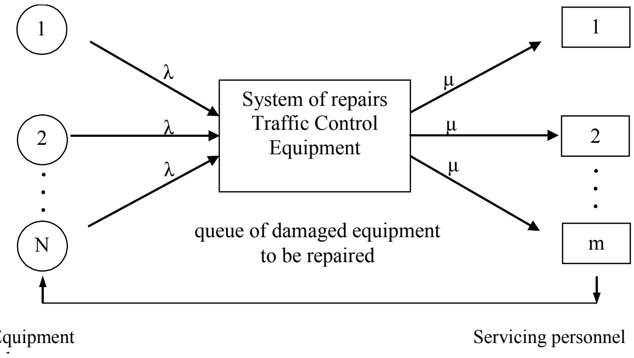

Figure 5. Visual diagram of transport traffic control equipment servicing

The system consists of one piece of equipment and one service employee. For the sake of considerations clarity, the denotations have been given once more:

– probability that the equipment is operational at a given moment amounts to p_0,
– probability that the equipment is damaged at a given moment amounts to p_1,
– intensity of equipment damage amounts to λ (the ratio of the number of defects in a given interval to the full time of equipment operation),
– intensity of servicing (performing repairs) amounts to μ (ratio of the repairs number to the full time of repairs duration).

The graph of states and transitions is presented in Figure 6.

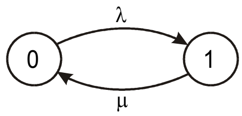

Fig. 6. Graph of transitions between the state of being operational and damage
1 piece of equipment,
1 service employee
where:
0 – means that the system is working properly, i.e. the equipment does not require a repair (it is operational),
1 – means that the equipment has been damaged and requires a repair.

When resolving this system we will obtain

$$p_1(t) = \frac{\lambda}{\lambda+\mu}\left[1 - e^{-(\lambda+\mu)t}\right] = \frac{\lambda}{\lambda+\mu} - \frac{\lambda}{\lambda+\mu} e^{-(\lambda+\mu)t}$$

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Probability $p_0(t)$ defines so-called availability of the system $A(t)$. Availability $A(t)$ is a probability that the system is operational (usable) in the future, assuming it is operational at the initial moment. For the exemplify system the course of availability is presented[3] in Figure 7.

The limit value of availability is interesting

$$\lim_{t\to\infty} A(t) = \frac{\mu}{\lambda + \mu}$$

which also defines system fitness in a steady state.

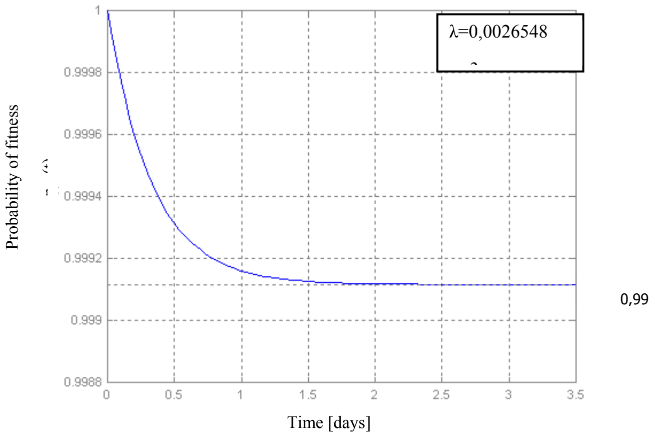

Fig. 7. Availability of the system

In example above only equipment repair was considered in servicing, while no preventive servicing (periodic inspections) was taken into account. Preventive servicing is a planned undertaking, carried out on an operational object to increase its reliability.

Inspection service allows earlier finding of defects (malfunctions), what enables preventing damages and increasing so-called availability of the system.

Introducing a possibility to carry out (from time to time) surveys and inspections of a given piece of equipment the graph of transitions (similar to the graph in Figure 6) will look as in Figure 8. State 00 is the state of using (availability), and the other two states – servicing: state 10 – a component is damaged and under repair, state 01 – the component is under inspection.

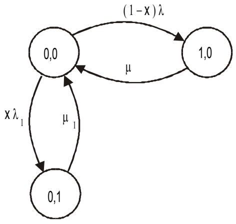

Figure 8. Graph of transitions between states of fitness (operational), damage and inspection
1 piece of equipment,
1 service employee

[3] for example for transport traffic control equipment, more specifically, for a signal

Intensity of equipment damage amounts to λ, while intensity of repair request servicing amounts to μ. Inspections intensity has been denoted by λ_1, while intensity of inspection request servicing, for each service employee, amounts to μ_1.

Resolving this system we obtain

$$p_{0,0}(t) = \frac{1}{\alpha - \beta}\left(\alpha e^{-\alpha t} - \beta e^{-\beta t}\right) - \frac{\gamma}{\alpha - \beta}\left(e^{-\alpha t} - e^{-\beta t}\right)$$
$$+ \frac{\delta}{\alpha\beta(\alpha - \beta)}\left[(\alpha - \beta) + \beta e^{-\alpha t} - \alpha e^{-\beta t}\right]$$

where:

$$A = \lambda + \lambda_1 + \mu + \mu_1$$
$$B = A^2 - 4C$$
$$C = \mu\mu_1 + \lambda\mu_1 + \lambda_1\mu$$
$$\alpha = \frac{A - \sqrt{B}}{2}$$
$$\beta = \frac{A + \sqrt{B}}{2}$$
$$\gamma = \mu + \mu_1$$
$$\delta = \mu\mu_1$$
$$\alpha + \beta = A$$
$$\alpha - \beta = -\sqrt{B}$$
$$\alpha\beta = C$$

Probability $p_{0,0}(t)$ defines also so-called availability of the system $A(t)$. For the exemplify system the course of availability is presented in Figure 9.

Boundary availability, specifying system availability in a steady state

$$\lim_{t\to\infty} A(t) = \frac{\delta}{\alpha\beta} = \frac{\mu\mu_1}{\mu\mu_1 + \lambda\mu_1 + \lambda_1\mu_1} = \frac{1}{1 + \rho + \rho_1}$$

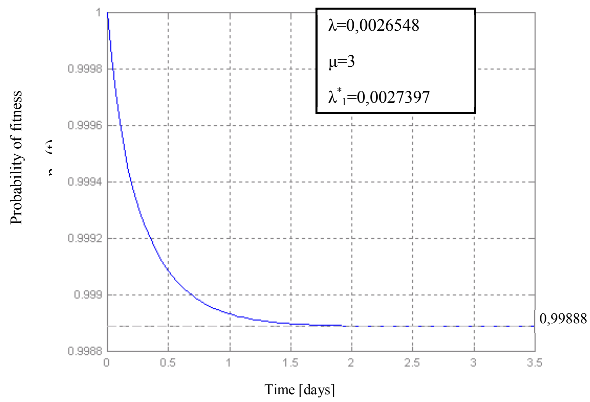

Figure 9. Availability of the system

The case of system discussed in example above, which task – apart from repair of damaged equipment – consisted also of carrying out periodic inspections of the equipment, did not take into account the fact that at the moment of switching the equipment off for inspection this equipment was not working. After all, an assumption is made (Zamojski 1980) that object's reliability characteristics are functions of working time that is the object may be damaged only during work. Hence the time of defect occurrence gets "elongated" and thereby in calculations the share of inspection in total intensity of defects and inspections shall be considered. Percentage of this share is determined by the selection coefficient

$$x = \frac{\lambda_1}{\lambda + \lambda_1}$$

In other words, in the case analysed, the selection coefficient specifies

$$\frac{\text{average inspections number}}{\text{(to) average number of repairs and inspections}}$$

This modification results in a change in the transitions graph model from Figure 8. Modified transitions graph is presented in Figure 10.

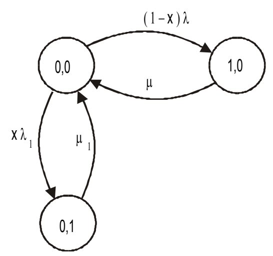

Fig. 10. Graph of transitions between states of fitness (operational), damage and inspection, taking into account the inspection time

1 piece of equipment,

1 service employee

Modified equation for probability that the system is operational in such a case is

$$p_{0,0} = \frac{1}{1 + (1-x)\rho + x\rho_1}$$

or after substitution of coefficient x

$$p_{0,0} = \frac{1}{1 + \left(\frac{\lambda}{\lambda + \lambda_1}\right)\rho + \left(\frac{\lambda_1}{\lambda + \lambda_1}\right)\rho_1}$$

and after expansion of relative intensities ρ and ρ_1

$$p_{0,0} = \frac{1}{1 + \left(\frac{\lambda}{\lambda + \lambda_1}\right)\left(\frac{\lambda}{\mu}\right) + \left(\frac{\lambda_1}{\lambda + \lambda_1}\right)\left(\frac{\lambda_1}{\mu_1}\right)} =$$

$$= \frac{(\lambda + \lambda_1)\mu\mu_1}{(\lambda + \lambda_1)\mu\mu_1 + \lambda^2\mu_1 + \lambda_1^2\mu}$$

The analysis of the course of probability function of system availability (fitness) is interesting. Was this function monotonously increasing, this would mean full advisability of preventive actions (by the way, for monotonously decreasing function preventive actions would turn out "harmful"). In the event that this function has an extremum, the introduction of preventive actions affects object's reliability in different ways, depending on preventive actions frequency (number) and on their duration.

Seeking for optimum inspections intensity that is such inspections frequency for which probability of correct operation would reach a maximum value, a derivative of this expression shall be determined, hence

$$\frac{dp_{0,0}}{d\lambda_1} = \frac{\mu\mu_1[(\lambda + \lambda_1)\mu\mu_1 + \lambda^2\mu_1 + \lambda_1^2\mu] - (\lambda + \lambda_1)\mu\mu_1(\mu\mu_1 + 2\lambda_1\mu)}{[(\lambda + \lambda_1)\mu\mu_1 + \lambda^2\mu_1 + \lambda_1^2\mu]^2}$$

This derivative is equal zero, if

$$\mu\lambda_1^2 + 2\lambda\mu\lambda_1 - \lambda^2\mu_1 = 0$$

$$\Delta = 4\lambda^2\mu^2 + 4\lambda^2\mu\mu_1; \qquad \Delta \rangle 0 \quad \text{always}$$

hence

$$\lambda_{1op} = \frac{-2\lambda\mu + 2\lambda\sqrt{\mu^2 + \mu\mu_1}}{2\mu} = \lambda\left(\sqrt{1 + \frac{\mu_1}{\mu}} - 1\right)$$

$$= \lambda\left(\sqrt{\frac{\mu + \mu_1}{\mu}} - 1\right)$$

Checking, what is the condition for probability of system availability (fitness) with inspections service to be higher than in the case of only repair service, consists in comparing appropriate expressions

$$\frac{1}{1 + \frac{\lambda^2}{(\lambda + \lambda_1)\mu} + \frac{\lambda_1^2}{(\lambda + \lambda_1)\mu_1}} \rangle \frac{1}{1 + \frac{\lambda}{\mu}}$$

The condition to satisfy this inequality is that

$$1 + \frac{\lambda}{\mu} \rangle 1 + \frac{\lambda^2}{(\lambda + \lambda_1)\mu} + \frac{\lambda_1^2}{(\lambda + \lambda_1)\mu_1}$$

$$\lambda(\lambda + \lambda_1)\mu_1 \rangle \lambda^2\mu_1 + \lambda_1^2\mu$$

$$\lambda\mu_1 \rangle \lambda_1\mu$$

that is that relative repairs intensity is higher than relative inspections intensity

$$\rho \rangle \rho_1$$

Calculations of system fitness in the event of inspections optimisation consist, having considered the selection coefficient in calculations with "new" defects intensity coefficients λ^* and inspections intensity λ^*_1

So the equipment defects intensity amounts then to

$$\lambda^* = \frac{\lambda^2}{\lambda + \lambda_1}$$

and the inspections intensity amounts then to

$$\lambda_1^* = \frac{\lambda_1^2}{\lambda + \lambda_1}$$

On the other hand, the intensity of repair request servicing amounts, as so far, to μ, and the intensity of inspection request servicing for each service employee amounts, as so far, to μ_1.

Resolving this system we obtain

$$p_{0,0}(t) = \frac{1}{\alpha - \beta}\left(\alpha e^{-\alpha t} - \beta e^{-\beta t}\right) - \frac{\gamma}{\alpha - \beta}\left(e^{-\alpha t} - e^{-\beta t}\right)$$
$$+ \frac{\delta}{\alpha\beta(\alpha - \beta)}\left[(\alpha - \beta) + \beta e^{-\alpha t} - \alpha e^{-\beta t}\right]$$

where

$$A = \lambda^* + \lambda_1^* + \mu + \mu_1$$
$$B = A^2 - 4C$$
$$C = \mu\mu_1 + \lambda^*\mu_1 + \lambda_1^*\mu$$
$$\alpha = \frac{A - \sqrt{B}}{2}$$
$$\beta = \frac{A + \sqrt{B}}{2}$$
$$\gamma = \mu + \mu_1$$
$$\delta = \mu\mu_1$$
$$\alpha + \beta = A$$
$$\alpha - \beta = -\sqrt{B}$$
$$\alpha\beta = C$$

Probability $p_{0,0}(t)$ defines also so-called availability of the system $A(t)$. The course of availability in the event of application of the principle of inspections optimisation is presented in Figure 11, while in the event of carrying out inspections with optimum intensity – Figure 12.

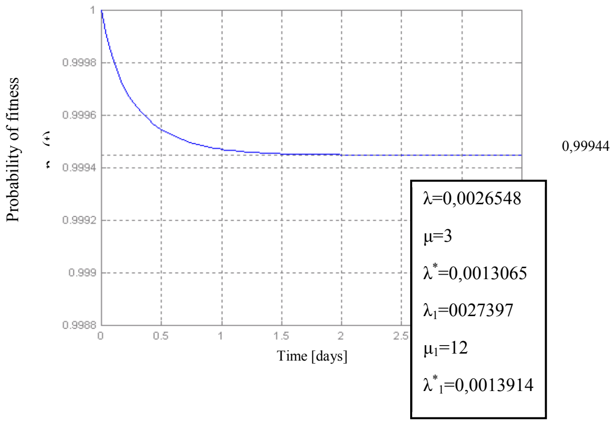

Figure 11. The course of availability in the event of application of the principle of inspections optimisation

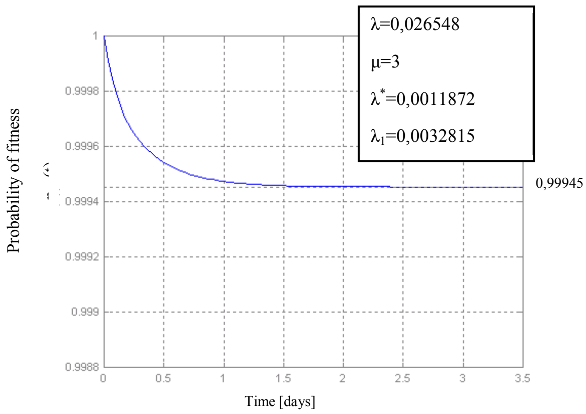

Figure 12. The course of availability in the event of carrying out inspections with optimum intensity

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