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A Note on Model-as-Usual for Leeway and Drift Track in Marine Navigation

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ABSTRACT: In this note, we briefly discuss and compare the simplified models of the water current and wind effects on ship's resultant track in marine navigation, which are represented by drift and leeway. Our focus is on a compensation of the lateral component of perturbation with respect to the ship heading line, which is included in the equations of resultant motion. The variable magnitude of the compensation affects ship navigation, in particular, total drift angle, speed and course over ground. The corresponding unit time fronts, speed changes and drift angle in relation to the various ranges of the cross compensation and own ship true course are also compared by some illustrative examples.

1 INTRODUCTION

Roughly speaking, there are in general distinguished two types of natural perturbations in marine navigation which influence a resultant ship track and are taken into account in chart work and navigational calculations, e.g. in the Electronic Chart Display and Information System (ECDIS). Namely, these are a leeway caused by wind and a drift due to a flow of water current or stream (For the sake of simplicity, the wave-induced motion is considered as included in the action of wind, and not distinguished as a separate type of perturbation). The basic data of the related motions, i.e. direction and speed (set and rate) are of navigational interest, however their actual effect on a specific ship track is even more essential. This is due to the fact that the same perturbation may have different impact on the vessels making way through the water and having different characteristics, for instance, draught, dimensions, displacement, shape, speed. As a consequence, the resultant velocity with respect to the ground and/or water may be subjected to substantial deviations. For more details and investigations concerning the leeway drift, see, for example, [2, 3, 5, 6, 8].

In this note, the main attention is paid to the way how the perturbation vector is included in the equations of motion of a navigating ship, taking into consideration the issue that the lateral effect of perturbation with respect to a true course line of own ship may differ measurably, depending on the type and force of natural phenomena represented by wind or water (sea, river) current as well as ship heading. In particular, the transverse impact can be taken in full, causing the maximum drift angle in given conditions. On the other edge, it can be compensated completely in some circumstances, e.g. by heeling a ship's hull, drag (fluid resistance). As a standard, the measurements of the real wind data are available on the vessel's bridge as well as the ship true velocity $(\vec{U})$ and the position over ground, making use of the global navigation satellite navigation sts (GNSS), gyrocompass, speed logs. Comparing the input data (wind, water stream/current) resulting in a perturbation $(\vec{W})$ and in consequence, the perturbation-induced force, as well as the output data referring to the resultant ship velocity $(\vec{V})$, one can observe that the common vector addition including the entire $\vec{W}$ -action, i.e. $\vec{V} = \vec{U} + \vec{W}$ may not hold explicitly, while deriving the total (angular) drift or the resultant (linear) deviation from the true course line in the presence of the perturbing natural forces which have a direct impact on ship (craft) navigation. In what follows, we aim at emphasizing that the entire perturbation-induced force (motion) that pushes a ship sideways is not necessarily distributed so that direction of its actual impact on the ship motion is collinear with $\vec{W}$. This implies that its components i.e. orthogonal and collinear with respect to the heading line can be subjected to reduction (scaling), which then affects the ship navigation.

2 DRIFT VERSUS LEEWAY

For simplicity, but without loss of generality, we apply the simplified model based on the Euclidean plane with the Cartesian coordinate system x0y and the kinematics of a material point, where axis y points downwards. Therefore, the unperturbed sailing in a calm sea reads $\dot{x}(t) = \cos\varphi$, $\dot{y}(t) = \sin\varphi$, where $\varphi \in [0, 2\pi)$ stands for a ship heading (true course) that is measured clockwise from the axis *x*, and the dot indicates derivative with respect to time *t*, i.e. $\dot{x} = \frac{dx}{dt}$ and $\dot{y} = \frac{dy}{dt}$. Moreover, the true speed is normalized, i.e. $|\vec{U}| = 1$ in what follows. In order to have a full control of navigation we assume that $|\vec{W}| < |\vec{U}|$, so the ship is able to reach any waypoint (point of destination) in the presence of perturbation. Furthermore, we consider a passive navigation, where the action of perturbation is not counteracted by a ship in order to keep the fixed (preplanned) track over ground. Namely, the heading is arbitral but fixed, and we let the ship to be drifted due to action of $\vec{W}$ in the exposition presented.

We begin with a simpler scenario, where the perturbation $\vec{W}$ is caused only by a horizontal motion of water, e.g. tidal stream, surface sea current, river (laminar) current. In this case, the set and rate are usually taken into a velocity triangle "as they are", clearly defining the drift track. Consequently, the resultant equation of motion reads

$$\vec{V} = \vec{U} + \vec{W} \tag{1}$$

Assuming that $\vec{W}(t,x,y) = [W_x(t,x,y), W_y(t,x,y)]$ points in the *x*-direction, the last relation in the model under consideration yields (Actually, W_y=0, since $\vec{W}$ is always pointing in the positive direction of axis x in the adopted model, and such set-up will be kept in the whole exposition

$$\dot{x}(t) = \cos\varphi + W_x, \quad \dot{y}(t) = \sin\varphi + W_y \tag{2}$$

The above means that the impact of the perturbation described by its velocity vector on the related drift effect of the ship is added entirely, i.e. with the maximum longitudinal and cross components of $\vec{V}$. The analogous situation is also described by a standard wind triangle in air navigation of the aircrafts, where wind plays the role of water current in marine navigation; see, for example, [7] in this regards. Obviously, the velocity vectors $\vec{U}$ and $\vec{V}$ are not collinear in general. For the sake of clarity, this is illustrated in Figure 1, where $\vec{V}_0$ determines the resultant destination, i.e. the estimated position *EP* (or the observed position Fix 1) as well as $\vec{U}$ indicates the dead reckoning position (*DR*).

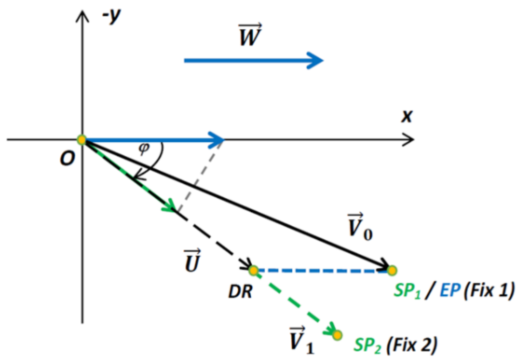

Figure 1. The effects of the standard drift-like perturbation $(\vec{V}_0)$ on the resultant ship's motion over the ground including the maximum cross component of $\vec{W}$ and the leeway-like perturbation $(\vec{V}_1)$ in the case, where the cross component is entirely compensated. The ship true velocity vector is denoted by $\vec{U}$ and the heading φ runs clockwise from the axis *x*.

In turn, a different situation may arise in the case of leeway being the effect of wind impact on an unperturbed ship's track along the heading line. This time the perturbation can in fact be compensated in practice, since a part of it can result in, for instance, heeling a ship (rolling), namely, it is not allocated entirely to altering the initial course, unlike the above-mentioned water drift. More precisely, the cross component with respect to the heading direction is subjected to compensation (reduction of magnitude) due to some physical phenomena, e.g. transverse drag, fluid resistance, while the longitudinal one is usually taken in full in the usual models. By analogy, the latter corresponds to a tailwind or a headwind in aviation, which increases or decreases the forward (a.k.a. effective) speed of a craft, respectively. Therefore, by contrast to the previous scenario with a water current, the resultant velocity in the case of the maximum compensation of the sideways movement (off the $\vec{U}$ -track deviation) leads to

$$\vec{V} = \vec{U} + proj_{\vec{U}}\vec{W} \tag{3}$$

where by $proj_{\vec{U}}\vec{W}$ we denote the perpendicular projection of $\vec{W}$ onto the true velocity $\vec{U}$. Recalling the local coordinate system applied, the last relation yields

$$\dot{x}(t) = \left(1 + \left|\vec{W}\right| \cos\varphi\right)\cos\varphi, \quad \dot{y}(t) = \left(1 + \left|\vec{W}\right| \cos\varphi\right)\sin\varphi. \tag{4}$$

In such edge case there is no lateral drift (or it is treated as negligible in practice), namely, the cross track distance (a.k.a crosswind leeway component) due to action of the perturbation is kept (almost) zeroed. For the sake of clarity, this is shown in Figure 1, where the velocity $\vec{V}_1$ determines the sea position *SP₂* (or the observed position *Fix 2*). In consequence, the resultant position varies in time of sailing along the true course line, which coincides with the water track in this case. In other words, the leeway angle is vanished and therefore $\vec{V}$ and $\vec{U}$ point in the same direction, where in general $\left|\vec{V}\right| \neq \left|\vec{U}\right|$. The effective ship speed is increased (as presented in Figure 1) or decreased, depending on direction of $\vec{W}$ with respect to the heading φ. For more details and advanced study on evaluation of resistance increase and speed loss of a vessel in wind and waves, see, e.g. [4].

There are some direct analogies to similar scenarios in other models, e.g. iceboat sailing under wind, driving a craft, skiing or running on a hill slope under gravity ([1]), where the requested direction of motion (over ground) is collinear with the control velocity (heading), although some natural forces (modelled as the perturbing vector fields) endeavour to push the moving craft or runner sideways off the initial track. However, due to the ability of applied wheel (tyre) or skid to hold the ground without sliding, i.e. traction, it is possible to resist the pushing side force and to continue the passage without any angular deviation.

Both above-mentioned settings yield different impacts on the resultant ship speed. The speed change for a given perturbation is expressed by the ratio $k(\varphi,t,x,y) = \dfrac{\left|\vec{V}_1\right|}{\left|\vec{V}_0\right|}$, which leads to

$$k(\varphi,t,x,y) = \frac{1 + \left|\vec{W}\right| \cos\varphi}{\sqrt{1 + \left|\vec{W}\right|^2 + 2\left|\vec{W}\right| \cos\varphi}}. \tag{5}$$

We mention that if $\vec{W}$ is of arbitrary (strong) force in (9), then the absolute value of the numerator should be applied instead. It is easily seen that the usual drift-like setting results in higher ground speed than the above leeway-like model, i.e. $\left|\vec{V}_0\right| \geq \left|\vec{V}_1\right|$ for all directions $\varphi \in [0, 2\pi)$; see also Figures 4 and 8 in this regards. Remark, however, that for any fixed (requested) course over ground, i.e. when $\vec{V}_0$ and $\vec{V}_1$ are collinear, the inequality holds in the opposite direction and both velocities correspond in general to different headings, i.e. $\varphi_0 \neq \varphi_1$.

With the above cases in mind we note that the perpendicular component of leeway track with respect to the velocity $\vec{U}$ can actually be compensated in arbitrary range (partially), since the effect is in particular subjected to speed and characteristics of ship, relative direction and speed of perturbation. Such approach represents more accurate model of a real perturbation effect, where the magnitude of compensation is defined more generally by a monotonic (increasing) function $f(a) \in [0,1]$ of a parameter $a \in [0,1]$ (say, a cross perturbation coefficient or leeway-like coefficient). In practice the formula for $f(a)$ (e.g. experimental, theoretical) depends on a specific ship characteristics and hydrometeorological conditions. We thus get $\vec{V}_a = \vec{U} + f(a)\, proj_{\vec{U}}\vec{W} + (1 - f(a))\vec{W}$. However, for our purposes and simplicity, this can be represented by an identity function, i.e. $f(a) = a$. Then we get the improved relation including the perturbation effect, namely

$$\vec{V}_a = \vec{U} + a \cdot proj_{\vec{U}}\vec{W} + (1-a)\vec{W} \tag{6}$$

which is also illustrated in Figure 2. Recalling the adopted coordinate system again, it follows that

$$\begin{aligned} \dot{x}(t) &= \left(1 + a\left|\vec{W}\right| \cos\varphi\right)\cos\varphi + (1-a)\left|\vec{W}\right| \\ \dot{y}(t) &= \left(1 + a\left|\vec{W}\right| \cos\varphi\right)\sin\varphi \end{aligned} \tag{7}$$

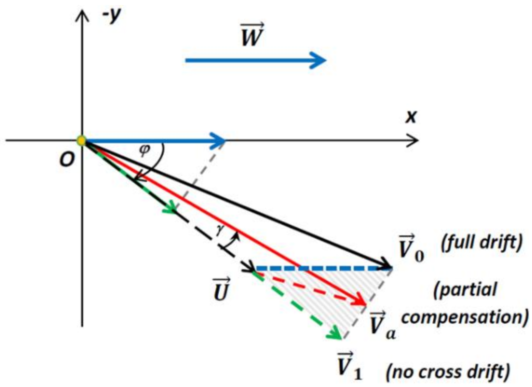

Figure 2. The unified model of the perturbation velocity vector ($\vec{V}_a$, red) including the arbitrary compensation of the lateral component which is determined by a parameter $a \in [0,1]$. In particular, if a=0, then this yields the standard full drift model $(\vec{V}_0)$ and if a=1, then this leads to the leeway-like model with the cross component vanished $(\vec{V}_1)$. The total drift angle is denoted by γ and measured from $\vec{U}$ to $\vec{V}_a$ (positive if clockwise and negative if counter clockwise)

The last outcome can be applied to modelling both drift-like and leeway-like effects with a given (partial) compensation of the sideways movement caused by a resultant perturbation (total drift), i.e. the cross component. In the extreme cases, if a=0, then (6) implies (2) and for a=1 we are led from (6) to (4). Scaling the component of $\vec{W}$ results in the changes of sea (estimated) position, which is then translated from *SP₁* (a=0) to *SP₂* (a=1) in the case of leeway as shown in Figure 1. The scaling factor $f(a)$ is specific for the given conditions and vessel, taking into account, in particular, the ship (hydro)dynamics in the advanced model.

The below formula shows the explicit dependence of speed over ground on the magnitude of compensation that is determined by the parameter $a \in [0,1]$. From $\left|\vec{V}_a\right| = \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}$ we obtain

$$\left|\vec{V}_a(\varphi, x(t), y(t))\right| = \sqrt{\left(1 + \left|\vec{W}\right| \cos\varphi\right)^2 + (1-a)^2 \left|\vec{W}\right|^2 \sin^2\varphi} \tag{8}$$

Furthermore, the angular deviation γ_a (a-drift angle) from the true course line due to the a-dependent effect of perturbation $\vec{W}$ is given as follows

$$
\begin{aligned}
\gamma_a\left(\varphi, x(t), y(t)\right) &= -\operatorname{sgn}(\sin\varphi)\cos^{-1}\left(\sqrt{\frac{\left|\vec{V}_1\right|}{\left|\vec{V}_a\right|}}\right) = \\
&= -\operatorname{sgn}(\sin\varphi)\cos^{-1}\left(\frac{1+\left|\vec{W}\right|\cos\varphi}{\sqrt{\left(1+\left|\vec{W}\right|\cos\varphi\right)^2+(1-a)^2\left|\vec{W}\right|^2\sin^2\varphi}}\right).
\end{aligned}
\tag{9}
$$

Remark that if one would like to consider arbitrary force of perturbation in (9), then the modulus of the numerator should be instead. Being in line with the standard definitions of leeway and drift angles in marine navigation, γ_a stands for the angular difference between a ship heading and resultant direction of motion, which is measured from $\vec{U}$ to $\vec{V}_a$ (positive if clockwise and negative if counter clockwise). This includes the notions of usual drift and leeway individually as well as it can be applied to express the total drift, referring to the resultant (combined) perturbation. If the perturbation is weak, i.e. $\left|\vec{W}\right| < \left|\vec{U}\right|$, then in general $\left|\gamma_a\right| < 90°$; see also Figure 5 in this regards.

3 UNIT TIME FRONTS BY EXAMPLES

In this section, we aim at presenting the influence of a-compensation on unit time front (UTF) that is a set of positions P at which a ship can arrive in equal (unit) time under the action of perturbation, commencing from a certain initial point O in all directions. More formally, we can write $TF = \left\{P \in \mathbb{R}^2 : (O,P) = 1\right\}$, where δ denotes a time distance. In the case of unit time, the front is generated by the rotating velocity vector $\vec{V}_a$ centred at O according to (6) – (8).

We first let the perturbation be constant, i.e. $\vec{W}_c = [c, 0]$, where $c \in \{0.25, 0.5, 0.75\}$, and the initial position is located at the origin. The outcome including a partial compensation, where a=0.5, together with the scenarios considering the extreme values of the parameter a is presented in Figure 3. Note that the time fronts create the Pascal's Snails (a>0) under the action of constant perturbation as well as the common Euclidean circle for a full drift case (a=0). Clearly, the stronger the wind force $\left|\vec{W}_c\right|$, the more the fronts differ from each other.

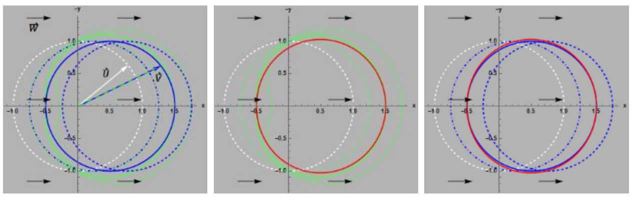

Figure 3. The unit time fronts created by the resultant velocities $\vec{V}_a$ for the full range of headings $\varphi \in [0, 360°)$ in the case of constant perturbation of force $\left|\vec{W}_c\right| \in \{0.25\,(\text{dashdotted}), 0.5\,(\text{solid}), 0.75\,(\text{dashed})\}$ for the standard drift-like perturbation (blue, a=0) and leeway-like perturbation with the cross component vanished (green, a=1). The unit time front generated by the true velocity of own ship is indicated in dashed white and the initial position is located at (0, 0). The effect of the partially compensated perturbation is presented in red, where a=0.5 and $\left|\vec{W}_c\right| = 0.5$

As mentioned in the preceding section, the a-compensation also affects the resultant speed changes. The related comparative example in the presence of $\vec{W}_c$ is analysed in Figure 4.

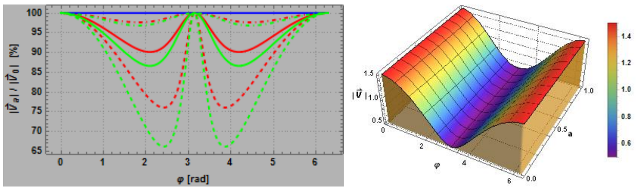

Figure 4. Left: The resultant speed decrease (percent) in relation to the ship's heading for the cases a=1 (green) and a=0.5 (red) in comparison to the standard full drift scenario (blue, 100%, a=0), i.e. $\left|\vec{V}_a\right| / \left|\vec{V}_0\right|$ in the presence of constant perturbation $\vec{W}_c = [c, 0]$, where $c \in \{0.25\,(\text{dashdotted}), 0.5\,(\text{solid}), 0.75\,(\text{dashed})\}$. Right: The resultant speed as the function of the ship's heading $\varphi \in [0, 2\pi)$ and the compensating parameter $a \in [0,1]$, where c=0.5. Remark that the obtained surface is not symmetric with respect to a, i.e. it is a-dependent along any fixed heading φ, although the graphical outcome looks very much like independent from a

Moreover, Figure 5 shows the impact of a-compensation on the drift angle γ. The maximum angular deviation of ca.14.5° (up to sign) from the true course track holds for the headings 104.5° (to port side) and 255.5° (to starboard side), while the perturbation acts along the axis x, i.e. 000.0°. On the other end, it is obvious that the a-drift is zero for any c if sailing exactly against or with the perturbation, i.e. $\gamma \in \{0, \pi\}$.

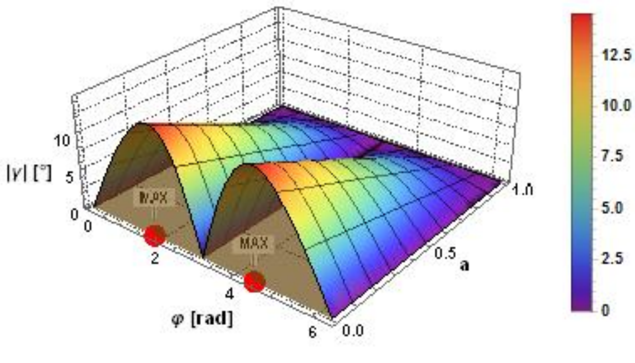

Figure 5. The modulus of the drift angle $|\gamma|$ as a function of φ and a in the case of constant perturbation, where $\left|\vec{W}_c\right| = 0.25$; $\left|\vec{U}\right| = 1$. The maximum value $|\gamma| \approx 14.5^o$ (red points) is reached for the standard drift-like perturbation (a=0), where the heading (is approximately equal to 104.5° (γ<0) or 255.5° (γ>0)

For the sake of completeness, we mention that the time front for a=0.5 remains convex if $\left|\vec{W}_c\right| < 1$. More generally, the fronts are convex for arbitrary $a \in [0,1]$ if $\left|\vec{W}_c\right| < 0.5$ [1].

Next, we take a look at the scenario with variable perturbation that depends on position. Namely, we have

$$\vec{W}_g = \left[b\exp\left(-0.9y^2\right), 0 \right], \text{ where } \ b \in (0,1) \qquad (10)$$

The corresponding time fronts are compared in Figures 6 and 7, however for different initial positions, i.e. (0, 0) and (0, 0.5), respectively. As above, the graphical outcomes are generated for various wind (current) forces $\left|\vec{W}_g\right|$, i.e. $b \in \{0.25, 0.5, 0.75\}$.

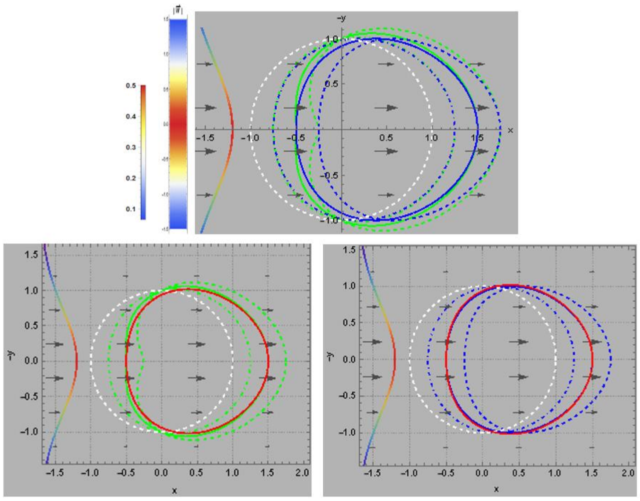

Figure 6. The unit time fronts of the ship's resultant motion commenced at (0, 0) for the edge cases of the cross compensation, i.e. a=0 (blue) and a=1 (green) in the presence of the position-dependent perturbation given by (10), where $b \in \{0.25\,(\text{dashdotted}), 0.5\,(\text{solid}), 0.75\,(\text{dashed})\}$. The outcome of the partially compensated lateral component of the perturbation (a=0.5, red) and the profile curve of $\vec{W}_g$ are shown only for b=0.5. The unit time front in the absence of perturbation is indicated in dashed white

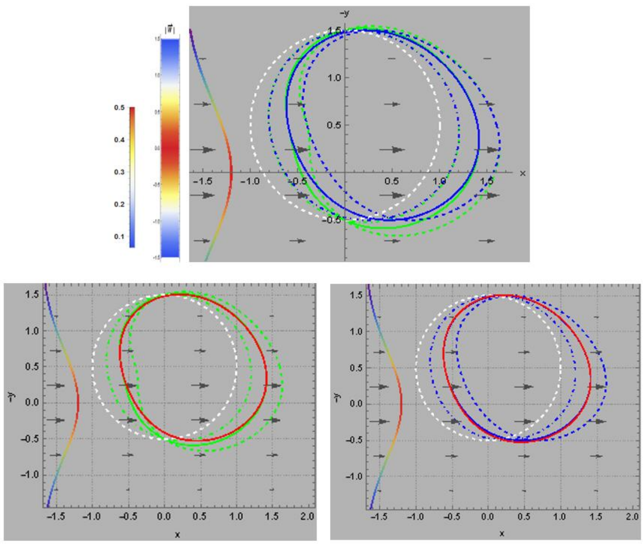

Figure 7. As in Figure 6, with the initial point at (0, 0.5) for all time fronts; t=1

Furthermore, the resultant speeds in relation to the ship heading for various compensations ($a \in \{0, 0.5, 1\}$) and $\vec{W}_g$-forces $(b \in \{0.25, 0.5, 0.75\})$ are juxtaposed in Figure 8.

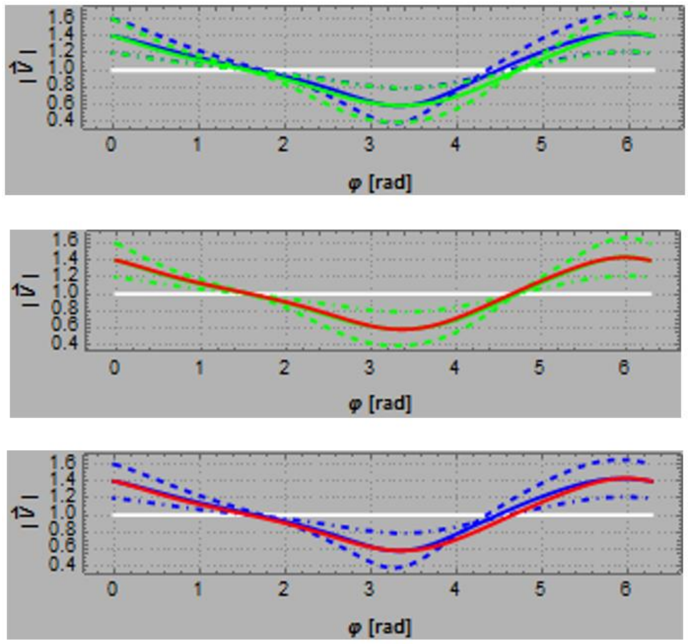

Figure 8. The resultant speeds with respect to the heading in the presence of the variable perturbation given by (10), corresponding to the scenario presented in Figure 7, where $b \in \{0.25\,(\text{dashdotted}), 0.5\,(\text{solid}), 0.75\,(\text{dashed})\}$, while a=0 (blue, full drift) and a=1 (green, drift with the entirely compensated cross component). The initial point (t=0) is located at (0, 0.5) in each case. The ship true speed $\left|\vec{U}\right|$ is indicated by a white horizontal line. The change of speed $\left|\vec{V}_a\right|$ in the case of the partial compensation is presented for a=b=0.5 (red)

4 CONCLUSIONS

The magnitude of sideways movement of a ship due to action of perturbation in the sense of wind and water current varies between specific conditions determined by the ship characteristics and hydrometeorological conditions, which include, among others, speed, draught, dimensions, displacement, stability and type of a ship, as well as the relative direction and speed of wind and/or water current. Due to interaction between the transverse water resistance (drag) and ship's hull in reality it is possible that the forward (backward) motion is influenced by the action of perturbation, i.e. increasing (decreasing) the effective ship speed through the water along the heading line, while the lateral drift track is in fact partially reduced and results instead in, for example, an angle of heel. In the brief analysis presented, the longitudinal component of perturbation with respect to true course line of a ship (craft) was included entirely in the equations of resultant motion, however it may also be subjected to compensation in the improved model by analogy to the cross component. Therefore, it is reasonable to take into consideration scaling both components of the perturbation at the same time and independently in the future investigation, since they have the substantial impact on ship track so that modelling of drift and leeway effects will involve a more accurate description of forward (backward) motion of a ship making way through the water as well.

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