1 INTRODUCTION

The modern radars are high-powered technical means of navigation and take important part to ensure maritime safety. The distinctive peculiarity of marine radar functioning is the necessity of reflected ships signals separation with essentially different radar cross section (RCS) in condition of close situated targets. In its term, that is demanded low side-lobes level and high directivity property (narrow enough main beam) of antenna diagram. These properties may be realized on the base of simple enough technique using antenna arrays with limited number of tuneable weight coefficients of spatial filter (antenna array elements), for example, when there are only two of such tuneable weight coefficients [1].

In this case the entire antenna array (spatial filter) weights coefficients $W_i$ of the processing, except two (first and last: $W_1, W_N$), are fixed (selected under the condition of providing the required antenna pattern average side lobe's level) $(W_2, W_3, ..., W_{N-1})$. Value of the two tuneable weights coefficients are selected for carried out the condition of providing zero values in two points $(\theta_1, \theta_2)$ of the reception pattern [1],[2],[3],[4]. Different algorithms for calculation fixed weights coefficients for providing the required average antenna pattern side lobes level are presented: on the base of weighting functions $\sin x$ and $(\sin x)^2$, which give the possibility of changing deep of modulation of weighing function in wide interval (from equal values maximum and minimum weight coefficients till the biggest possible difference between them) and correspondingly different levels of the average side lobe suppression and losses in antenna’s directivity. Numerical examples were considered which demonstrated efficiency of weighing functions suggested for fix coefficients. Partial diagram were calculated for this purpose. Modified weighing functions are also considered which provided increasing level of suppression side-lobes nearby main lobe. As all numerical examples in this work were considered for 10th array and for 26th array, so partial diagram were calculated for $N-2=8$ and for $N-2=24$ element array with equal weight coefficients values and other weighing functions with unequal weights coefficients values. Because the algorithms of reception diagram side
lobes suppression in given points by means tuned weights and decreasing average level of side lobes in other points by means weighing correction of fix element leads to increasing the width of main lobe of antenna diagram, the possibilities of decreasing of main lobe width are investigated.. The algorithm was considered for providing decreasing main lobe width on the base of approach suggested in [13]. This algorithm includs the forming of two partial diagrams and using special interaction between them [5, 6]. Investigations of full diagrams for N-element antenna array including tuned elements with different situation suppresed points of diagram are also provided.

2 MATHEMETICAL DISCRIPTION

The expressions, which are describing the reception pattern of the antenna array \(G(\theta)\), may be written in the following form:

\[
G(\theta) = \sum_{i=1}^{N} W_i e^{-j2\pi(i-1)\frac{\phi}{2}\sin\theta} \\
(1)
\]

The expressions for tuneable weight coefficients may be presented in the form [1]:

\[
W_i = G_{n=2}(\theta_1)e^{-j2\pi(n-1)\frac{\phi}{2}\sin\theta_1} - G_{n=2}(\theta_2)e^{-j2\pi(n-1)\frac{\phi}{2}\sin\theta_2} \\
(2)
\]

\[
W_N = \frac{G_{n=2}(\theta_N) - G_{n=2}(\theta_2)}{e^{-j2\pi(n-1)\frac{\phi}{2}\sin\theta_N} - e^{-j2\pi(n-1)\frac{\phi}{2}\sin\theta_2}} \\
(3)
\]

where \(G_{n=2}(\theta)\) – partial diagram for untuneable (fixed) weight coefficients, which has the following form:

\[
G_{n=2}(\theta) = \sum_{i=1}^{N-1} W_i e^{-j2\pi(i-1)\frac{\phi}{2}\sin\theta} \\
(4)
\]

Function \(G(\theta)(1)\) (with consideration (2), (3)) may be represented in the next form:

\[
G(\theta) = G_{n=2}(\theta) - \gamma_1(\theta)G_{n=2}(\theta_1) - \gamma_2(\theta)G_{n=2}(\theta_2) \\
(5)
\]

were

\[
\gamma_1(\theta) = \frac{e^{-j2\pi(d/\lambda)(N-1)\sin\theta} - e^{-j2\pi(d/\lambda)(N-1)\sin\theta_1}}{e^{-j2\pi(d/\lambda)(N-1)\sin\theta} - e^{-j2\pi(d/\lambda)(N-1)\sin\theta_2}} \\
(6)
\]

\[
\gamma_2(\theta) = \frac{e^{-j2\pi(d/\lambda)(N-1)\sin\theta} - e^{-j2\pi(d/\lambda)(N-1)\sin\theta_1}}{e^{-j2\pi(d/\lambda)(N-1)\sin\theta} - e^{-j2\pi(d/\lambda)(N-1)\sin\theta_2}} \\
(7)
\]

\(\phi = 2\pi\sin\theta/\lambda\) — signal phase; \(\lambda\) — wave’s length; \(d\) — distance between antenna’s array elements; \(\theta\) — angle between the normal to the axis of the array antenna and direction of the coming signal.

In this paper we consider the different weight functions effect on reception diagram, which allow transforming the reception diagram properties.

The expressions for the untuneable weight coefficient are describing by the next:

\[
W^{(1)}_n = \sin \left[\pi \left(\frac{y}{N-1} + \frac{n-2}{N+z-2}\right)\right] \\
(8)
\]

\[
W^{(2)}_n = \left[\sin \left(\frac{y}{N-1} + \frac{n-2}{N+z-2}\right)\right]^2 \\
(9)
\]

where: \(n=2:N-1\);

\(y = 1: \frac{N-1}{2} ; \ z = \frac{2y(N-2)-(N-1)}{N-1-2y}\).

The main lobe widening may be regulated by parameter ‘y’. The bigger value of parameter ‘y’ corresponds to the less main lobe widening. For example, the reception diagrams calculated with (8) are shown in Figure 1.

![Figure 1. Partial reception diagram \(G^{(1)}_{N-2}(\theta)\) (N=10), y=1, y=2, y=3 (\(W^{(1)}_n\))](image)

The reception diagram \(G^{(2)}_{N-2}(\theta)\) with equal weight coefficients \(W_N\) (equable correction), which corresponds the condition with absence of main lobe widening is practically coincide with the case \(W^{(1)}_n\) shown in Figure 1.

Calculation reception diagram \(G^{(2)}_{N-2}(\theta)\) according to (9) are shown in Figure 2.
So, we can see that the weight function, described by (8) and (9), allows correcting the reception diagram properties widely.

Modified weighing functions may be suggested which provided the increasing level of side-lobes suppression nearby main lobe. Modified weighing functions are described by the next expressions:

\[
W_n^{(k)} = \begin{cases} 
W_n^{(k)} & n = 2 \\
W_{n-1}^{(k)} & n = 3 \\
W_n^{(k)} & n = 4 \div \left[ \frac{N+1}{2} \right] 
\end{cases}
\]  

(10)

where \([x]\) - is integer part of \(x\); \(k=1; 2\); \(W_n^{(k)}\) - is determined by (8) and (9), under \(k=1\) and \(k=2\) correspondingly,

\[
W_n^{(k)} = W_{N-(n-1)}^{(k)} \text{, if } n = \left[ \frac{N+1}{2} \right] + 1 + N - 1.
\]

The reception diagrams, calculated with weight coefficients (10) are shown in Figures 3-4.

It follows from figures 3 and 4 that modified weighing function decreasing diagram side-lobes level nearby main lobe area. The using weighing functions lead to the widening of the main lobe and decreases directional properties of antenna array. Decreasing the main lobe width and increasing antenna directivity may be gotten by means increasing the number of antenna array elements. For example in figures 5 – 8 represented the results of calculations for \(N=26\) antenna array diagram (partial diagram is formed by means \(N-2=24\) elements).
For comparison in figures 5 and 6 in addition are shown the reception diagrams for equal weighting \( W[1] \) (all weights coefficients are equal 1). The results of calculations reception diagrams represented in the figures shows the high efficiency of using weighing functions, especially modified weighing functions for the area nearby main lobe.

Losses in antenna’s directivity due to weighing functions described by the next expression [3]:

\[
\rho = \frac{|G_{N-2}(\theta)|^2}{N \sum_{n=2}^{N-1} |W_n|}
\]  

(11)

So, we can correct the regulated part of reception diagram by different weights functions. This approach does not require the implementation of numerical optimization procedures as were described in [2]. Choosing of such kind weighting functions we can get additional average side-lobe suppression, but with widening main lobe. This widening is decreased with increasing number of the array antenna elements. The method of increasing angle selectivity without losses in average side-lobe level [5], [6] may be considered, which based on approach suggested in [13].

In considered case at the output of N-2 antenna array elements (no tunable part of reception antenna (1) (see (4)) we have complex signals \( X_i \) [1]. By the first N-2 signals are created the sum:

\[
Z_i = \sum_{n=1}^{N-2} W_n X_i
\]  

(12)

Beside (12) the second sum is created:

\[
Z_2 = \sum_{n=1}^{N-1} W_n X_i
\]  

(13)

where:

\[X_i = S_i + \mathcal{N}_i, S_i = |S_i| e^{j(\phi - i\theta)} + \phi = 2\pi d \sin \theta, \]

\[\mathcal{N}_i - \text{thermal noise. If root-mean-square value of thermal noise is negligible small and } |S_i| = 1 \text{, (12) coincides with (4)} \text{ and } Z_2 = Z_2 e^{2j\phi}. \]

Then the sum is created [5]

\[
G_{R}^{N-2}(\phi) = \left[ 1 - e^{-j\phi} \right] \left[ \begin{array}{c} Z_1 \\ Z_2 \end{array} \right] G_R(\phi) = \left[ 1 - e^{-j\phi} \right] \left[ \begin{array}{c} Z_1 \\ Z_2 \end{array} \right]
\]  

(14)

\[e^{-j\phi} = \frac{1 - \mu e^{-j\phi}}{1 - \mu e^{j\phi}}, \]

(15)

where: \( \mu - \text{coefficient } (0 \leq \mu \leq 1) \),

\[\sin 2\phi = \text{Im} \frac{Z_1 Z_2^*}{|Z_1|^2}, \cos 2\phi = \text{Re} \frac{Z_1 Z_2^*}{|Z_1|^2}. \]

From (14), using (15), after some transformations we can get:

\[
G_{R}^{N-2}(\phi) = \frac{G_{N-2}(\phi) [\mu - \cos \phi]}{1 - 2\mu \cos \phi + \mu^2}
\]  

(16)
Considering (16) may be represented in the form:

$$G_{s}^{(N-2)}(\theta) = G_{s,-}(\theta) \cdot G_{s}(\theta), \quad G_{s}(\theta) = G_{s,-}(\theta) \cdot G_{s}(\theta),$$

(17)

$$G_{s}(\varphi) = \frac{2|\mu - \cos \varphi|}{1 - 2\mu \cos \varphi + \mu^2},$$

(18)

where: $G_{s,-}(\theta)$ is determined by (4),

$$\varphi = 2\pi \frac{d}{\lambda} \sin \theta.$$

As we can see from (17) angle selectivity of antenna may be essentially increased by means proper choose the value of $\mu$.

The width of the main beam of (18) on the level 0,5 $G_{s}(\theta)$ is:

$$\Delta \theta_{0.5} = \frac{\arccos \frac{\mu}{2} + \sqrt{\frac{2 \mu - \mu^2}{2}}}{\pi \frac{d}{\lambda}}$$

(19)

From (16) it follows that if $\mu \to 1$, so $\Delta \theta_{0.5} \to 0$.

So Supperdirectivity may be provided by means (18).

Consider some peculiarities of antenna array working with diagram (17). Due to functional transform (14) linearity of processing and Principe of superposition are breaking under affecting a few signals from different direction. If two interfering signals have close angels of arrival, may be provided good enough suppression of both signals. If the difference of arrival angels is big and intensity one is bigger enough than another, we get the suppression of the bigger signal. These considerations are stay in force for the case of more, then two signals. Thus for providing functionality of proposed principia of selection for multitarget situation special condition should be provided. Which suppose that signals with approximately equal intensity would have small difference of arrival angels, and for signals with essential different angels of arrival would be provided corresponding difference in their intensities.

It may be realise by means antenna with diagram $G_{s}^{(N-2)}(\theta)$ (17). So, approximately equal intensities will be took place only in narrow angels interval, determined by main lobe beam width of diagram $G_{s,-}(\theta)$. For the signals which have essential difference of arrival directions, weighting of their intensities would be provided by the same diagram ($G_{s,-}(\theta)$).

Thermal noise and errors of practical realization are limited the maximal value of $\mu$ and thus limited the minimal value of main lobe beam width for real antenna design. If $\eta = \text{noise/interference ratio}$ (supposed equivalent noise, which included thermal noise and technology errors), so $\mu \leq 1/1 + \eta$ [5]. Using this value in (16) we can get restriction on main lobe beam width.

Calculations $G_{s}^{(N-2)}(\theta)$ for different $\mu$ and different weighing functions for $G_{s,-}(\theta)$ (N=10) are presented at figures 9 – 11. For comparison are shown the reception diagrams for equal weighing W [1] (all weights coefficients are equal 1 for both sums (12), (13) and $\mu \to 0$)
The results of calculations partial diagrams are represented in Figures 9-11 show the possibility of narrowing the width of main lobe by means proper choosing of parameter $\mu$ for different kinds of weighing functions. Modified weighing functions in this case additionally decrease the level of side-lobes nearby the main lobe area. For $N=26$ the result of such calculations are represented in Figures 12-14.

Under increasing number of antenna elements for forming partial diagram, increasing the efficiency of using the different kinds of weighing functions together with approach for increasing the spatial directivity of antenna array (see Figures 12, 13). The special role under this plays using of modified weighing functions. That is demonstrated in Figure 14.

The reception diagrams with suppressions in given points, calculated with weights coefficients $W_1$ and $W_2$ according to (2) and (3) for $N=10$ are shown in Figures 15-17.
As we can see, the suppression in given points for these cases is high enough, and the side lobes level between the suppressed points is also small enough (about −80 dB). The possibilities of suppression Cross Ambiguity Function in discrete points were considered also in [8],[9],[11],[12],[14]. Estimates the level of side-lobes between suppression points had been gotten analytically in [14].

The results of calculations for N=26 are represented in Figures 18-20.
Figure 20. Reception diagram $G(\theta)$ (N=26) with suppression in points $\theta_1=1.2001$, $\theta_2=1.1718$; ($Y_2$).

So, we have not only suppression in given points of reception diagram, but and given value for main lobe width and high enough average side-lobes suppression.

The block diagram of the proposed N element radar antenna array algorithm is shown in Figure 21.

Figure 21. Block diagram of the proposed algorithm

3 CONCLUSIONS

In this paper antenna array design capable of obtain the given side-lobe suppression with controlled value of directivity coefficient is suggested. The antenna with super selectivity properties is also suggested. The approach is simple enough for calculations and does not require the implementation of numerical optimization procedures, such as [10]. It’s very useful for practical implementation, when it’s necessary to get the given side lobes suppression with given main lobe properties.

REFERENCES


