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A New Definition of a Collision Zone For a Geometrical Model For Ship-Ship Collision Probability Estimation

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ABSTRACT: In this paper, a study on a newly developed geometrical model for ship-ship collisions probability estimation is conducted. Most of the models that are used for ship-ship collision consider a collision between two ships a physical contact between them. The model discussed in this paper defines the collision criterion in a novel way. A critical distance between two meeting ships at which such meeting situation can be considered a collision is calculated with the use of a ship motion model. This critical distance is named the minimum distance to collision (MDTC). Numerous factors affect the MDTC value: a ship type, an angle of intersection of ships' courses, a relative bearing between encountering ships and a maneuvering pattern. They are discussed in the paper.

1 LITERATURE REVIEW

A number of models for ship-ship collision probability estimation can be found in the literature. They can be divided into two major groups namely: static and dynamic models. The static models are simpler and less time consuming for computation, however their accuracy can be questioned. The dynamic models are more complex, in principle they need more input data than static models, but their results are comprehensive. In this section the short overview of existing models will be made, and our contribution to the existing knowledge will be put forward.

1.1 *Static models*

The most known approaches were introduced by (Fujii et al. 1970) and (MacDuff 1974). Models of this kind have been commonly used in recent decades and won the popularity among researches mainly due to simplicity and robustness. However they have also some drawbacks, lack of ship dynamics or assumption regarding a collision between two ships. A collision is defined as a meeting of two ships in a distance named the "collision diameter", which means almost the physical contact. Such an assumption may lead to an understanding that in any ship-ship encounter at a distance greater than the "collision diameter" these ships are able to avoid a collision, which in most cases is not true. Despite the drawbacks the model was adopted by (Pedersen 1995), and with minor modifications was used to determine the safety of navigation in many European waters: (Otto et al. 2002), (Sfartsstyrelsen 2008). Hence in Europe it is mostly known as Pedersen model. Another method for the frequency of collision estimation, making an assumption regarding uncorrelated traffic, was outlined by (Fowler and Sorgrad 2000). A critical situation is assumed to occur when ships come to close quarters to a distance of 0.5 Nm of each other, which is constant regardless of a meeting scenario. A model for encounter probability estimation proposed by (Kaneko 2002) defines a critical area of an optional form of a closed boundary, around a ship which violation means collision. Kaneko in his model recognizes two shapes of the critical area: rectangular and circular, but again the size of the area is fixed. A series of papers utilizing the ship domain approach to ship-fixed object collision assessment was published also by (Gluver and Olsen 1998) and (Pedersen 2002).

However, none of the model listed above takes ship dynamics into consideration.

1.2 *Dynamic models*

Another group of models utilize marine traffic simulations. A group of researches led by Merrick proposed a risk analysis methodology for maritime traffic in coastal areas based on system simulation

(Merrick et al. 2002), (Merrick et al. 2003). Maritime traffic is simulated in the time domain based on routes obtained from expert opinion and vessel arrival records. Finally, these were combined with the simulation output in order to carry out a risk analysis (van Dorp and Merrick 2009).

Another probabilistic model for the assessment of navigational accidents in an open sea area was outlined by (Gucma and Przywarty 2007). The method makes use of a simplified model of maritime traffic, which is simulated in the time domain. A recent model, introduced by (Goerlandt and Kujala 2011) is based on an extensive time-domain simulation of maritime traffic in a given area. Vessel movements are modelled based on data obtained from a detailed study of route-dependent vessel statistics. The collision candidates are detected by a collision detection algorithm which assesses the spatio-temporal propagation of the simulated vessels in the studied area.

Markov, semi-Markov and Random Field theory based models for maritime traffic safety estimation were introduced recently (Smalko and Smolarek 2009), (Smolarek and Guze 2009), (Smolarek 2010), (Guze and Smolarek 2010). However the main assumption of the models proposed is that traffic flow is stationary, which is not applicable to areas with scheduled traffic. Recently a geometrical model for estimation the probability of ship collisions while overtaking were introduced by (Lizakowski 2010), his model considers human factor and the fairway and ship dimensions. However all these models are advanced mathematically they do not take into account ship dynamics nor human factors.

A multicomponent model for an inland ship safety estimation was presented by (Galor 2010). However each of the proposed model's component is essential, the model itself constitutes rather an introduction to the further quantitative analysis of the problem.

For the first time the idea of ship manoeuvrability implementation into a collision assessment model was presented by (Curtis 1986). However, this model was limited to one ship type, which was a very large crude carrier (VLCC), and only overtaking and head-on situations were considered.

1.3 *Authors' contribution*

A new criterion for ship-ship collision probability estimation and a new model have been introduced by (Montewka et al. 2010). The model considers ship maneuverability and traffic parameters; the new collision criterion is named the Minimum Distance To Collision (MDTC). MDTC is a critical distance between two ships being on collision courses, at which they must perform collision evasive actions, in order to pass safely. The MDTC is estimated by means of ship motion model and series of experiment for various ship meeting scenarios.

This paper is a continuation of our previous research, it consists of the detailed analysis of the MDTC values for a wide ranges of input variables (they are defined in the following Chapters) and two patterns of performing collision evasive action. The maneuvering patter one means that own ship is performing a collision evasive action and the other ship is not acting, in the maneuvering patter two both ships are involved in avoiding a collision. Performance of turning circle is considered a collision evasive action.

2 INTRODUCTION TO MDTC MODEL

The MDTC model introduced in a previous work of (Montewka et al. 2010) and developed further in this paper, is based on an initial assumption, that two ships collide if the distance between them becomes less than a certain value, named a MDTC. This MDTC value is not a fixed number, but it is calculated dynamically for each type of vessel and encounter individually. Thus it changes with the situation. The main factors affecting the MDTC value are: the vessels maneuverability, the angle of intersection labelled α in Figure1a, the relative bearing from one vessel to the other labelled β in Figure1b and a pattern of evasive maneuvers (one vessel swinging or both). In the previous study, a simplified methodology was applied, which assumed that two vessels met at a constant relative bearing while proceeding with their service speeds. Presented study considers a wide range of relative bearings, varying from 10 do 80 degrees (counting from the own ship's bow) and takes into account two different engine settings for each ship type, therefore providing more detailed results.

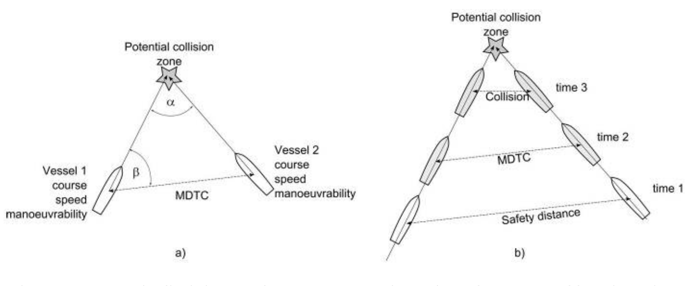

Figure 1: A definition of MDTC and major factors affecting it
Source: (Montewka et al. 2010)

3 RESEARCH MODEL

The theory of the model and preliminary research aiming to define the "collision zone" were presented by (Montewka et al. 2010). In this paper the results of studies with respect to different ship types and ship speeds and varying meeting angles are shown. However only planar motion of a ship is taken into account and assumption regarding ship navigating through deep water is made. We also assume, that the prevailing weather conditions do not deteriorate significantly the maneuverability of ships sailing in the analyzed part of the Gulf of Finland. In order to validate it, we simulated a maneuver of turning circle to starboard side, for the chosen ship type, which was a RoPax (for ship particulars see Table 1), for two different wave conditions (no wave, and an average wave height for the Gulf of Finland). According to (Pettersson et al. 2010) and (Raamet et al. 2010) the average monthly weave height recorded in the analyzed area (sea between Helsinki and Tallin) does not exceed 2 meters, and as a such was adopted for the simulation. For this purpose the Laidyn ship motion model was adopted (Matusiak 2007).

The results allowed us to keep our assumptions, as a difference between the trajectories of a ship in two different heights of a wave seems to be negligible for the purposes of our research (Figure 2).

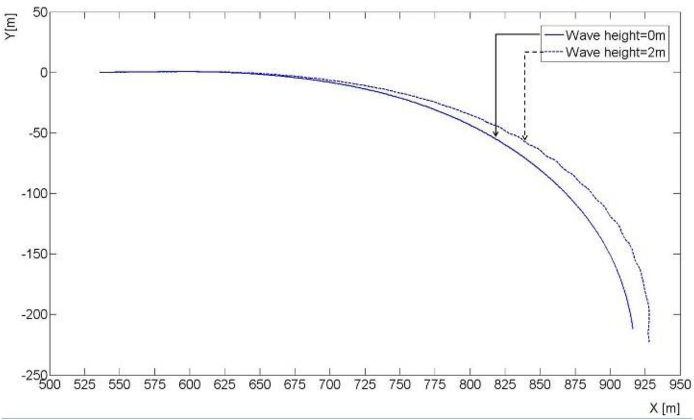

Figure 2: Turning circles of RoPax performed for two different wave heights

3.1 *Ships considered*

In the course of our analysis we are considering four major ship types: a passenger ship, a containers carrier, a RoPax and a tanker. In each scenario, ships are assumed to proceed with two different engine settings (except for a passenger vessel which is assumed to sail always at a maximum speed) which result in forty two encountering scenarios, as depicted in Figure 4. The following abbreviations are used: 'FA' is full ahead and 'HA' means half ahead. The 'FA' abbreviation corresponds to a mean speed of a ship of given type as obtained from recorded AIS data. The abbreviation 'HA' does not correspond to an actual engine setting, it rather reflects a spread of recorded speed values for a given class of ships in the analyzed area. The value of 'HA' for given ship type was calculated by subtracting the standard deviation from the mean value for a given type of ship. The main particulars of the analyzed vessels are listed in Table 1.

Table 1: Ships particulars.

Ship type	LOA [m]	B [m]	T [m]	v [kn]
Container carrier	150.0	27.2	8.5	20;17
RoPax	158.6	25.0	6.1	20;18
Tanker	139.0	21.0	9.0	14;11
Passenger	185.0	27.7	6.5	25

3.2 *Encountering scenarios*

Each of an encountering scenario is run for seventeen different crossing angles (α), varying from 010 to 170 degrees with 10 degrees increment. Where 010 degrees means almost overtaking (vessel B on a course of 350deg) and 170 stands for almost head-on meeting (Vessel B on a course of 190deg), as depicted in Figure 3. The situation shown there considers own ship seeing another at 45 degrees relative bearing. In the course of the experiment, each crossing angle is calculated for a range of relative bearings, from 10 to 80 degrees, counting from the own ship's bow.

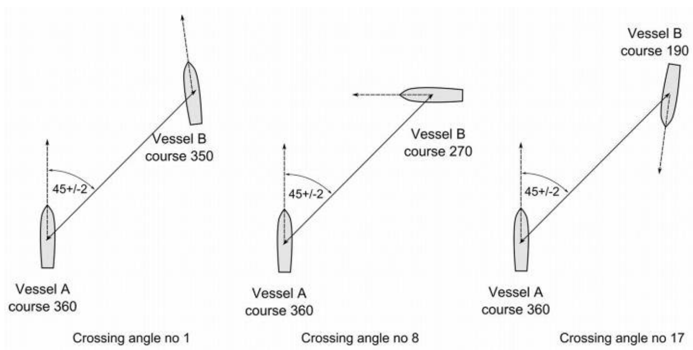

Figure 3: Relative positions of vessels, with three chosen crossing angles, before they start to maneuver, (Montewka et al.2010)

For each ship-ship encounter at a given intersection angle (α) and at a given relative bearing (β), one MDTC value is obtained. As specified in a block diagram depicted in Figure 4, in total 5712 MDTC values are obtained. Then for each intersection angle (α) the maximum MDTC value among eight (as there are eight relative bearings considered) is drawn. Also the relative bearing which is the most inconvenient from a collision evasive point of view, and which requires the most space to make an action is indicated. For further statistical analysis 714 out of all 5712 MDTC values are selected for each maneuvering pattern.

3.3 *Maneuvering patterns*

In case of a maneuvering pattern number one, the own ship is performing an evasive action, by turning circle, and another ship is following her initial course. In case of maneuvering pattern number two, two ships are performing turning circles in order to avoid collisions. The following simplifications in the presented methodology are done:

- in case of evasive pattern where two vessels perform turning circles, they both start their maneuvers at the same time;
- ships are turning away from each other, which implies course alteration away from each other to avoid collision and to shorten the time at close quarters (such assumption meets requirements of the COLREG, which states, that ships must avoid altering courses towards each other if in close quarters);
- the settings of ships' engines and rudders are constant during maneuvers;
- the influence of weather conditions is omitted;
- the hydromechanical ship-ship interactions are omitted.

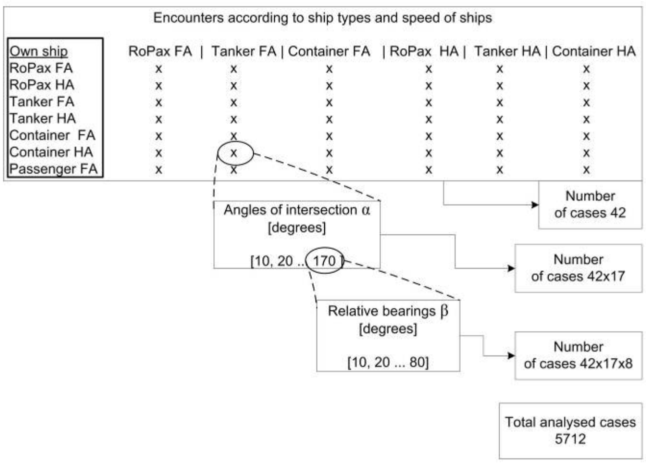

Figure 4: Research model

3.4 *MDTC estimation*

In order to calculate the value of MDTC for a given pair of vessels, an iterative algorithm is used, as depicted in Figure 6. The basic assumption is that the two ships collided at a time instant t0. Then starting from this time the reverse iterative algorithm is applied. It uses backward calculation method in a space-time domain. Two trajectories of two ships are drawn and the consecutive positions of ship's centre of gravity are plotted every second (*dt*=1s). If corresponding ships' contours following the trajectories have at least one common point, indicating that they both collided, the algorithm increases the initial distance between these two ships by constant value of $0.1\mathrm{LOA}_{average}$. The trajectories are redrawn starting from the new initial positions of the ships. This process is repeated until the two contours of ships have no overlaps at any time instant for a given relative bearing.

New initial positions are defined by moving ship B from ship A away. For a given meeting scenario (a given angle of intersection α and a given relative bearing β) the ships are moved away along a line of a given relative bearing (β line). For the simplicity of calculations it is assumed, that own ship holds her initial position, while the other ship is moved away along the β line.

In the situation where two trajectories have no common points and the contours of the ships do not over-lap, the initial position of vessel B is recorded (as the initial position of own ship A was always (0,0)), and the distance between these two positions is calculated and stored. This distance, is named MDTC for a given relative bearing. As each meeting scenario is analyzed for a range of relative bearings (from 10 to 80 degrees), the procedure presented is repeated for all relative bearings, yielding eight values of MDTC for each angle of intersection α. Finally, the maximum value of MDTC among these eight is drawn. This maximum value is considered a MDTC value for a given angle of intersection. This procedure is repeated for all angle of intersection, then for each maneuvering patterns. Thus the MDTC charts are obtained.

In order to determine the MDTC charts, all routines are encoded in MATLAB. As a polygonal region, which could represent a ship contour an ellipse is chosen. To determine, whether two contours of the ships (represented as ellipses) overlap, the following MATLAB function is applied (MathWorks 2010):

$$\mathbf{IN} = inpolygon(X,Y,xv,yv), \tag{1}$$

it returns a matrix **IN** of the same size as X and Y. Each element of (**IN**) is assigned the value 1 or 0 depending on whether the point ($X(p,q),Y(p,q)$) is inside the polygonal region whose vertices are specified by the vectors *xv* and *yv*.

For the sake of computation effectiveness each ellipse is transformed into discrete form and the number of points that represent the ellipse is 24. The ellipse's axes are defined in the following way:

$$a = 0.5LOA$$

$$b = 0.5B, \tag{2}$$

where a denotes a major axis, b is a minor axis, LOA means length overall of a ship and B is a ship's breadth.

A MDTC value for a given encounter implies a safe passage of two vessels, which corresponds to a situation where these two vessels approximated by the ellipses, will always be separable and will not touch each other at any time step of a collision evasive action. A graphical interpretation of above is depicted in Figure 5, where both ships are at the closest distance in the time step 81sec., however they are still separable. A block diagram showing an algorithm applied in the study to estimate a MDTC chart for a given meeting scenario is depicted in Figure 6.

4 DATA ANALYSIS

The data obtained in the course of MDTC calculations (see Figure 6) considers different ship types, different engine settings and two different maneuvering patterns, as stated in Figure 4. In the next step the statistical analysis of the obtained data is performed.

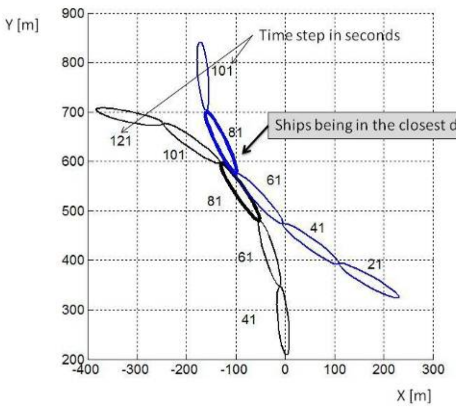

Figure 5: Ships as ellipses and interpretation of a non-contact passage

In Figure 7 data sets concerning MDTC assuming a maneuvering pattern number one (own ship involved in collision evasive action), according to a ship type, are presented. Whereas the data depicted in Figure 8 shows appropriate values for MDTC, according to a ship type for maneuvering pattern number two (both vessels are involved).

To determine whether the differences which can be noted visually are significant from the statistical point of view, the appropriate statistical tests are performed. The following are hypothesized:

– H_0: the obtained values of MDTC, for a range of intersection angles α, are drawn from the same population (or equivalently, from different populations with the same distribution), thus MDTC do not depend on a ship type.
– H_1: the medians of analyzed variables are not all equal, thus the MDTC values do not originate from the same population, and they are a ship type dependent.

In order to validate these hypotheses we performe a nonparametric Kruskal-Wallis test which compares samples from two or more groups, as the obtained data do not follow a normal distribution. In the case presented here we analyze 42 different encounters, each consisting of 17 crossing angles, as depicted in Figure 4. We form a 42-by-17 matrix, where each column of the matrix represent an independent sample containing 42 mutually independent observations, and a number of columns is equivalent to a number of crossing angels α. The function that Kruskal-Wallis test is based on compares the medians of the samples in a matrix, and returns the *p*-value for the null hypothesis.

In the course of the analysis the obtained *p*-value vary for two maneuvering patterns concerned. In the case where both vessels make a turn (the maneuvering pattern No 2) the *p*-value yields 0.9988. This shall not cast any doubt on the null hypothesis, and suggests that all sample medians come from the same population.

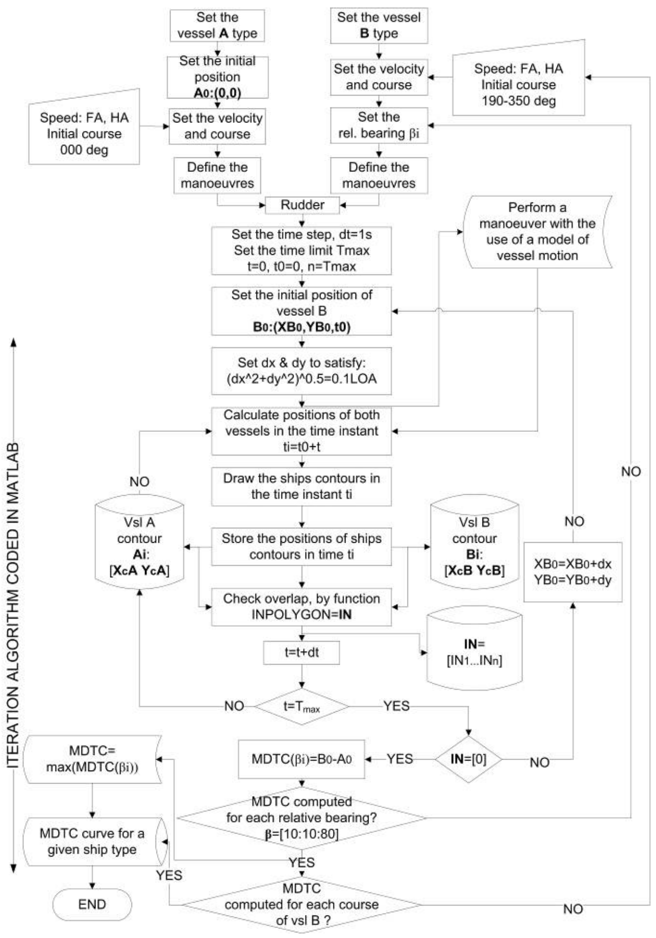

Figure 6: Block diagram for MDTC calculation

However, the results obtained for the maneuvering pattern No 1, where the own ship performs collision evasive action only, are more scattered therefore not so straightforward in inference. The results of the statistical tests concerning both maneuvering patterns are gathered in Table 2. Analyzing a full range of intersection angles, for maneuvering pattern No 1, there is no evidence for not rejecting the null hypothesis. This can lead to thinking that at least one data set originates from a different population that the other data sets.

Table 2: Results of statistical tests ordered by the angle of intersection

Maneuvering pattern	Segment [*deg*]	*α* value	*p* value	Hypothesis rejected
No 1	10-170	0.05	0.002	H_0
No 2	$10-170$	0.05	0.9988	H_1

However dividing the intersection angles range into segments, and analyzing them separately, makes it feasible to defend the null hypothesis.

In order to made a cross check of the results obtained in this analysis, in the next step we order a data set according to a ship type, and run the Kruskal-Wallis tests on smaller samples. The results obtained are presented in Table 3.

Table 3: Results of statistical tests according to a ship type - the maneuvering pattern No 1.

Ship type	Segment [*deg*]	*α* value	*p* value	Hypothesis rejected
Container	10-170	0.05	0.4424	H_1
RoPax	10-170	0.05	0.9891	H_1
Tanker	10-170	0.05	0.1237	H_1
Passenger	10-170	0.05	0.9307	H_1

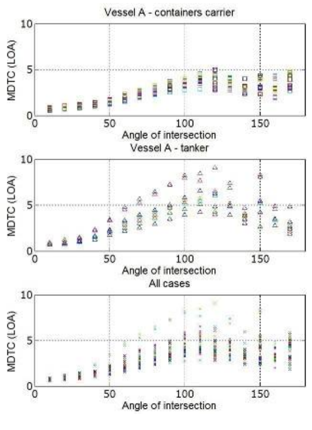

Figure 7: MDTC values for a given ship type - maneuvering pattern No 1 (own vessel performing an evasive action)

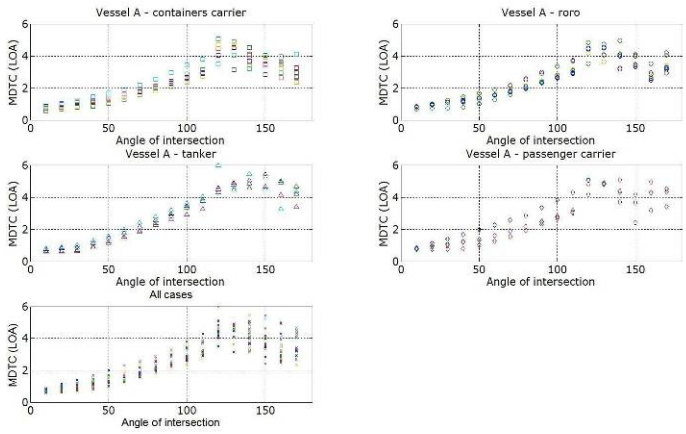

Figure 8: MDTC values for a given ship type - maneuvering pattern No 2 (both vessels performing collision evasive maneuvers)

It can be noticed that two cases (”RoPax” and ”Passenger”) defend the null hypothesis for a full range of intersection angles. In a case of ”Container” the null hypothesis can not be rejected, but the $p-value$ obtained is not as high as in the previous cases, however still acceptable. In a case of ”Tanker”, although the $p-value$ obtained is greater than the adopted level α, however the null hypothesis can be rejected as early as a confidence level α is 0.13. This obviously can cast some doubts on the null hypothesis. In order to have an insight into a data set regarding variable ”Tanker” we divide a range of intersection angles into three segments, and run again the statistical test. The results obtained are presented in Table 4.

Table 4: Results of statistical tests according to a ship type and the angle of intersection - the maneuvering pattern No 1.

Ship type	Segment [*deg*]	*α* value	*p* value	Hypothesis rejected
Tanker	10-120	0.05	0.4411	H_1
Tanker	120-140	0.05	0.6671	H_1
Tanker	140-170	0.05	0.8306	H_1

Having variable ”Tanker” excluded the p-value for the null hypothesis for the maneuvering pattern No 1 becomes higher than adopted level α.

Table 5: Results of statistical tests according to the angle of intersection, the maneuvering pattern No 1, tankers excluded.

Maneuvering pattern	Segment [*deg*]	*α* value	*p* value	Hypothesis rejected
No 1	10-170	0.05	0.20	H_1
No 1	10-120	0.05	0.98	H_1
No 1	120-140	0.05	0.70	H_1
No 1	140-170	0.05	0.39	H_1

The following conclusions can be made:

– for the maneuvering pattern No 1, if the variable ”Tanker” is excluded, the statistical tests prove, that the data for other three variables (”Container”, ”RoPax” and ”Passenger”) are drawn from the same population;
– for maneuvering pattern No 1, the MDTC values for tankers are obtained in the course of a separate analysis;
– for maneuvering pattern No 2, there are strong evidences, that all data come from the same population, thus MDTC is not a ship type depended variable.

5 RESULTS

The obtained MDTC values are categorized according to an intersection angle α, and we make attempts to define distributions of MDTC values for each angle α and for given maneuvering patterns. Because of the limited survey sample, and data scatter none of commonly known distributions, neither continuous nor discreet, fit the data. Thus further analysis is conducted using one of the sampling methodology, namely a non parametric bootstrap procedure.

For each maneuvering pattern, the following values are estimated: a mean and a standard deviation of a MDTC for a given angle of intersection (α). In order to obtain these parameters, the following procedure is adopted (Vose 2008):

– to collect the data set of n samples $\{x_1 \ldots x_n\}$ in our case n=42
– to create B bootstrap samples $\{x_1{*} \ldots x_n{*}\}$ where each $x_i{*}$ is a random sample with replacement from $\{x_1 \ldots x_n\}$, in our case $B = 10^6$;
– to estimate, for each bootstrap sample $\{x_1{*} \ldots x_n{*}\}$, the required statistics sˆ. The distribution of these B estimates of s represents the bootstrap estimate of uncertainty about the true value of s.

The outcome of the bootstrap analysis are as follows:

– the mean MDTC values, for each intersection angle α;
– the 0.95 confidence intervals around the mean values (represented as dotted lines in a Figure 6).

Then using the upper confidence interval of the mean value, the 0.95 prediction interval is calculated. The upper prediction band obtained is shown as a solid line with diamonds. The results obtained, for two maneuvering patterns are depicted in the following Figures: 9, 10, 11.

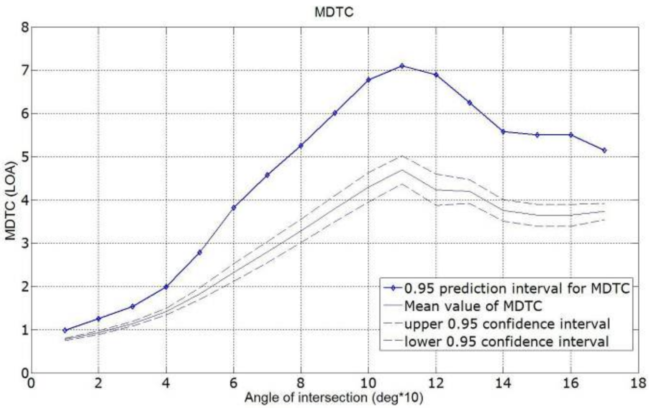

Figure 9: The obtained MDTC chart for the maneuvering pattern No 1

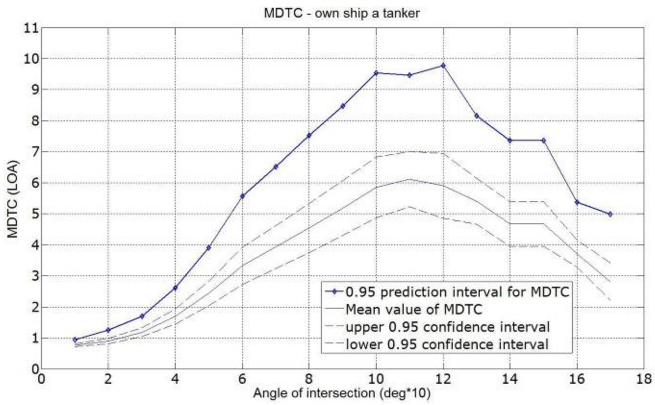

Figure 10: The obtained MDTC chart for tankers - the maneuvering pattern No 1

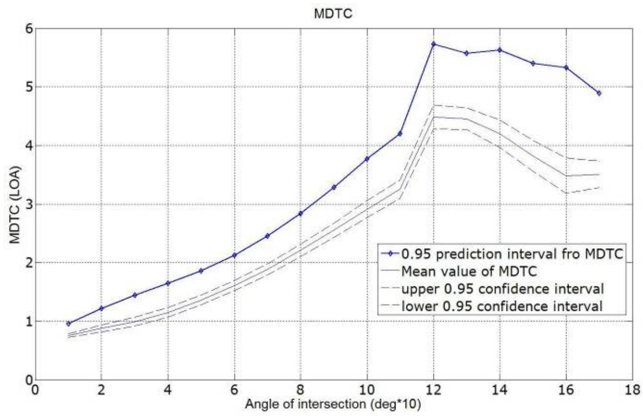

Figure 11: The obtained MDTC chart for the maneuvering pattern No 2

6 CONCLUSIONS

This paper addresses a chosen aspects of marine traffic safety modelling. This means a novel method for definition a collision-zone for ship-ship meetings. This parameter named MDTC is an input for a model that estimates the probability of ship-ship collision. It takes into account ship dynamics, traffic patterns and indirectly the human actions (the maneuvering patterns).

In the course of the analysis presented in this paper three different charts representing three types of collision zones were obtained. The statistical analysis shows that the dimension of a collision zone depends mostly on a maneuvering pattern. In case where both ships perform collision evasive actions, one chart describes all types of ships analyzed. However in case where only one ship performs a collision evasive maneuver, two charts are obtained, where one considers tankers and another the remaining ship types.

The experiment leading to MDTC chart estimation is based on a ship planar motion model, and the assumptions concerning deep water and lack of external forces and hydromechanical ship-ship interactions are made. However the size of the vessels under consideration allows the statement, that the sea conditions prevailing in the Baltic Sea, and especially in the Gulf of Finland do not affect the results significantly.

Another important factor affecting the actual number of modelled accidents is a causation factor. This topic is not addressed by research presented in this paper.

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