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Asymptotic Theory of Ship Motions in Regular Waves Under Shallow Water Conditions

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ABSTRACT: The hydrodynamic theory of ship motions in shallow water under the action of regular waves is discussed. The boundary value problem for velocity potential is solved using the matched asymptotic expansion method (MAEM). The solution is based on Fourier – Michell integral transformation technique and characteristics of Helmholtz and Klein – Gordon equations. Using the obtained results formulae for hydrodynamic characteristics are derived. The application of these formulae demonstrated good coincidence of the results of calculations and model experiments carried out in towing tank of Odessa National Maritime University.

1 INTRODUCTION

The intensification of shipping gave rise to complication of navigating conditions at sea roots and in recent years over 75% of navigation accidents occurred in restricted waters and bounded waterways. The growth of merchant ship dimensions during the last decades led to the situation in which vast regions of oceans and seas become comparatively shallow.

The wrecks of modern large vessels getting stranded or collided are accompanied by serious economical losses and negative ecological consequences.

The estimation of ship motion characteristics in restricted waterways is necessary not only for eliminating the possibilities or minimizing the number of accidents, but for substantiation of sea routes dimensions in the proximity of ports as well.

Modern theoretical and experimental hydrodynamics provides us with a great amount of information for predicting seakeeping qualities of ships in open deep sea. On the contrary such an information for a vessel sailing in shallow water is comparatively poor and the proper methods are not widely developed. Such a situation may be explained by virtue of the additional difficulties arising in the theoretical investigation of the potential boundary value problems for a ship propagating in shallow water conditions. First of all the complicacy of the singularities method is ten times higher for shallow water potential problems in comparison with the unbounded sea ones. Then the strip method widely used in practical calculations is inconsistent with the physical reality and often causes insoluble problems when using in shallow water cases with clearly expressed three dimensional water flow phenomena.

Thus a new approach for investigating ship hydrodynamic problems free from difficulties of classical method of singularities and shortcomings of strip method is vital. Such approach is demonstrated in this paper.

2 BOUNDARY VALUE PROBLEM FOR VELOCITY POTENTIAL

If the water around a ship is considered as inviscid incompressible fluid the important hydrodynamic information is derived from the solutions of boundary value problems for the velocity potential. Founded on basic physical principles the nonlinear problems with apriori unknown boundaries are simplified by linearization and the solutions of corresponding linear problems are practically used. Consider a vessel floating with a zero forward speed in shallow water with the depth H under the action of regular waves $\zeta_в = r_в e^{i\sigma t}$, $r_в$ and σ being wave amplitude and circular frequency accordingly.

The longitudinal x and transverse y axes of the Cartesian coordinate system are taken on the free surface of water and the vertical axe z is pointed downward.

The potential function $\Phi(x, y, z, t)$ can be divided into cosine $\Phi^{\tilde{n}}$ and sine Φ^s parts

$$\Phi(x, y, z, t) = \mathrm{Re}\left[\Phi^c(x, y, z) - i\Phi^s(x, y, z)\right]e^{i\sigma t} \qquad (1)$$

It is systematically demonstrated in investigations of Y.L. Vorobyov (Vorobyov, 2002), that estimation of added inertia, damping, coupling coefficients and exciting forces can be done using asymptotic values of radiation potential. So we can avoid the necessity of treating the wave scattering problem and difficulties of integration in the hull proximity.

Consider the ship performing longitudinal harmonic oscillations with circular frequency σ. The potential functions $\Phi_j^{\tilde{n},s}(x,y,z)$, $j=1,3,5$ must satisfy the following differential systems

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}\right)\Phi_j^{c,s}(x,y,z)=0,(x,y,z)\in E_0; \quad (2)$$

$$\left(\frac{\partial}{\partial z}+\frac{\sigma^2}{g}\right)\Phi_j^{c,s}(x,y,0)=0,(x,y)\in\Sigma_0; \quad (3)$$

$$\frac{\partial}{\partial n}\Phi_j^{c,s}(x,\pm y,z)=u_j(x,z),\ (x,y,z)\in S; \quad (4)$$

$$\frac{\partial}{\partial z}\Phi_j^{c,s}(x,y,H)=0, \quad (-\infty<x,y<\infty). \quad (5)$$

If the ship is performing transverse harmonic oscillation with circular frequency σ, the potential functions $\Phi_j^{c,s}(x,y,z)$, $j=2,6$ must satisfy the differential systems (2), (3), (5) and hull conditions

$$\frac{\partial}{\partial n}\Phi_j^{c}(x,\pm y,z)=\pm v_j(x,z), \frac{\partial}{\partial n}\Phi_j^{s}(x,\pm y,z)=0, \quad (x,y,z)\in S. \quad (6)$$

Both systems must satisfy radiation conditions in the infinity.

If the velocity of oscillations is taken to be unity, $u_j(x,z)$ and $v_j(x,z)$ are for longitudinal oscillations $j=1,3,5$

$$\begin{aligned}&u_1(x,z)=\cos(N,x), u_3(x,z)=\cos(N,z),\\&u_5(x,z)=z\cos(N,x)-x\cos(N,z),\end{aligned} \quad (7)$$

and for transverse oscillations $j=2,6$

$$\begin{aligned}&v_2(x,z)=\cos(N,y), v_6(x,z)=\\&=y\cos(N,x)-x\cos(N,y)\end{aligned} \quad (8)$$

Now let us consider ship as slender body, supposing that $B/L=O(\varepsilon)$, $T/L=O(\varepsilon)$, L,B,T being her length, beam and draft, $\varepsilon<<1$ and the hull varies slowly along the longitudinal axe. Under this assumption matched asymptotic expansion method (MAEM) is used for solving the potential problems.

According to MAEM the flow field is divided into two zones: far field zone where $y/L=O(1)$ and near field zone in which $y/L=O(\varepsilon)$. The condition along the boundary between the zones are not formulated and satisfied in the process of matching the solutions in far and near fields along the their boundary.

3 FAR FIELD SOLUTIONS

If the observation point is located in the far field as $\varepsilon\to 0$ the hull degenerates into a cut $\delta=-L/2\le x\le L/2$ of free surface plane $z=0$. The potential functions $\Phi_j^{c,s}$ are harmonic (2) in the layer $0\le z\le H$ with ship centerplane $y=0$ excluded, satisfy free surface (3) and radiation conditions. The boundary conditions on the centerplane $y=\pm 0$, that is on the inner boundary of the outer zone are not formulated as soon as the hull (with its centerplane) belongs to inner zone. The only identities come from the physical considerations

$$\begin{aligned}&\Phi_j^{c,s}(x,-y,z)=\Phi_j^{c,s}(x,+y,z); \frac{\partial}{\partial y}\Phi_j^{c,s}(x,-y,z)=\\&=-\frac{\partial}{\partial y}\Phi_j^{c,s}(x,+y,z),\ y>0, j=1,3,5;\end{aligned} \quad (9)$$

$$\begin{aligned}&\Phi_j^{c,s}(x,-y,z)=-\Phi_j^{c,s}(x,+y,z); \frac{\partial}{\partial y}\Phi_j^{c,s}(x,-y,z)=\\&=\frac{\partial}{\partial y}\Phi_j^{c,s}(x,+y,z),\ y>0, j=2,6.\end{aligned} \quad (10)$$

In accordance with (9), (10) the boundary conditions on the centerplane $y=\pm 0$ are taken in the form

$$\frac{\partial}{\partial y}\Phi_j^{c}(x,\pm 0,z)=\pm f_j(x,z); \frac{\partial}{\partial y}\Phi_j^{s}(x,\pm 0,z)=0, j=1,3,5; \quad (11)$$

$$\Phi_j^{c}(x,\pm 0,z)=\pm g_j(x,z); \Phi_j^{s}(x,\pm 0,z)=0, j=2,6, \quad (12)$$

where unknown functions $f_j(x,z)$ and $g_j(x,z)$ are taken as known ones for a moment.

Let us find the solution of the outer problem (2), (3), (11), (12), (5) for cosine amplitude $\Phi_j^c(x,y,z)$ of velocity potential $\Phi_j(x,y,z,t)$. Using the Fourier method for the outer differential problem we find the expansions for $\Phi_j^c(x,y,z)$

$$\Phi_j^c(x,y,z)=F_j^0(x,y)Z_0(z)+\sum_{m=1}^{\infty}F_j^m(x,y)Z_m(z); \quad (13)$$

$$F_j^0(x,y)=\frac{1}{H}\int_0^H \Phi_j^c(x,y,z)Z_0(z)\mathrm{d}z;$$
$$F_j^m(x,y)=\frac{1}{H}\int_0^H \Phi_j^c(x,y,z)Z_m(z)\mathrm{d}z. \tag{14}$$

The eigen functions $Z_0(z)$, $Z_m(z)$ form a complete orthogonal set in $[0,H]$ with mean square value of 1:

$$Z_0(z)=(\mathrm{N}_0)^{-\frac{1}{2}}ch\alpha_0(z-H), Z_m(z)=(\mathrm{N}_m)^{-\frac{1}{2}}\times$$
$$\times ch\alpha_m(z-H); \mathrm{N}_0(z)=\frac{1}{2}\left(1+\frac{sh2\alpha_0 H}{2\alpha_0 H}\right); \tag{15}$$
$$\mathrm{N}_m(z)=\frac{1}{2}\left(1+\frac{\sin 2\alpha_m H}{2\alpha_m H}\right),$$

where α_0 = real positive root of the equation

$$\frac{\sigma^2}{g}=\alpha_0 th\alpha_0 H \tag{16}$$

and $\alpha_1<\alpha_2<\alpha_3<$ = subsequence of real positive roots of the equation

$$\alpha_m tg\alpha_m H+\frac{\sigma^2}{g}=0. \tag{17}$$

As soon as $\Phi_j^c(x,y,z)$ is harmonic and the eign function system is orthogonal, $F_j^0(x,y)$ and $F_j^m(x,y)$ satisfy the Helmholtz and Klein-Gordon equations

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\alpha_0{}^2\right)F_j^0(x,y)=0,$$
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}-\alpha_m{}^2\right)F_j^m(x,y)=0. \tag{18}$$

Taking in mind, that for $j=1,3,5$ $\frac{\partial}{\partial y}\Phi_j^c(x,\pm 0,z)=\pm f_j(x,z)$, the boundary conditions for equations (18) according to (14) are to be taken in the form

$$\frac{\partial}{\partial y}F_j^0(x,\pm 0)=\pm\frac{1}{2}\gamma_j^0(x),$$
$$\frac{\partial}{\partial y}F_j^m(x,\pm 0)=\pm\frac{1}{2}\gamma_j^m(x),\ -L/2\le x\le L/2, \tag{19}$$

where

$$\gamma_j^0(x)=\frac{2}{H}\int_0^H f_j(x,z)Z_0(z)\mathrm{d}z;$$
$$\gamma_j^m(x)=\frac{2}{H}\int_0^H f_j(x,z)Z_m(z)\mathrm{d}z. \tag{20}$$

According to Green theorem and conditions (19), (20), after using radiation conditions we find for $j=1,3,5$

$$\Phi_j^c(x,y,z)=\frac{1}{2H}Z_0(z)\int_{-L/2}^{L/2}\mathrm{N}_0(\alpha_0 R)\int_0^H f_j(\xi,\zeta)Z_0(\zeta)d\xi d\zeta-$$
$$-\frac{1}{\pi H}\sum_{m=1}^{\infty}Z_m(z)\int_{-L/2}^{L/2}\mathrm{K}_0(\alpha_m R)\int_0^H f_j(\xi,\zeta)Z_m(\xi)d\xi\, d\zeta, \tag{21}$$

$$\Phi_j^s(x,y,z)=-\frac{1}{2H}Z_0(z)\int_{-L/2}^{L/2}J_0(\alpha_0 R)\int_0^H f_j(\xi,\zeta)Z_0(\zeta)d\xi\, d\zeta, \tag{22}$$

where $J_0(\alpha_0 R)$, $\mathrm{N}_0(\alpha_0 R)$, $\mathrm{K}_0(\alpha_m R)$=Bessel functions, $R=\sqrt{(x-\xi)^2+y^2}$.

The last formula is based on Green theorem, equations (18), boundary condition $\frac{\partial}{\partial y}\Phi_j^s(x,\pm 0,z)\equiv 0$ and radiation conditions.

Now returning to (18) we find that for $j=2,6$ $\frac{\partial}{\partial y}\Phi_j^c(x,\pm 0,z)=\pm g_j(x,z)$, the boundary condition for equation (18) must be taken in the form

$$F_j^0(x,\pm 0)=\pm\frac{1}{2}\gamma_j^0(x), F_j^m(x,\pm 0)=\pm\frac{1}{2}\gamma_j^m(x). \tag{23}$$

where

$$\gamma_j^0(x)=\frac{2}{H}\int_0^H g_j(x,z)Z_0(z)\mathrm{d}z;$$
$$\gamma_j^m(x)=\frac{2}{H}\int_0^H g_j(x,z)Z_m(z)\mathrm{d}z, j=2,6. \tag{24}$$

Now taking solutions of (18) that satisfy boundary conditions (23), (24) along the cut $|x|\le L/2$ after using radiation conditions we have

$$\Phi_j^c(x,y,z)=\frac{1}{2H}Z_0(z)\frac{\partial}{\partial y}\int_{-L/2}^{L/2}\mathrm{N}_0(\alpha_0 R)\int_0^H g_j(\xi,\zeta)Z_0(\zeta)d\xi\, d\zeta-$$
$$-\frac{1}{\pi H}\sum_{m=1}^{\infty}Z_m(z)\frac{\partial}{\partial y}\int_{-L/2}^{L/2}\mathrm{K}_0(\alpha_m R)\int_0^H g_j(\xi,\zeta)Z_m(\xi)d\xi\, d\zeta, \tag{25}$$

$$\Phi_j^s(x,y,z) = -\frac{1}{2H} Z_0(z) \times$$
$$\times \frac{\partial}{\partial y} \int_{-L/2}^{L/2} J_0(\alpha_0 R) \int_0^H g_j(\xi,\zeta) Z_0(\zeta) d\xi d\zeta, \; j = 2,6. \quad (26)$$

4 NEAR FIELD SOLUTIONS. MATCHING

To study flow phenomena in the near field the transverse coordinates are stretched $\eta = y/\varepsilon$, $\zeta = z/\varepsilon$ and as $\varepsilon \to 0$ omitting terms of $O(\varepsilon^2)$ we obtain the totality of two dimensional boundary value problems in $x = const$ planes for $\Phi_j^c(\eta,\zeta)$

$$\left(\frac{\partial^2}{\partial\eta^2} + \frac{\partial^2}{\partial\zeta^2}\right)\Phi_j^c(\eta,\zeta) = 0, \; (\eta,\zeta) \in e(x);$$
$$\left(\frac{\partial}{\partial\zeta} + \kappa_1\right)\Phi_j^c(\eta,0) = 0; \; \kappa_1 = \varepsilon\frac{\sigma^2}{g}, |\eta| > \frac{1}{2}b(x);$$
$$\frac{\partial}{\partial N}\Phi_2^c(\eta,\zeta) = \cos(N,\eta); (\eta,\zeta) \in L^+(x); \quad (27)$$
$$\frac{\partial}{\partial N}\Phi_3^c(\eta,\zeta) = \cos(N,\zeta); (\eta,\zeta) \in L^+(x);$$
$$\frac{\partial}{\partial\zeta}\Phi_j^c(\eta,h) = 0; |\eta| < \infty, \; j = 1,2,3,5,6.$$

Here $e(x)$ = a fluid domain in a form of strip $\{-\infty < \eta < \infty, 0 \le \zeta \le h\}$ with frame contour $L^+(x)$ excluded, $b(x)$ = the width of this contour.

The problem for $\Phi_j^s(\eta,\zeta)$ is uniform and has a trivial zero solution. The boundary value problem (27) has to be discussed keeping in mind that for matching procedure the asymptotics of solutions when η tends to infinity are needed. These asymptotics are found using specially worked out procedure. In addition to harmonic potential $\Phi_j^c(\eta,\zeta)$ let us introduce the conjugate harmonic stream function $\Psi_j^c(\eta,\zeta)$ and multi-valued analytical function $U_j(\chi = \eta + iy,) = \Phi_j^c(\eta,\zeta) + \; + i\Psi_j^c(\eta,\zeta)$ being determined outside the close contour $L^+(x) \mathrm{U} L^-(x)$. Various branches of $U_j(\chi)$ differ one from another by function

$$\Delta[U_j] = \frac{ch\lambda_0 h ch\lambda_0(\zeta - h)}{2(\lambda_0 h + sh\lambda_0 h ch\lambda_0)} \begin{cases} A, & j = 2,6; \\ B, & j = 1,3,5, \end{cases} \quad (28)$$

where $A = P_j(x,\lambda_0)\cos\lambda_0\eta$, $B = -Q_j(x,\lambda_0)\sin\lambda_0\eta$, λ_0 = real positive root of the equation

$$\varepsilon\frac{\sigma^2}{g} = \lambda_0 th\lambda_0 h. \quad (29)$$

We notice that outer boundaries of inner zone $\eta \to \pm\infty$ are at the same time the inner boundaries of the outer zone $y = \pm 0$. We have from (28) and (29)

$$f_j(x,\pm 0,z) = \pm\frac{1}{2}Q_j(x,\alpha_0)\frac{\alpha_0 ch\alpha_0 H ch\alpha_0(z - H)}{\alpha_0 H + sh\alpha_0 H ch\alpha_0 H}, j = 1,3,5, \quad (30)$$

$$g_j(x,\pm 0,z) = \mp\frac{1}{2}P_j(x,\alpha_0)\frac{ch\alpha_0 H ch\alpha_0(z - H)}{\alpha_0 H + sh\alpha_0 H ch\alpha_0 H}, j = 2,6. \quad (31)$$

Functional coefficients $Q_j(x,\alpha_0)$ and $P_j(x,\alpha_0)$ are determined in the form

$$Q_3(x,\alpha_0) = 4\int_0^{\frac{1}{2}b(x)} \exp\left[-\frac{\sigma^2}{g}Z_0(t)\right] \times$$
$$\left\{[\alpha_0 A_+(t) + B_+(t)s(t)]\cos\alpha_0 t - \alpha_0 A_+(t)\frac{dZ_0(t)}{dt}\sin\alpha_0 t\right\}dt, \quad (32)$$

$$P_3(x,\alpha_0) = 4\int_0^{\frac{1}{2}b(x)} \exp\left[-\frac{\sigma^2}{g}Z_0(t)\right] \times$$
$$\left\{[\alpha_0 A_+(t) + B_+(t)s(t)]\sin\alpha_0 t - \alpha_0 A_+(t)\frac{dZ_0(t)}{dt}\cos\alpha_0 t\right\}dt, \quad (33)$$

where $A_+(t)$ = values of potential function under determination on the contour with the equation $z = Z_0(t)$ and $S(t) = \sqrt{1 + \left[\frac{dZ_0(t)}{dt}\right]^2}$.

As soon as $A_+(t)$ is unknown, it is proposed to take its approximate value when the frequency of oscillations tends to infinity. Values of $B_+(t)$ are values of normal derivative of potential function according to the hull boundary conditions. The value of $A_+(t)$ can be easily found using the standard integral equation procedure.

For $f_5(x,\alpha_0)$ and $f_6(x,\alpha_0)$ we find

$$f_5(x,\alpha_0) = -xf_3(x,\alpha_0); \; f_6(x,\alpha_0) = -xf_2(x,\alpha_0) \quad (34)$$

Inserting (30)-(34) into (21), (22) and (25), (26) we actually performed matching of solutions in far and near field zones upon their boundary and get an approximate solutions for five radiation potentials uniformally valid in the whole water domain.

5 HYDRODYNAMIC COEFFICIENTS OF SHIP MOTIONS

It is convenient to find damping and exciting forces according to Haskind-Newman approaches where asymptotic expansions of radiation potentials are used. Thus we avoid the necessity of solving the diffraction problem and simplify calculations because of the simplicity that asymptotics of potential functions have far from ship hull. According to (Haskind, 1973, Newman, 1961) wave exciting forces and moments acting on a vessel may be calculated using such expressions

$$X_j(\beta)=2\gamma r_0 e^{i\sigma t}\left[F_c(\beta)-iF_s(\beta)\right],\ j=1,3,5. \quad (35)$$

$$\left.\begin{matrix}F_c(\beta)\\F_s(\beta)\end{matrix}\right\}=\int_{-L/2}^{L/2}\int_0^H f_j(\xi,\zeta)(ch\alpha_0\zeta - \\ -th\alpha_0 H sh\alpha_0\zeta)\left\{\begin{matrix}\cos(\alpha_0\xi\cos\beta)\\ \sin(\alpha_0\xi\cos\beta)\end{matrix}\right\}d\zeta\, d\xi \quad (36)$$

$$X_j(\beta)=\frac{1}{2}\gamma r_0\sin\beta e^{i\sigma t}\left[F_s(\beta)+iF_c(\beta)\right],\ j=2,6. \quad (37)$$

Functions $F_{c,s}(\beta)$ are determined by (36), but for $j=2,6$ instead of $f_j(x,z)$ $g_j(x,z)$ is taken. Functions $f_j(x,z)$ $j=1,3,5$ and $g_j(x,z)$ $j=2,6$ are given by (30) - (34).

In expressions (35) - (37) r_0 - incoming wave amplitude, β - angle between longitudinal axe of ship hull and vector of wave crests propagation.

The real parts of (35) and (37) must to be taken into account.

Damping forces and moments are calculated analyzing the energy flow carried of to infinity from ship hull by outgoing waves. According to (Haskind, 1973, Newman, 1959) damping coefficients μ_{ij} are given by formulae

$$\mu_{ij}=q\frac{\rho\sigma\alpha_0\Psi(\alpha_0)}{16}\times \\ \times\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(-x)^{\frac{i-3}{2}}Q_i(x,\alpha_0)(-\xi)^{\frac{j-3}{2}}\times \\ \times Q_j(\xi,\alpha_0)J_0(\alpha_0|x-\xi|)d\xi dx,\ i=3,5,\ j=3,5. \quad (38)$$

$\left(\frac{i-3}{2}\right),\left(\frac{j-3}{2}\right)$ are taken equal to zero for $i=j=1$.

$$\mu_{ij}=-q\frac{\rho\sigma\alpha_0\Psi(\alpha_0)}{32}\times \\ \times\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(-x)^{\frac{i-2}{4}}P_i(x,\alpha_0)(-\xi)^{\frac{j-2}{4}}\times \\ \times P_j(\xi,\alpha_0)\left[J_0(\alpha_0|x-\xi|)+J_2(\alpha_0|x-\xi|)\right]d\xi dx, \\ i=2,6,\ j=2,6, \quad (39)$$

where $q=2$ if $i=j$, otherwise $q=1$, $\Psi(\alpha_0)=\dfrac{1}{\dfrac{\alpha_0 H}{ch^2\alpha_0 H}+th\alpha_0 H}$, functions $P_i(x,\alpha_0)$ and $Q_i(x,\alpha_0)$ are given by (32) – (34), J_0 and J_2 are Bessel functions.

For calculation of inertia forces acting on an oscillating vessel potential functions in the near field must be used. To avoid the difficulties of integration the source-like functions in the vicinity of ship hull an alternative method is used. The method is based on the fact proved in (Landau, Lifshits, 1964), discussed and used in (Kotic, Mangulis, 1962).

It was demonstrated that added masses and damping coefficients are proportional to integral sine and cosine transformations of identical functions. It is enough to find a couple of transformations

$$\lambda_{ij}(\sigma)=\lambda_{ij}(\infty)+\frac{2}{\pi}\int_0^\infty\frac{\mu_{ij}(x)-\mu_{ij}(\infty)}{x^2-\sigma^2}dx, \quad (40)$$

$$\mu_{ij}(\sigma)=\mu_{ij}(\infty)-\frac{2\sigma}{\pi}\int_0^\infty\frac{\left[\lambda_{ij}(x)-\lambda_{ij}(\infty)\right]}{x^2-\sigma^2}xdx, \quad (41)$$

where $\lambda_{ij}(\sigma)$, $\lambda_{ij}(\infty)$, $\mu_{ij}(\sigma)$, $\mu_{ij}(\infty)$ = added mass and damping coefficients for frequency σ and infinite frequency consequently.

Integrals in (40), (41) are introduced us principle value integrals. It is known that mostly $\mu_{ij}(\infty)\equiv 0$. The value of $\lambda_{ij}(\infty)$ for a ship can easily be calculated using strip method and solving standard integral equation in the layer $0\le z\le H$.

The hydrodynamic characteristics of 200000 DWT tanker (Oortmerssen, 1976) for motions in shallow water conditions $\frac{H}{T}=1.2$ are demonstrated in Fig. 1-5. The values calculated using the results of paper are given by solid lines, while the results of experiments conducted in towing tank Odessa National Maritime University are presented by dots. The coincidence of theoretical and experimental results is satisfactory for practical uses.

Coefficients of added mass, exciting forces and damping are plotted against undimensional frequency $\nu=\sigma\sqrt{\frac{L}{g}}$.

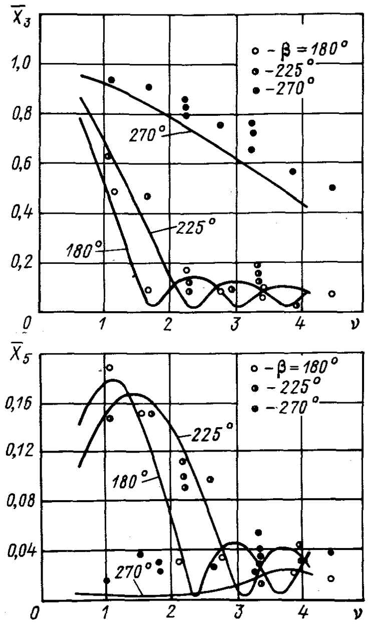

Figure 1. Longitudinal exciting forces

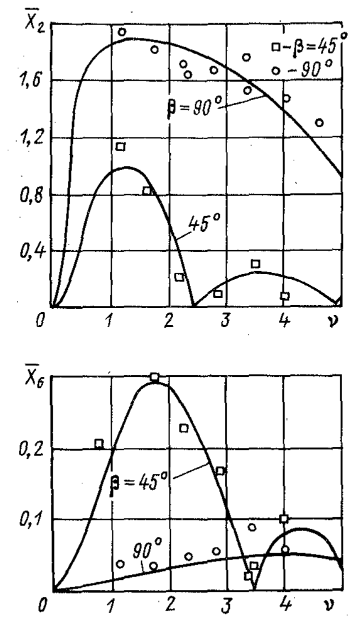

Figure 2. Transverse exciting forces

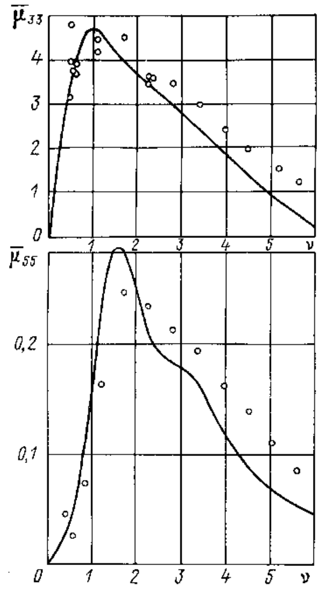

Figure 3. Longitudinal damping

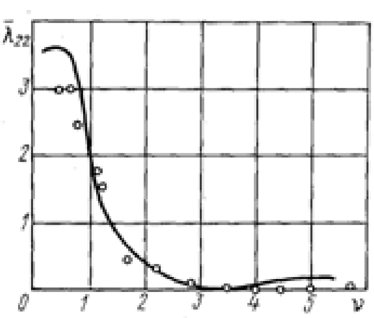

Figure 4. Sway added mass

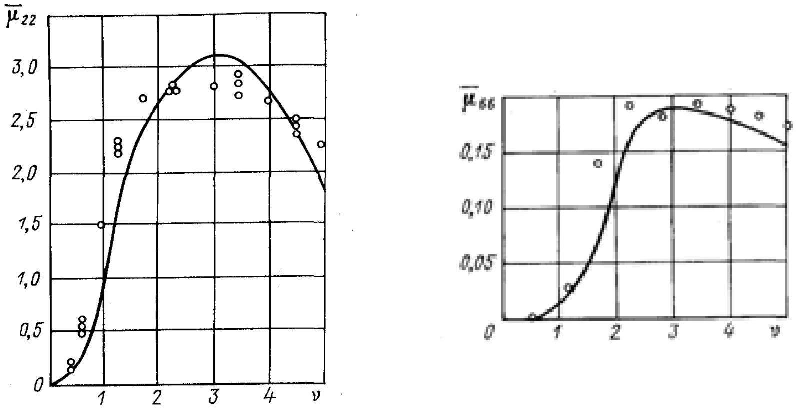

Figure 5. Transverse damping

6 CONCLUSION

The results derived on the base of MAEM were used for systematic calculations of hydrodynamic characteristics for a ship floating in regular waves under shallow water conditions.

The calculated values demonstrated good agreement with the results of model experiments conducted in towing tank of Odessa National Maritime University.

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