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A Comparison of the Least Squares with Kalman Filter Methods Used in Algorithms of Fusion with Dead Reckoning Navigation Data

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ABSTRACT: Different calculation methods and configurations of navigation systems can be used in algorithms of navigational parameter fusion and estimation. The article presents a comparison of two methods of fusion of dead reckoning position with that from a positioning system. These are the least squares method and the Kalman filter. In both methods the minimization of the sum of squared measurement deviations is the optimization criterion. Both methods of navigation position parameter measurements fusion are illustrated using the data recorded during actual sea trials. With the same probabilistic model of dead reckoning navigation, the fusion of DR results with positioning data gives similar outcome.

1 INTRODUCTION

The use of only one method and one navigation system typically results in lower accuracy, credibility and reliability of navigational information. This lowers the level of navigational safety and economic efficiency of a voyage. In order to overcome this, various independent methods and systems for determining navigational parameters are used. These primarily include, in accordance with the SOLAS convention, dead reckoning navigation systems (compasses, logs, accelerometers), positioning systems (GNSS or terrestrial systems) and electronic chart display and information systems (ECDIS) or paper charts. Various measurements and navigational parameter data are combined through joint algorithms and information-measurement systems. Uptodate examples of such systems are integrated navigation systems and integrated navigation bridge.

Figure 1 illustrates differences in ship's trajectory, obtained from dead reckoning (DR) navigation system and independently used positioning system (GPS). There are visible differences between the two trajectories obtained on the basis of measurements from these systems. Due to the influence of drift (systematic error) in dead reckoning navigation, that was not measured, the DR trajectory is moved to starboard in relation to GPS fixes.

The algorithms of integrated navigation systems carry out the fusion of different methods, in particular the parametric method is combined with the dead reckoning method.

This process requires the combined processing of measurement data, which allows us to optimize the use of navigational information. Multi-sensor fusion of navigational data is widely discussed in the literature, e.g. [10], while GPS data integration with other navigational measurements is described in [3], [5].

These authors present selected variants of the integration of navigational data obtained from different navigation systems. The method of least squares and the classic Kalman filter were used as the mathematical model of measurements integration. The analyzed methods are illustrated with an example of actual measurements recorded during sea trials. Both cases of measurement data integration were based on the same measurements and similar assumptions concerning their statistical distributions. This made their comparative analysis objective.

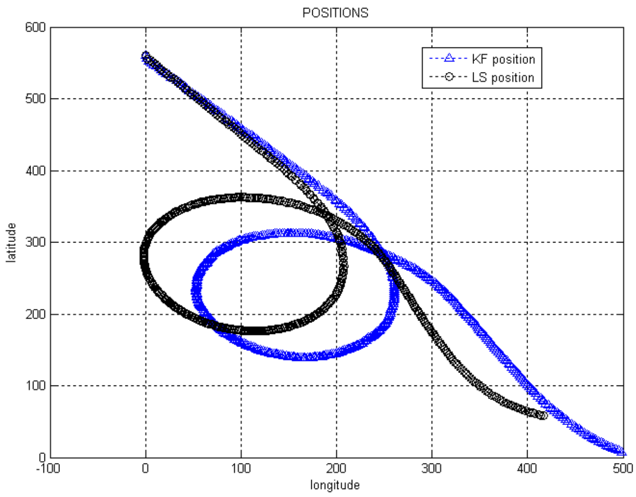

Figure 1. Comparison of ship's trajectory according to DR and GPS.

Briefly listed below are the employed methods of position coordinates estimation method, the method of least squares and the Kalman filter. Dead reckoning navigation, used in these algorithms, was described in [2], [3] and other works.

2 THE METHOD OF LEAST SQUARES

Let us assume that we have measurements of varying accuracy and we will use the method of least squares with weights for their fusion [11]. In this case, the vector of state (position coordinates) is described by this formula:

$$\mathbf{x} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{z}, \quad (1)$$

and its covariance matrix is written as this relation:

$$\mathbf{P} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{R}^{-1}\mathbf{G}\right)^{-1}, \quad (2)$$

$$\mathbf{x} = \left[\varphi, \lambda\right]^{\mathrm{T}}, \quad (3)$$

$$\mathbf{z} = \left[\varphi_{GPS}, \lambda_{GPS}, \varphi_{DR}, \lambda_{DR}\right]^{\mathrm{T}}, \quad (4)$$

where
$\mathbf{x}$ - vector of state,
$\mathbf{z}$ - vector of measurements,
$\mathbf{G}$ - a matrix binding the vector of state with the vector of measurements,
$\mathbf{P}$ - covariance matrix of the state vector,
$\mathbf{R}$ - covariance matrix of the measurements vector.

One of the simple measurement situations is a combination of GPS position coordinates with *dead reckoning* (*DR*) position. In this case $\mathbf{G}$, the matrix will be a block matrix in the following form:

$$\mathbf{G} = \left[\mathbf{I}_{2\times2} \vdots \mathbf{I}_{2\times2}\right]^{\mathrm{T}}. \quad (5)$$

The matrix of measurements covariance $\mathbf{R}$ will also be a block matrix

$$\mathbf{R} = \begin{bmatrix} \mathbf{P}_{GPS} & 0 \\ 0 & \mathbf{P}_{DR} \end{bmatrix}, \quad (6)$$

where $\mathbf{P}_{GPS}$ the matrices $\mathbf{P}_{DR}$ are, respectively, matrices of GPS and DR covariances. With these assumptions,, the matrix inverse to the covariance matrix of measurements will have this form

$$\mathbf{R}^{-1} = \begin{bmatrix} \mathbf{P}_{GPS}^{-1} & 0 \\ 0 & \mathbf{P}_{DR}^{-1} \end{bmatrix}. \quad (7)$$

Ultimately, with the above assumptions, we get the vector of state (position coordinates)

$$\mathbf{x} = \left(\mathbf{P}_{GPS}^{-1} + \mathbf{P}_{DR}^{-1}\right)^{-1}\left[\mathbf{P}_{GPS}^{-1} \vdots \mathbf{P}_{DR}^{-1}\right]\mathbf{z}, \quad (8)$$

and its covariance matrix

$$\mathbf{P} = \left(\mathbf{P}_{GPS}^{-1} + \mathbf{P}_{DR}^{-1}\right)^{-1}. \quad (9)$$

In this case, the DR position is treated as a separate measurement (along with its evaluation of accuracy). It should be noted that DR is performed at short, one second intervals, which makes the DR error comparable to the fix error.

3 KALMAN FILTER

Kalman filtering is commonly used today [5], [6], [7], [10], [12]. It is implemented at various levels of navigational information processing, from physical measurements by sensors (preliminary processing), through the combination of measurements from different sensors (intermediate processing), to the estimation of position coordinates and other navigational parameters (final processing). At each of these levels we use the same mathematical tools and the same computing algorithm.

The discrete Kalman filter, in a particular case, describes the system of two equations [1], [8], [9]:

– the state equation (structural model)

$$\mathbf{x}_{i+1} = \mathbf{A}_{i+1,i}\mathbf{x}_i + \mathbf{w}_i, \quad (10)$$

– measurement equation (measurement model)

$$\mathbf{z}_{i+1} = \mathbf{C}_{i+1}\mathbf{x}_i + \mathbf{v}_i \tag{11}$$

where
$\mathbf{x}$ - n-dimensional vector of state,
$\mathbf{w}$ - r-dimensional vector of state disturbances,
$\mathbf{z}$ - m-dimensional vector of measurements,
$\mathbf{v}$ - p-dimensional vector of measurement disturbances (identified with measurement noise),
$\mathbf{A}$ - n×n-dimensional transition matrix,
$\mathbf{C}$ - m×n-dimensional measurement matrix,
r ≤ n, p ≤ m.

We assume that the vectors of disturbances **w** and **v** are Gaussian noise with normal distribution, with zero mean vector and are mutually non-correlated. In the case of colour noise (with a trend) the extended Kalman filter is applied, where the disturbance trend is included as additional components of the state vector.

The equation of state describes the evolution of the dynamic system described in the state space, and the model of measurements functionally combines measurements with the system state. The solution to the equations (10), (11), taking into account the constraints imposed on the vectors of disturbances, is the Kalman filter. Calculation of the state vector in the Kalman filter is described by the following algorithm:

– projection of the state vector:

$$\tilde{\mathbf{x}}_{i+1,1} = \mathbf{A}_{i+1,i}\hat{\mathbf{x}}_i \, , \tag{12}$$

where $\tilde{\mathbf{x}}$ is the projected value of the state vector, $\hat{\mathbf{x}}$ is estimated value of the state vector,

– covariance matrix of the projected state vector

$$\mathbf{P}_{i+1,i} = \mathbf{A}_{i+1,i}\mathbf{P}_i\mathbf{A}^{\mathrm{T}}_{i+1,1} + \mathbf{Q}_i \, , \tag{13}$$

where **Q** is the covariance matrix of disturbances of the state (of vector w),

– innovation process

$$\boldsymbol{\varepsilon}_{i+1} = \mathbf{z}_{i+1} - \mathbf{C}_{i+1}\tilde{\mathbf{x}}_{i+1,i} \, , \tag{14}$$

– covariance matrix of the innovation process

$$\mathbf{S}_{i+1} = \mathbf{R}_{i+1} + \mathbf{C}_{i+1}\mathbf{P}_i\mathbf{C}^{\mathrm{T}}_{i+1} \, , \tag{15}$$

where **R** is the covariance matrix of measurement disturbances (of vector **v**),

– filter gain matrix (Kalman matrix)

$$\mathbf{K}_{i+1} = \mathbf{P}_{i+1,1}\mathbf{C}^{\mathrm{T}}_{i+1}\mathbf{S}^{-1}_{i+1} \, , \tag{16}$$

– estimated value of the state vector from filtering after measurement $\mathbf{z}_{i+1}$

$$\hat{\mathbf{x}}_{i+1} = \tilde{\mathbf{x}}_{i+1,i} + \mathbf{K}_{i+1}\boldsymbol{\varepsilon}_{i+1} \, , \tag{17}$$

– covariance matrix of the estimated state vector

$$\mathbf{P}_{i+1} = \left(\mathbf{I} - \mathbf{K}_{i+1}\mathbf{C}_{i+1}\right)\mathbf{P}_{i+1,i} \, . \tag{18}$$

4 THE STRUCTURE OF THE INTEGRATING FILTER

The adopted mathematical model of ship movement and the configuration of navigational devices affect the structure of the Kalman filter algorithm. Let us assume, as in the position estimation algorithm by the method of least squares, that position coordinates are determined using GPS (parametric navigation), and measurements in dead reckoning navigation are obtained from a gyroscopic compass and Doppler log.

Let us define the state vector as:

$$\mathbf{x} = \left[\varphi, \lambda, V_N, V_E, COG, SOG\right]^{\mathrm{T}} , \tag{19}$$

where
φ - latitude,
λ - longitude,
V_N, V_E - projections of the vector of speed over ground on the parallel and the meridian,
COG - course over ground,
SOG - speed over ground.

The quantities measured (measurement model) are the following parameters: position coordinates from a GPS system $\left(\varphi_{GPS}, \lambda_{GPS}\right)$, projections of the vector of speed over ground on the parallel and meridian (V_N, V_E), course over ground (*COG*) and speed over ground (*SOG*). Hence, the vector of measurements will take this form:

$$\mathbf{z} = \left[\varphi_{GPS}, \lambda_{GPS}, V_N, V_E, COG, SOG\right]^{\mathbf{T}} . \tag{20}$$

Each of the system matrices was appropriately selected for vectors (19) and (20) [4], [5], [6]. DR navigation in this algorithm is an element of a model (trend) of ship movement, but also - as is apparent from (19) - some of its parameters are estimated.

5 AN EXAMPLE

For a comparison of the two methods of measurements fusion, simulation tests have been carried out. The same measurement conditions were adopted for the two methods of data fusion with DR results: LS - DR and KF - DR. Therefore, the tests were based on the same series of measurements of position coordinates from a GPS, gyrocompass course and log speed. In this case, the following values of variance of specific measurements were taken [5], [6]:

– DGPS system: $\sigma_\varphi = 2,0$ m, $\sigma_\lambda = 1,5$ m, the coordinates are non-correlated;
– course over ground: $\sigma_{COG} = 1,5^0$;
– speed over ground: $\sigma_{SOG} = 0,5$ knot.

For these data the DR position errors for one second interval were equal to $\sigma_\varphi = \sigma_\lambda = 1,414$ m. These values have been adopted in both cases of the estimated position calculations. Fig. 2 shows that in the macroscopic sense both methods of coordinates estimation yield the same result, i.e. both trajectories coincide.

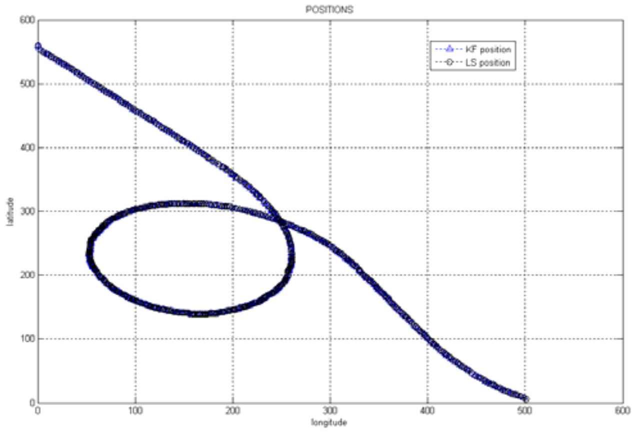

Figure 2. Macroscopic compatibility of the two trajectories.

The same initial position in both cases was adopted (Figure 3). Only in the next steps the particular positions, KF, LS start to differ significantly. These differences become more visible where larger changes of the velocity vector are made, in this case during the turning circle (Figure 4). This is because the KF algorithm identifies the velocity vector trend, while the LS algorithm does not.

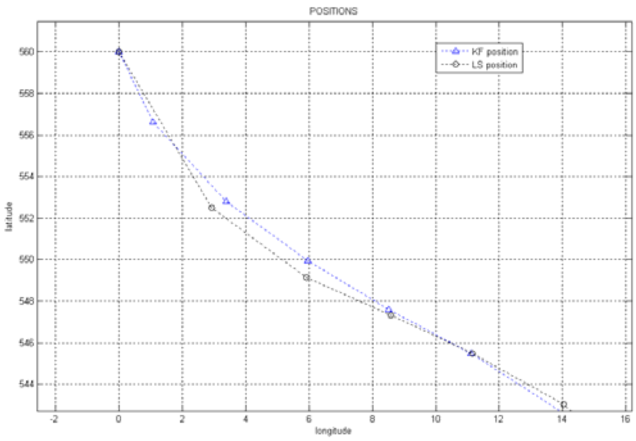

Figure 3. The initial section of the two trajectories.

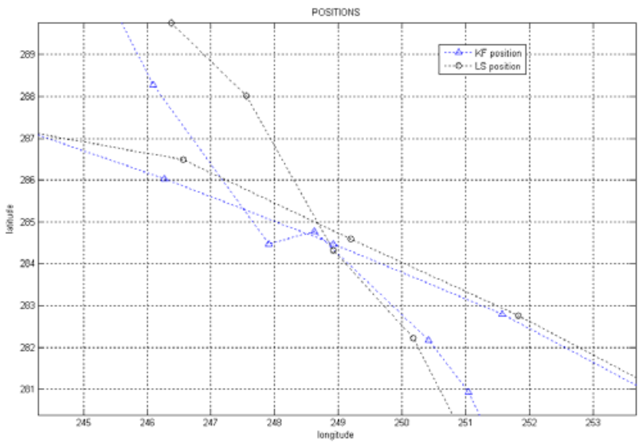

Figure 4. The two trajectories at a point of intersection with the turning circle.

Figure 5 depicts magnified differences between KF and LS trajectories on a steady course (uniform rectilinear movement).

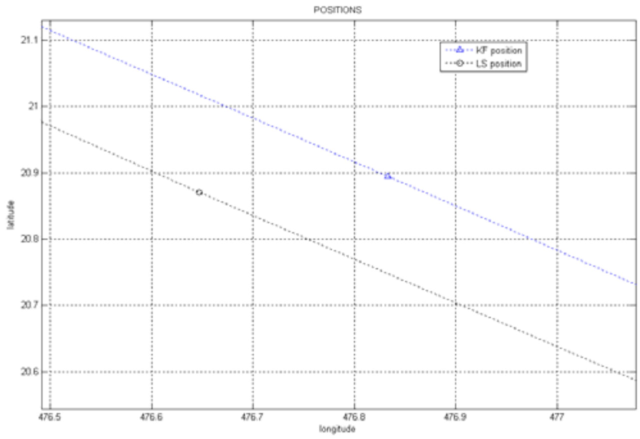

Figure 5. Example differences between the two trajectories of a ship on steady course (magnified).

Figure 6 presents distances between KF and LS positions along a trajectory (in subsequent iterations). The maximum deviation is 6 metres, the minimum 0.5 meter. The average distance between estimated positions (for the whole set) from KF and LS is 2.747 meters.

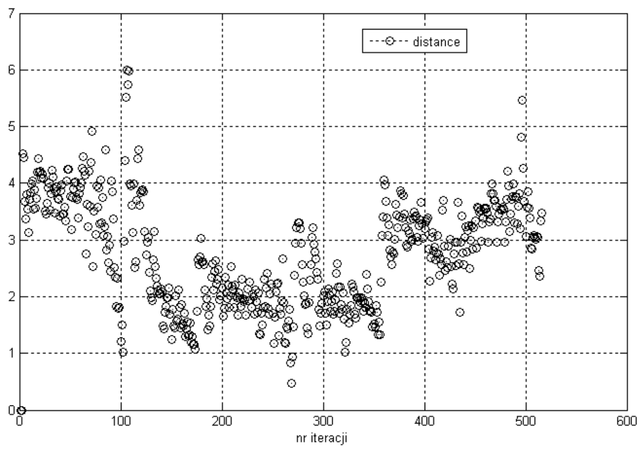

Figure 6. The distances between KF and LS positions.

6 SUMMARY

The presented models and algorithms illustrate two of many possibilities of the navigational application of the method of least squares and the Kalman filter for the integration of navigational data. In the Kalman filter the state vector reproduces the system evolution (movement trend) on the basis of dead reckoning navigation. The main advantages of Kalman filter, in this case, are its recurrence, which is a natural necessity in case of ship navigation and the possibility of using the ship movement data (its trend).

The fusion of position parameters with DR navigation by the LS method gives similar results to those obtained by the KF method, in which data from DR are also used. The differences between the position coordinates obtained from using these methods are almost insignificant (on average less than 3 meters). One advantage of the KF over the LS method, owing to the structure of the former, is that it offers a possibility to identify a trend of ship movement parameters. On the other hand, the KF algorithm requires more complex computations. However, to achieve the same level of functionality of the LS method, the estimation model would have to be expanded to include the identification, which in turn would increase significantly the computational complexity of the algorithm.

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