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A Method and a Model for Risk Assessment of GNSS Utilisation with a Proof-of-Principle Demonstration for Polar GNSS Maritime Applications

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ABSTRACT: The GNSS positioning performance is commonly defined and described in terms unspecified to particular GNSS-based application. The approach causes difficulties to GNSS application developers, operators, and users, rendering the impact assessment of GNSS performance on the GNSS application Quality of Service (QoS) particularly difficult. Here the Probability of Occurrence (PoO) Model is introduced, which allows for a risk assessment of the probability for the GNSS positioning accuracy failure to meet the requirements of the particular GNSS-based application. The proposed PoO Model development procedure requires a large set of position estimation errors observations, which shall cover a range of classes of positioning environment (space weather, troposphere, multi-path etc.) disturbances affecting GNSS positioning accuracy. As result, the PoO Model becomes a tool that returns the probability of failure in meeting the positioning accuracy requirements of the GNSS applications considered, thus providing the input for a GNSS deployment risk assessment. The proposed PoO Model and its development procedure are demonstrated in the case of polar region positioning environment, with raw GNSS pseudorange observations taken at the International GNSS Service (IGS) Network reference station Iqualuit, Canada are used for the PoO Model development. The PoO Model proof-of-principle is then used to estimate the probability of the unmet required positioning accuracy for a number of polar maritime navigation applications. Manuscript concludes with a discussion of the PoO Model benefits and shortcomings, a summary of contribution, and intentions for the future research.

1 INTRODUCTION

Satellite navigation has mature to become a public goods, an essential component of the national infrastructure, and the enabling technology for growing number of technical and socio-economic systems [1, 2, 3]. Global Navigation Satellite System (GNSS) comprises several global satellite navigation systems, including Global Positioning System (GPS), operated by the US, which provide the Positioning, Navigation, and Timing (PNT) services in an interoperable and co-operative manner [4, 5, 6]. Matured enough to be one of the cornerstones of modern society, GNSS still keeps traditions that constraint developments and utilisations [6, 7]. Among them is the system-centric view of GNSS positioning performance assessment and declaration, which does not concern with the potential application and its requirements [2, 6]. Consequentially, GNSS application developers, operators, and users cannot either assess systematically the effects and risks the GNSS positioning performance degradation renders to GNSS application Quality of Service, or develop the GNSS application QoS resilient to potential short-term GNSS performance degradations or outages [2, 3, 6, 8, 9, 10]. The problem becomes more emphasised in the wide range of disciplines utilising GNSS, including the maritime sector [3, 6, 8, 9, 10].

Disciplines, such as air or maritime transport and traffic, attempt to assist GNSS applications with standards of required GNSS performance [11, 12, 13, 14]. An excellent review of current standards with direct amendments proposals from expert panels has been performed by European Agency for Space Programme (EUSPA), which conducts bi-annual re-assessment of GNSS user needs and requirements per disciplines, based on expert panel opinions [8]. Still, the current state-of-the-art does not overcome the gap between the GNSS operator's expression of GNSS performance, which naturally considers it without knowledge of the actual positioning conditions around GNSS users, and the GNSS application's needs to assess the risk of GNSS positioning failure in actual conditions of usage [6, 7, 8, 11, 13].

Here we propose the solution for the growing problem by introduction of the method and the model/index for the risk assessment of GNSS utilisation for the particular GNSS application with known requirement for GNSS position accuracy. Called the Probability of Occurrence (PoO), the index is capable of advising GNSS application developer, operator, and user on the risk of GNSS not meeting the required positioning accuracy level at the certain conditions of usage. Such a knowledge may assist GNSS application developers, operators, and users in making the objective inference on the need for suitable alternative when the GNSS PNT underperform in relation to GNSS application, thus rendering GNSS application resilient. The proposed method and model are demonstrated in this manuscript on the case of risk assessment of the GPS PNT ionospheric effects for single-frequency commercial-grade GPS positioning in the Arctic region.

This manuscript is structured, as follows. This chapter introduces the problem and outlines the research hypothesis. Chapter 2 describes (2.1) the theoretical foundations and the proposal for PoO model and the methodology for its development, (2.2) the experimental observations needed for the PoO model development, (2.3) a proof-of-principle case study the PoO model development, and (2.4) demonstrates and discusses PoO model utilisation for risk assessment for a particular maritime task/application. Chapter 3 concludes the manuscript with the outline of contributions and findings, and proposals for future research.

2 GPS UTILISATION RISK PROBABILITY-OF-OCCURANCE (POO) METHOD AND MODEL

The Probability of Occurrence (PoO) model is proposed in this research as a single GPS application-centric model for the GPS utilisation risk estimation in defined GPS positioning conditions and utilisation scenario. This Section outlines the method and describes the material required for the PoO model development and deployment for the GPS utilisation risk assessment.

2.1 *Method*

The proposed GPS/GNSS Probability of Occurrence (PoO) risk index is defined based on empirical identification of the GPS/GNSS positioning degradation [2, 3, 4, 6, 9, 10, 15] risk in characteristic positioning environments [3, 6, 9, 10, 16, 17, 18], and in accordance to a fundamental statistical principles [19, 20], as follows.

Let X be a statistical random variable, and x its value. Experimental observations of a variable may be considered values x of a statistical variable X. Statistical distribution of a variable X serve as its statistical model, characterised with two essential functions: the cumulative distribution function (CDF) and the probability density function (PDF) [19, 20].

The probability density function $f_X(x)$ of statistical variable X is defined as the function that returns the probability of X acquiring the value exactly equal to x [19, 20], as shown in (1).

$$F_X(x) = P(X = x), F : \mathbb{R} \to [0,1] \tag{1}$$

The cumulative distribution function (CDF) $F_X(x)$ is defined as the function that returns the probability that X will acquire the value less then, or equal to x [19, 20], as expressed in (2). Two essential functions of a statistical variable X are mutually related [19, 20].

$$F_X(x) = P(X \le x), F : \mathbb{R} \to [0,1] = \int_{-\infty}^{x} f_X(x)\,dx \tag{2}$$

The complementary cumulative distribution function (CCDF), or tail distribution, is derived from the cumulative distribution function, and defined as expressed in (3).

$$\bar{F}_X(x) = P(X > x) = 1 - F_X(x) \tag{3}$$

The CCDF outcome may be interpreted as the probability of X exceeding (being larger than) x. Introduction of CCDF may serve as a numerical indicator of probability of risk that an observed variable exceeds a critical value, established in the domain of interest.

Statistical distribution functions may be estimated analytically using various statistical methods [19, 20], implemented within either stand-alone software packages, such as *CumFreq* [21], or in programming environments, such as the open-source R environment for statistical computing [22]. Software libraries, such as the R-based *fitdistrplus* [23], allow for estimation of experimental PDF and CDF, fitting them to theoretical statistical distributions [19, 20]. Furthermore, statistical tests, such as Kolmogorov-Smirnov and Shappiro-Wilk, may be uitilised to confirm compliance of an experimental statistical distribution to the particular theoretical one, thus completing the identification of the experimental statistical distribution [19, 20]. Once estimated and identified, the experimental CDF may be used for derivation of the CCDF for the process/data pool in question.

The CCDF approach is applied in the research presented for definition of the Probability of

Occurence model development method, which utilises a massive data set of GPS positioning error observations, as a statistical variable. The PoO model emerges as the CCDF of the observed GPS positioning errors over a long time in the specified conditions of positioning environment and GPS application requirements for the particular scenario of utilisation.

The PoO model application for risk estimation requires understanding and specification of means of utilisation in an GPS application-oriented sense, as well as the targeted effect as the source of the risk. The PoO model is derived from a large data set of experimental GPS position observations in positioning conditions and the particular scenario of usage, of interest for a particular GPS-based application.

The GPS positioning environment effects that may cause GPS positioning degradation may include [3, 4, 5, 6, 24, 25]: (i) GPS ionospheric delay, (ii) GPS ionospheric scintillation, (iii) GPS multipath in a specified class of terrain (forest, mountain, urban, semi-urban, rural, ocean), (iv) GPS tropospheric delay etc.

The GPS utilisation scenario should be described by specification of user equipment (for example: commercial-grade single-frequency GPS receiver, as found in most smartphones still), utilisation of SBAS (for example: EGNOS) or other advanced positioning techniques (such as Real-Time Kinematics, RTK) instead of essential GPS, expected utilisation environment (indoors vs outdoors, urban, rural, mountains, forrest, maritime, air, vehicle etc.) [3, 4, 5, 6, 24, 25].

Once the targeted risk effects is defined, and GPS positioning environment and GPS application utilisation scenario specified, a massive set of GPS positioning error observations should be obtained, either by long-term data collection using individual equipment, or by utilisation of trusted data. Section 2.2 should be consulted for more details.

With the massive GPS positioning error data set is at hand, the CCDF should be estimated, using the experimental PDF and CDF estimation methods. The experimental CCDF then serves as the Probability of Occurrence (PoO) model. Te PoO model returns the probability of GPS positioning failure for a given GPS positioning error threshold. With the GPS positioning error threshold set as a maximum acceptable positioning error of the GPS application in consideration, the PoO will return the probability of occurrence the event in which the GPS positioning process will fail to meet the requested positioning accuracy threshold for particular GPS application.

The PoO model development method is developed in a formal manner to be implemented easily within a programming environment of a choice. The lack of 'ground truth' complicates the validation of the PoO model performance. An alternative cross-validation approach [20] is taken for this research, thus allowing for objective assessment of the PoO model correctness.

2.2 *Material*

Material required for the PoO model development comprises GPS positioning errors taken over a long time with the specified satellite navigation receiver/position estimation method in the geographical, terrain, and positioning environment (space weather, ionospheric, tropospheric, multipath, satellite visibility) conditions [6]. The concept of material collection aims at creation of a large GPS positioning error observations as a statistical sample that represent correctly the population of GPS positioning errors which can occur in the real-time GPS-based application usage. It is essential that the representation (sample) resembles frequency of GPS positioning degradation at different levels as best as possible [19, 20].

The user GPS equipment specifications, and GPS application requirements should be defined by a risk assessor in collaboration with interested parties (GPS application developers, operators, and users) [2, 6, 7]. Positioning environment conditions and GPS positioning error datasets may be acquired either through a tailored long-term field campaigns, or obtain from databases operated by trusted third parties.

As an example, in the case of GPS ionospheric effects risk assessment, the 24 hours-a-day GNSS raw pseudorange observations at reference stations across the world have been continuously collected, with exposed ionospheric effects and mitigated all the others, for the exact purpose of evaluation of the ionospheric effects on GNSS positioning performance. Available from the International GNSS Service (IGS) database [26], the GNSS raw pseudorange observation may be processed with a suitably configured GNSS Software-Defined Radio in the post-processing mode [2, 6] to allow for generation of GNSS positioning estimates and estimation of GNSS positioning error [4, 5, 6, 25, 27], as required for the PoO model development. Other sources of GNSS raw pseudorange observations include the internet-based open-access databases, such as Sonel [28], and the EUREF Permanent GNSS Network [29]. The GPS positioning environment is systematically described from the perspective of space weather, geomagnetic, and ionospheric conditions with a number of the internet-based open-access archives of global data and wide range of indices [3, 6, 9, 30], provided by different national and international organisations and agencies. The US NASA maintains a well-structured observation-rich OMNIWeb repository [31] with a Graphical User Interface (GUI) aimed at selection of data [32]. The International Service of Geomagnetic Indices [33] allows for a free access to archived derived values of geomagnetic descriptors. The Intermagnet [34] network offers a free access to structured global observations of geomagnetic conditions, as observed with a geographically spread network maintained to serve scientists, researchers, and engineers. Alternative third-party sources of observations are also available, with the APIs supporting the computer-based data access.

2.3 *A case study of practical PoO model development*

The research presented is established a method for PoO model development, and defined the required inputs. The case scenario of the PoO model development for the risk assessment of the single-frequency commercial-grade GPS utilisation in maritime sector in polar regions is developed, and presented as a proof-of-principle demonstration.

This research is driven by the rising interest in sailing in polar region, a geographical area with known exposure to dynamic space weather, geomagnetic, and ionospheric conditions known to produce considerable degradation of the GPS positioning performance [3, 4, 10, 11]. Consequently, a case scenario is established, resembling market conditions with prevailing share of the single-frequency commercial-grade unassisted GPS receivers. The common GPS receiver is assumed to be utilising just the standard correction models provided by GPS operator, thus correcting the ionospheric, tropospheric, and satellite clock effects in the standardised manner [6, 27]. A suitably configured Software-Defined Radio GNSS receiver is used to produce the GPS position, based on the raw GPS pseudorange observations taken at the reference station in the polar region. The position of the reference station was determined by precise geodetic methods. The GPS positioning residuals xresidual(t) are used as the GPS positioning error estimates xe(t) at the time instant of GPS positioning t, with xGPS(t) denoting the vector of GPS position components estimates, and xref(t) denoting the true position vector of GPS receiver, as determined by a precise geodetic method as given with (4).

$$x_{residual}\left(t\right)=x_{e}\left(t\right)=x_{GPS}\left(t\right)-x_{ref}\left(t\right) \tag{4}$$

The GPS observations used in the proof-of-principle PoO model development were taken at the IGS [26] reference station Iqualuit, Canada, using a stationary GNSS receiver collecting continuously the raw GNSS pseudoranges 24-hours-a day at 30 s sampling interval. Observations taken throughout 2014 are selected as a representative sample of the population. Total of 1 028 713 GPS position estimates ate derived from the massive data set of raw GPS pseudorange observation, after those were fed into RTKLIB [35], a, SDR GNSS receiver, to produce GPS positioning, and GPS positioning error estimates.

Space weather, geomagnetic, and ionospheric disturbances at various scales occurred in that year, with their frequency of occurrence resembling the long-standing pattern, as confirmed with the examination of the Dst index [3, 9, 36, 37] of geomagnetic storms/disturbances in 2014, as depicted in Figure 1.

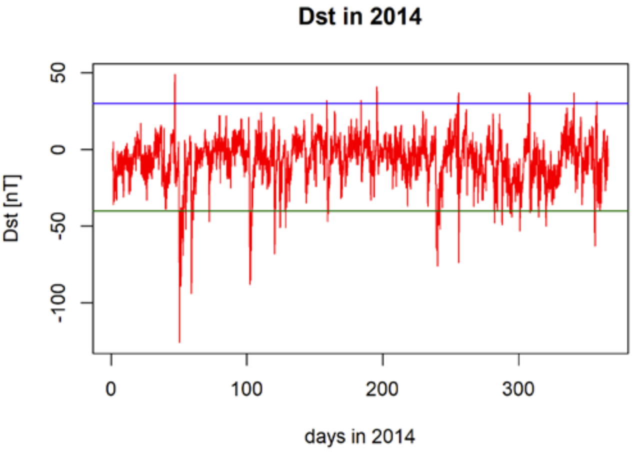
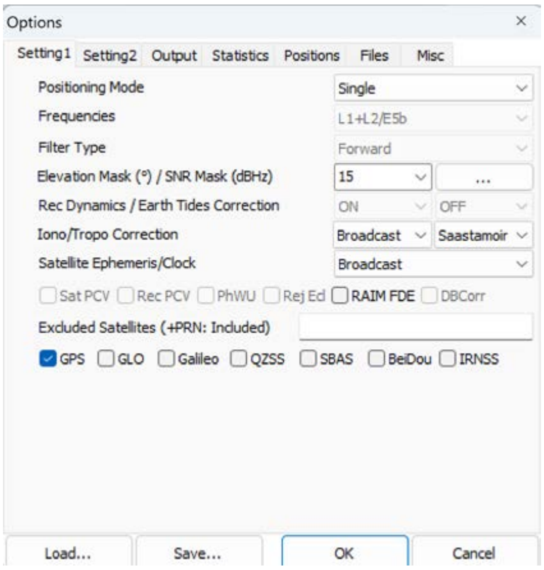


Figure 1. Time series of Dst, geomagnetic disturbance index, throughout 2014.

Dst data set is obtained from [32, 33], for the purpose of identification of particular classes of the GPS positioning environment: (i) (relatively) quiet geomagnetic condition, (ii) positive (first) phase of the geomagnetic storm, (iii) negative (deep through, and recovery) phase of the geomagnetic storm [36, 37]. Such classification allows for establishing three cross-validation [20] scenarios for the PoO model performance validation.

The raw GPS pseudorange observations are then fed into the RTKLIB GNSS SDR receiver, set in the post-processing mode and with configuration as a single-frequency commercial-grade sole-GPS receiver [27, 35], as outlined in Figure 2.



Positioning Mode: Single

Frequencies: L1+L2/E5b

Filter Type: Forward

Elevation Mask (°) / SNR Mask (dBHz): 15 | ...

Rec Dynamics / Earth Tides Correction: ON | OFF

Iono/Tropo Correction: Broadcast | Saastamoir

Satellite Ephemeris/Clock: Broadcast

☐ Sat PCV ☐ Rec PCV ☐ PhWU ☐ Rej Ed ☐ RAIM FDE ☐ DBCorr

Excluded Satellites (+PRN: Included)

☑ GPS ☐ GLO ☐ Galileo ☐ QZSS ☐ SBAS ☐ BeiDou ☐ IRNSS

Figure 2. Configuration of RTKLIB as a single-frequency sole-GPS satellite navigation receiver

The RTKLIB receiver returns the GPS positioning estimates, which are then transformed into GPS positioning errors using the method (4). The method is applied in the tailored software developed for the purpose of this research in the R environment for statistical computing. The GPS positioning error statistical analysis and the PoO model development are performed with the R-based software developed under this research.

The PoO model in the presented proof-of-principle is targeting the horizontal GPS positioning accuracy, derived from the positioning accuracy of horizontal components of position, as the index referred to in GPS-based application requirements for maritime [7]. Before the PoO model development takes place, the obtained components of GPS positioning error vector are examined for their statistical properties, as outlined with box-plot diagrams in Figure 3.

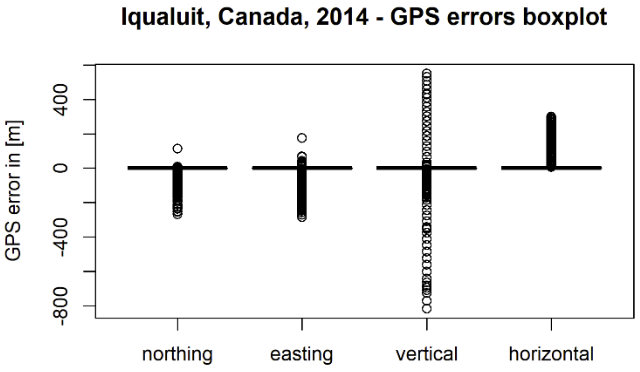


Figure 3. Box-plot of GPS positioning error vector components

Figure 3 reveals a number of outliers, caused by extensive ionospheric storms in the region, which were affecting the GPS positioning quality with the resulting risks for GPS-based applications.

A cross-validation method is developed for the PoO model validation, based on classification of GPS positioning error vector into one of the three classes of geomagnetic conditions [36, 37]: (i) quiet conditions, (ii) positive phase of geomagnetic storm, (iii) negative phase of geomagnetic storm. Essential statistical properties for classes (ii) and (iii), and the total sample are given in Table 1.

Table 1. Statistical properties of the total set of horizontal GPS positioning errors, and the two subsets related to geomagnetic storm development

GPS horizontal error [m]	No. of observations in 2014	Mean	Median	Variance
Total	1019769	1.9549	1.6300	4.89951
Dst > 30 nT	24	2.0077	1.5496	1.629124
Dst < -40 nT	222	1.8110	1.6709	1.328492

The box-plots of horizontal GPS positioning errors remain balanced in regard to the Dst value ranges during quiet geomagnetic conditions. However, it is evident from Figures 4, and 5, respectively, that the median of GPS positioning error rises in regard to the absolute value of Dst during disturbed geomagnetic conditions, thus justifying the classification approach.

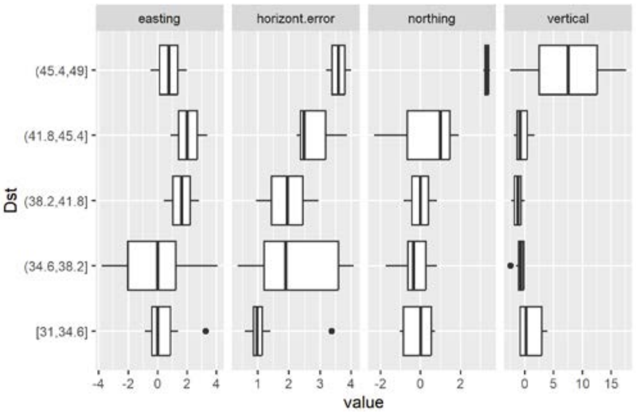


Figure 4. Box-plots of GPS positioning errors per ranges of Dst values, positive phase of geomagnetic storm

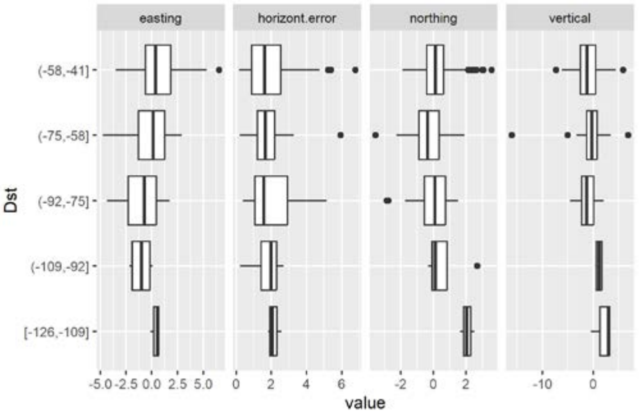


Figure 5. Box-plots of GPS positioning errors per ranges of Dst values, negative phase of geomagnetic storm

Cross-validation approach is justified further with the comparison of horizontal GPS positioning error data sets for geomagnetic disturbance classes (ii) and (iii). Related statistical tests [19, 20] reveal no similarities in two subsets of the original observations.

Table 2. Statistical tests results of comparison of GPS positioning error subsets during phases of geomagnetic storms

Compared with General	t-test, p-value, H0: means are equal	F-test, p-value, H0: variances are equal
Dst > 30 nT	0.8411	0.00227
Dst < -40 nT	0.06421	< 2.2e-16

Finally, the method outlined in Section 2 is applied to yield three PoO models, for the whole sample (total) of horizontal GPS positioning errors, and for the subsets relating to geomagnetic conditions classes (ii), and (iii). The method application results in three PoO models, respective to geomagnetic conditions, as depicted in Figure 6.

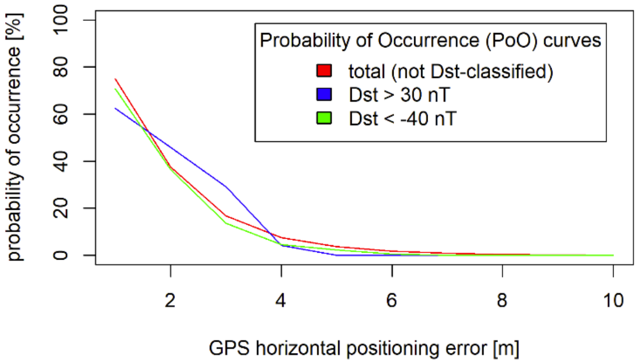


Figure 6. Three PoO models, for the whole set of horizontal GPS positioning errors (total, in red), for the positive phase of geomagnetic storm (Dst > 30 nT, in blue) subset, and for the negative phase of geomagnetic storm (Dst < -40 nT, in green) subset

In consideration of the PoO model performance, the negative phase PoO fits well the total PoO, thus confirming the total PoO model success. The reason may be found in the fact that statistical properties of the horizontal GPS positioning errors during the negative phase of the geomagnetic storms are clearly identified within the total pool of horizontal GPS positioning errors. A relatively small subset of horizontal GPS positioning errors observed during the positive phase of geomagnetic storm creates a unique statistical pattern, resulting in a slightly different PoO. Difference is particularly visible in the 1 m – 6 m range of horizontal GPS positioning errors. However, the size of related sample may lead to inference of neglecting the observed difference in PoOs.

2.4 *PoO model application on particular GPS application*

The total PoO model, expressed either in graphical or analytical form may be utilised to asses the risk of GPS utilisation, in the presented positioning environment conditions and the scenario of utilisation, in a specific GPS application defined by its requirements for horizontal GPS positioning accuracy [7, 38].

Let us assume the PoO is expressed in the analytical form, as given in (5).

$$P_{risk} = f\left(requested\ horizontal\ accuracy\right) \tag{5}$$

A specific GPS-based application should define its request for the highest acceptable horizontal GPS positioning error, and use it as the value for requested horizontal accuracy. Applied to the PoO model (5), the GPS-based application receives the probability of horizontal GPS positioning accuracy not meeting its request, and may consider potential alternatives for periods of degraded GPS positioning performance. Determination of the PoO/risk for a particular requested horizontal GPS accuracy may be performed using the analytical model, or graphically, as shown in Figure 7.

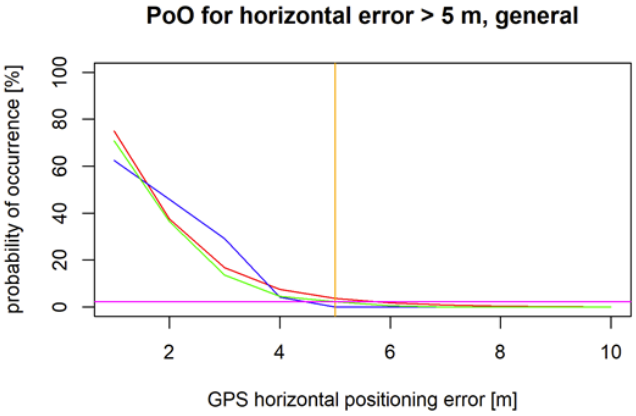


Figure 7. Graphical determination of PoO/risk of horizontal GPS positioning accuracy not meeting the requested level, based on the PoO curve

Considering the proof-of-principle demonstration scenario, a maritime GPS-based application requiring the positioning accuracy of 5 m may find the probability of approx 4% that the required accuracy level will not be met. Reference [7] states such a request may be set for the navigation in port operations. GPS application operator and user may consider implementation and operation of a redundant positioning system in a confined area (port) to overcome the risk, or the utilisation of integrated navigation (for example, GPS+INS), for the period of degraded GPS positioning performance.

The proposed method, and PoO model demonstrated in the proof-of-principle scenario, may be generalised towards any positioning indicator requested, as well as to utilisation multiple GNSS position estimation.

3 CONCLUSIONS

The lack of objective and systematic risk assessment of GPS/GNSS utilisation renders raising number of GPS/GNSS-based applications uncertain of potential derogation of their Quality of Service (QoS), or even failure to deliver, due to potentially unacceptable GPS/GNSS positioning performance degradation.

The research presented addresses the problem with the proposal for a method for development of Probability of Occurrence model, a statistical model, for risk assessment of GPS utilisation in particular positioning environment and for a specific GPS-based application that extends its requirements for GPS positioning accuracy.

Contributions of the presented research summarise, as follows:

1. A method of the Probability of Occurrence (PoO) model of GPS utilisation.
2. Assemblage of a year-long massive database of GPS positioning errors at Iqualuit, Canada in Arctic region, presented in the open-source access manner.
3. A cross-validation method for the PoO validation, based on geomagnetic conditions classification.
4. A method of the PoO utilisation for the GPS utilisation risk assessment for the specified GPS-based application.

5. An R-based software for PoO model development and validation.

The proposed method and the PoO model are demonstrated in the case scenario of a sole-GPS, single-frequency commercial-grade GPS positioning in the Arctic polar region. As the result, the PoO model is developed based on the experimental data taken in the real and statistically representative conditions. The PoO model utilisation for risk assessment is demonstrated in the case of a maritime GPS application. The research will continue with improvement and advancement of developed R-based software that will allow for specification of positioning environment and GPS-based application utilisation scenario, as well as with generalisation of the PoO method development.

REFERENCES

[1] HM Government Office for Science. (2018). Satellite-derived Time and Position: A Study of Critical Dependencies. HM Government. London, UK. Available at: https://bit.ly/2MEeBy6 (open access)

[2] Filjar, R, Damas, M C, Iliev, T B. (2020). Resilient Satellite Navigation Empowers Modern Science, Economy, and Society. CIEES 2020. IOP Conf. Ser: Mater Sci Eng 1032, 012001 (10 pages). Borovets, Bulgaria. doi:10.1088/1757-899X/1032/1/012001 (open access)

[3] Filić, M, Filjar, R. (2018). Modelling the Relation between GNSS Positioning Performance Degradation, and Space Weather and Ionospheric Conditions using RReliefF Features Selection. Proc of 31st International Technical Meeting ION GNSS+ 2018, 1999-2006. Miami, FL. doi: 10.33012/2018.16016

[4] Sanz Subirana, J. et al. (2013). GNSS Data Processing – Vol. I: Fundamentals and Algorithms. European Space Agency (ESA). Nordwijk, The Netherlands. ISBN978-92-9221-886-7. Available at: https://tinyurl.com/wbhu57us (open access)

[5] Teunissen, P J G, Montenbruck, O. (eds). (2017). Springer Handbook of Global Navigation Satellite Systems. Springer International Publishing AG. Cham, Switzerland. ISBN: 978-3-319-42928-1

[6] Filjar, R. (2022). An application-centred resilient GNSS position estimation algorithm based on positioning environment conditions awareness. Proc ION ITM 2022, 1123 - 1136. Long Beach, CA. doi: 10.33012/2022.18247

[7] Renfro, B A, Stein, M, Reed, E B, Villalba, E J. (2021). An Analysis of Global Positioning System Standard Positioning Service Performance for 2020. Space and Geophysics Laboratory, Applied Research Laboratories, The University of Texas at Austin. Austin, TX. Available at: https://www.gps.gov/systems/gps/performance/2020-GPS-SPS-performance-analysis.pdf

[8] EUSPA. (2021). Report on Maritime and Inland Waterways User Needs and Requirements: Outcome of the EUSPA Consultation Platform. European Agency for Space Programme (EUSPA). Prag, Czechia. Available at: https://www.gsc-europa.eu/sites/default/files/sites/all/files/Report_on_User_Needs_and_Requirements_Maritime.pdf

[9] Sikirica, N, Dimc, F, Jukić, O, Iliev, T B, Špoljar, D, Filjar, R. (2021). A Risk Assessment of Geomagnetic Conditions Impact on GPS Positioning Accuracy Degradation in Tropical Regions Using Dst Index. Proc ION ITM 2021, 606-615. San Diego, CA. doi: 10.33012/2021.17852

[10] Špoljar, D, Jukić, O, Sikirica, N, Lenac, K, Filjar, R. (2021). Modelling GPS Positioning Performance in Northwest Passage during Extreme Space Weather Conditions. TransNav, 15(1), 165-169. doi:10.12716/1001.15.01.16 (open access)

[11] Thomas, M et al. (2011). Global Navigation Space Systems: reliance and vulnerabilities. The Royal Academy of Engineering. London, UK. Available at: https://tinyurl.com/55vnk8tn

[12] Filjar, R, Sikirica, N, Iliev, T B, Jukić, O. (2022). A Risk Assessment of Space Weather-caused GPS Positioning Accuracy Degradation for GPS Applications in Polar Regions. Presentation at 21st International Beacon Satellite Symposium. Boston College, Chestnut Hill, MA.

[13] Jukić, O, Iliev, T B, Sikirica, N, Lenac, K, Špoljar, D, Filjar, R. (2020). A method for GNSS positioning performance assessment for location- based services. Proc of 28th Telecommunications Forum TELFOR 2020 (4 pages). Belgrade, Serbia. doi: 10.1109/TELFOR51502.2020.9306548

[14] Volpe. (2001). Vulnerability Assessment of the Transportation Infrastructure Relying on the Global Positioning System. John A. Volpe National Transportation Systems Center. Cambridge, MA. Avalaible at: https://rosap.ntl.bts.gov/view/dot/8435

[15] Filić, M, Filjar, R. (2018). A South Pacific Cyclone-Caused GPS Positioning Error and Its Effects on Remote Island Communities. TransNav, 12(4), 663-670. doi: 10.12716/1001.12.04.03 (open access)

[16] Heđi, I, Malić, E, Sikirica, N, Musulin, M, Šimag, D, Filjar, R. (2022). An analysis of GNSS TEC predictability during a rapidly developing short-term geomagnetic storm using Shannon entropy. Proc of 30th TELFOR, 31 - 35. Belgrade, Serbia. doi: 10.1109/TELFOR56187.2022.9983679

[17] Špoljar, D, Štajduhar, I, Lenac, K, Filjar, R. (2021). A Predictive Model of Multipath Effect Contribution to GNSS Positioning Error for GNSS-based Applications in Transport and Telecommunications. The Journal of CIEES, 1(2), 7–13. doi: 10.48149/jciees.2021.1.2.1 (open access)

[18] Rumora, I, Sikirica, N, Filjar, R. (2018). An Experimental Identification of Multipath Effect in GPS Positioning Error. TransNav, 12(1), 29-32. doi: 10.12716/1001.12.01.02 (open access)

[19] Maindonald, J., Brown, W. J. (2010). Data Analyisis and Graphics Using R: An Example-Based Approach (3rd ed). Cambridge University Press. Cambridge, UK. ISBN 978-0521762939

[20] Forsyth, D. (2018). Probability and Statistics for Computer Science. Springer International Publishing AG. Cham, Switzerland. ISBN 978-3-319-64409-7

[21] Institute for Land Reclamation and Improvement (ILRI). (2022). CumFreq, a tool for cumulative frequency analysis of a single variable and for probability distribution fitting (freeware). Executables and documentation avaiable at: https://www.waterlog.info/cumfreq.htm (open access)

[22] R-project. (2023). The R environment for statistical computing, ver. 4.2.2. R-project. Vienna. Austria. Available at: https://www.r-project.org

[23] Delignette-Muller, M L, & Dutang, C. (2015). fitdistrplus: An R Package for Fitting Distributions. Journal of Statistical Software, 64(4), 1–34. doi: 10.18637/jss.v064.i04 (open access)

[24] Parkinson, B W, Spilker Jr, J J. (1996). Global Positioning System: Theory and Applications (Vol. I.). AIAA, Washington, DC. ISBN 978-1-56347-106-3

[25] Filić, M, Grubišić, L, Filjar, R. (2018). Improvement of standard GPS position estimation algorithm through utilization of Weighted Least-Square approach. Proc of 11th Annual Baška GNSS Conference, 7-19. Baška, Krk Island, Croatia. Available: at: https://www.pfri.uniri.hr/web/hr/dokumenti/zbornici-gnss/2018-GNSS-11.pdf (open access)

[26] IGS. (2023). International GNSS Service database. Maintained by NASA. Available at (free registration

required): https://cddis.nasa.gov/archive/gnss/data/daily/

[27] Filic, M, Filjar, R, Ruotsalainen, L. (2016). An SDR-Based Study of Multi-GNSS Positioning Performance During Fast-Developing Space Weather Storm. TRANSNAV, 10(3), 395-400. doi: 10.12716/1001.10.03.03 (open access)

[28] Sonel. (2023). Sonel database of GNSS pseudorange observations. Available at: https://www.sonel.org/-GPS-.html (open access)

[29] EUREF. (2023). EUREF Permanent GNSS Network database. Available at: https://www.epncb.oma.be/_networkdata/datacalendar.php?station=BORR00ESP&year=2020&month=3&rv=2&c=epn (open access)

[30] Sikirica, N, Zhen, W, Filjar, R. (2022). Statistical properties of mid-latitude TEC time series observed during rapidly developing short-term geomagnetic storms: A contribution to GNSS-related TEC predictive model development. Proc 3rd URSI AT-AP-RASC. Gran Canaria, Spain. doi: 10.23919/AT-AP-RASC54737.2022.9814229

[31] NASA. (2023). OMNIWeb database of space weather observations. Space Physics Data Facility. NASA Goddard Space Center. Available at: https://omniweb.gsfc.nasa.gov/ow.html (open access)

[32] NASA. (2023). OMNIWeb Data Explorer. NASA Goddard Space Center. Available at: https://omniweb.gsfc.nasa.gov/form/dx1.html (open access)

[33] SIIG-ISGI. (2023). International Service of Geomagnetic Indices database. Available at: https://isgi.unistra.fr/data_download.php (open access)

[34] Intermagnet. (2023). Intermagnet network database of geomagnetic field observations. Available at: https://www.intermagnet.org/data-donnee/download-eng.php (open access)

[35] Takasu, T. (2023). RTKLIB: A Software-Defined GNSS Receiver. Available at: https://github.com/tomojitakasu/RTKLIB (open source)

[36] Filjar, R, Kos, S, Krajnovic, S. (2013). Dst index as a potential indicator of approaching GNSS performance deterioration. Journal of Navigation, 66(1), 149-160. doi:10.1017/S037346331200029X

[37] Filjar, R. (2022). A contribution to short-term rapidly developing geomagnetic storm classification for GNSS ionospheric effects mitigation model development. Proc ICEASE 2021 Conference. Islamabad, Pakistan. doi: 10.1109/ICASE54940.2021.9904168

[38] IMO. (2001). Resolution A 22/Res.915, adopted on 29 November 2001 (Agenda item 9). International Maritime Organisation (IMO). London, UK.