Multivariable Adaptive Controller for the Nonlinear MIMO Model of a Container Ship

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ABSTRACT: The paper presents an adaptive multivariable control system for a Multi-Input, Multi-Output (MIMO) nonlinear dynamic process. The problems under study are exemplified by a synthesis of a course angle and forward speed control system for the nonlinear four-Degrees-of-Freedom (4-DoF) mathematical model of a single-screw, high-speed container ship. The paper presents the complexity of the assumed model to be analyzed and a synthesis method for the multivariable adaptive modal controller. Due to a strongly nonlinear nature of the ship movements equations a multivariable adaptive controller is tuned in relation to changeable hydrodynamic operating conditions of the ship. In accordance with the given operating conditions controller parameters are chosen on the basis of four measured auxiliary signals. The system synthesis is carried out by linearization of the nonlinear model of the ship at its nominal operating points in the steady-state and by means of a pole placement control method. The final part of the paper includes results of simulation tests of the proposed control system carried out in the MATLAB/Simulink environment along with conclusions and final remarks.

1 INTRODUCTION

Nonlinear control systems are commonly encountered in many different areas of science and technology. In particular, problems difficult to solve arise in motion and/or position control of various vessels such as drilling platforms and ships, sea ferries, container ships, etc. Complex motions and/or complex-shaped bodies moving in the water, and in the case of ships also at the boundary between water and air, give rise to resistance forces dependent in a nonlinear way on velocities and positions, thus causing the floating bodies to become strongly nonlinear dynamic plants.

In general, there are two basic approaches to solve the control problems for nonlinear plants. The first one called “nonlinear” includes synthesizing a nonlinear controller that would meet certain requirements over the entire range of control signals variability (Fabri & Kadrikanathan 2001; Huba et al. 2011; Khalil 2001; Tzirkel-Hancock & Fallside 1992; Witkowska et al. 2007). The popular methods of predictive control (MPC) employ nonlinear or on-line linearized models of the plant (Maciejowski, 2002; Rawlings & Mayne, 2009; Limon et al., 2005; Qin & Badgwell, 2003). However, in the case of MIMO nonlinear processes such nonlinear control algorithms are too complex for computations to be performed online. Such tasks are particularly difficult when additional constraints on the control signals are considered, which demands using some numerical procedures to solve optimization problems with constraints. When a nonlinear description of the plant is not known accurately, predictive controllers employing artificial intelligence, for example neural
networks (Akesson & Toivonen, 2006; Lawrynczuk, 2010; van der Boom et al., 2005) can be used.

The second approach called “linear” consists in designing an adaptive linear controller with varying parameters to be systematically tuned up corresponding to changing plant operating conditions determined by system nominal operating points. Here, linearization of nonlinear MIMO plants is a prerequisite for the methods to be employed. As a result of the linearization local linear models are obtained and they are valid for small deviations from operating points of the plant.

Since properties exhibited by linear models at different (distant) “operating points” of the plant may vary substantially the controllers used should be either robust (Ioannou & Sun 1996) (usually of a very high order as has been observed by (Gierus 2005)) or adaptive with parameters being tuned in the process of operation (Aström & Wittenmark 1995).

If the mathematical description of the nonlinear plant is known, then it is possible to make use of systems with linear controllers prepared earlier for possibly all operating points of the plant. Such controllers can create either a set of controllers with switchable outputs from among which one controller designed for the given system operating point (Barúka et al. 2010; Barúka et al. 2010b; Dworak & Pietrusiewicz 2010) is chosen, or multi-controller structures from which the control signal components are formed. One example is weighted means of outputs of a selected controller group according to Takagi-Sugeno-Kang (TSK) rules, i.e. with weights being proportional to the degree of their membership of appropriately fuzzied areas of plant outputs or other auxiliary signals (Tanaka & Sugeno 1992; Tatjewski 2007; Dworak et al. 2012a; Dworak et al. 2012b).

What all the above-mentioned multi-controller structures, have in common is that all controllers employed in these structures must be stable by themselves, in distinction to a single adaptive controller with varying (tuned) parameters. This means that system strong stability conditions should be fulfilled (Vidyasagar 1985).

In the presented paper an adaptive modal MIMO controller with (stepwise) varying parameters in the process of operation is studied. The controller can be physically realized as a multi-controller structure of modal controllers with switchable outputs. The considered adaptive control system will be designed for all possible “operating points” of the plant. In the simulation study a 4-DoF nonlinear model of a single-screw high-speed container vessel has been used as a nonlinear MIMO plant. The main goal of the paper is a synthesis of the course-keeping adaptive control system for a container vessel assuming two controlled variables: yaw angle and forward speed of the ship relative to water.

2 NONLINEAR MODEL OF A CONTAINER SHIP

2.1 Ship dynamics

The considered course-keeping control system structure has been studied by means of a 4-DoF nonlinear mathematical model of a container vessel (Son & Nomoto 1981, Fossen 1994 The vessel is 175m long (L), 25.4m wide in beam (B) with an average draught of 8.5m (H). In order to describe movements of the ship two reference systems are defined. The yaw angle and the ship position are defined in an Earth-based fixed reference system. On the contrary, force and speed components with respect to water are determined in a moving system related with the ship’s body and the axes directed to the front and the starboard of the ship with the origin placed in its gravity center (G) (shown in Fig. 1).

Designations for the linear and angular speed of the ship, in the considered degrees of ship motion are as follows: \( u \) (surge velocity), \( v \) (sway velocity), \( p \) (roll rate) and \( r \) (yaw rate). Corresponding designations of the position coordinates of the ship are as follows: \( x \) (ship position in N-S), \( y \) (ship position in W-E), \( \phi \) (roll angle), \( \psi \) (yaw angle).

![Figure 1. Ship's co-ordinate systems.](image)

General nonlinear equations of motion in surge, sway, roll and yaw (Son & Nomoto 1981, Fossen 1994) are as follows:

\[
\begin{align*}
(m+m_j)\dot{u}-(m+m_j)vr &= X \\
(m+m_j)v+(m+m_j)ur+m_j\alpha_j\dot{r}-m_j\dot{I}_z\dot{p} &= Y \\
(I_z+J_z)p-m_j\dot{l}_y-m_jur+WG\dot{M}\phi &= K \\
(I_z+J_z)\dot{r}+m_j\alpha_j\dot{v} &= N-Y\dot{x}_G.
\end{align*}
\]

Here \( m \) denotes the ship mass. The \( m_j \), \( m_r \), \( J_z \), \( I_z \) denote the added mass and added moment of inertia in the \( x \) and \( y \) directions and about the \( x \)-axes and \( z \)-axes, respectively. \( I_z \) and \( I_r \) denote moment of inertia about the \( x \)-axes and \( z' \)-axes, respectively. Furthermore, \( \alpha_j \) denotes the \( x \)-
coordinates of the center of \( m_i \), whereas \( l_i \) and \( l_{i}^{'} \) denote the \( z \)-coordinates of the centers of \( m_i \) and \( m_{i}^{'} \), respectively. \( x_G \) is the location of the center of gravity in the \( x \)-axes, \( GM \) is the metacentric height and \( W \) is the ship displacement.

The hydrodynamic forces \( X \), \( Y \) and moments \( K \), \( N \) in above equations are given as:

\[
X = X_{\text{ml}} l' u + (1-t) T + X_{v_{r}} v_{r} + X_{v_{v}} v^{2} + X_{v_{r}} r^{2} + X_{\theta} \phi + c_{m_{\phi}} F_{N} \sin(\delta),
\]

(2)

\[
Y = Y_{v} v + Y_{r} r + Y_{p} p + Y_{\phi} \phi + Y_{v_{v}} v^{3} + Y_{v_{r}} r^{3} + Y_{\theta} v_{r}^{2} + Y_{v_{\phi}} \phi \phi^{2} + Y_{\theta} v_{\phi}^{2} + (1 + a_{n}) F_{N} \cos(\delta),
\]

(3)

\[
N = N_{v} v + N_{r} r + N_{p} p + N_{\phi} \phi + N_{v_{v}} v^{3} + N_{v_{r}} r^{3} + N_{\phi_{v}} v_{r}^{2} + N_{v_{\phi}} \phi \phi^{2} + N_{\phi_{\phi}} \phi \phi^{2} + (x_{r} + a_{n} x_{\phi}) F_{N} \cos(\delta),
\]

(4)

\[
K = K_{v} v + K_{r} r + K_{p} p + K_{\phi} \phi + K_{v_{v}} v^{3} + K_{v_{r}} r^{3} + K_{\phi_{v}} v_{r}^{2} + K_{v_{\phi}} \phi \phi^{2} + K_{\phi_{\phi}} \phi \phi^{2} + (1 + a_{n}) z_{r} F_{N} \cos(\delta).
\]

(5)

Here, the rudder force \( F_{N} \) can be resolved into:

\[
F_{N} = \frac{-6.13A_{1} \Delta K_{r}}{\Delta + 2.2S} \frac{A_{1}}{L} (u_{r}^{2} + v_{r}^{2}) \sin(\alpha_{r}),
\]

(6)

where:

\[
\alpha_{r} = \delta + \tan^{-1}(v_{r}/u_{r}),
\]

(7)

\[
u_{r} = u_{p} e^{\sqrt{1+8kK_{r}/(\pi J)}},
\]

(8)

\[
u_{v} = v_{r} + c_{v_{v}} r^{3} + c_{v_{v}} v^{3} + c_{v_{v}} v^{3} v_{r},
\]

(9)

where:

\[
J = u_{p} v / (nD),
\]

(10)

\[
K_{r} = 0.527 - 0.455 J,
\]

(11)

\[
u_{p} = \cos(v) \left[ (1-w_{p}) + w_{r} \left( v + x_{r} r^{2} + c_{p_{r}} v + c_{p_{r}} r^{2} \right) \right].
\]

(12)

The actual speed of the vessel is designated as \( V = \sqrt{u^{2} + v^{2}} \). The control signals of the nonlinear MIMO model of the ship (1) are: \( \delta \) (rudder angle) and \( n \) (propeller shaft speed).

2.2 Actuators dynamics

In order to synthesize the control system, the steering machine model based on (Fossen 1994) is represented by the first-order dynamic system with time constant \( T_{m} = 1.8 \) s and gain \( K_{g} = 1 \), whereas the shaft model is represented by the linear model with average time constant \( T_{m} = 10.48 \) s and gain \( K_{m} = 1 \). Therefore, the actuators block shown in Fig. 6 can be described in the state-space form:

\[
\dot{x}_{i}(t) = A_{i} x_{i}(t) + B_{i} u_{i}(t)
\]

(13)

\[
y_{i}(t) = x_{i}(t),
\]

where:

\[
A_{i} = \begin{bmatrix} -0.556 & 0 \\ 0 & -0.095 \end{bmatrix}, B_{i} = \begin{bmatrix} 0.556 & 0 \\ 0 & 0.095 \end{bmatrix}
\]

Here \( u_{i}(t) = [\delta_{i}(t) \ n_{i}(t)]^{T} \) is a vector of commanded control signals and \( x_{i}(t) = [\delta(t) \ n(t)]^{T} \) is a vector of control signals. In the simulations the following limitations of control signals are assumed: maximum speed of the screw \( n_{\text{max}} = 160 \) rpm, maximum rudder angle \( \delta_{\text{max}} = 15 \) deg and maximum rudder angular velocity \( \dot{\delta}_{\text{max}} = 5 \) deg/s.

3 MULTIVARIABLE ADAPTIVE CONTROL SYSTEM

The dynamic model of the container ship (1) can be described in the state-space nonlinear form:

\[
\dot{x}_{2}(t) = f(x_{2}, u)
\]

(14)

\[
y(t) = g(x_{2}, u),
\]

with the semi-state vector \( x_{2}(t) \) defined as shown in Fig 1.

\[
x_{2}(t) = [u \ v \ p \ r \ \phi \ \psi]^{T}
\]

(15)

and output and control signals defined as:

\[
y(t) = [u(t) \ \psi(t)]^{T}
\]

(16)

\[
u(t) = [\delta(t) \ n(t)]^{T}.
\]

In order to synthesize the control system the resulting model is linearized in the nominal operating points of the ship, defined as \( x_{i}(t) = x_{2x}(t) \). The nominal state vector of the model (1) in the nominal operating regimes is defined as:
\[
x_{2n}(t) = \begin{bmatrix} u_n \ v_n \ 0 \ r_n \ \phi_n \end{bmatrix}^T.
\] (17)

The values of state variables \((u_n, v_n, r_n, \phi_n)\) are defined in the turning circle simulation tests carried out in MATLAB/Simulink for various control signals: \(\delta_n\) and \(n_v\). The range of changes of these signals is as follows: \(\delta_n = [-15 + 15] \) deg with the resolution of 1 deg and \(n_v = [5 + 160] \) rpm with the resolution of 5 rpm, which results in a set of 992 operating points.

Each combination of the control signals and their corresponding parameters of the ship movements: \(u_n, v_n, r_n\) and \(\phi_n\) determines the nominal operating point of the ship. The resulting functions \(u_n(\delta,n), v_n(\delta,n), r_n(\delta,n), \phi_n(\delta,n)\) are shown in Figures 2, 3, 4 and 5, respectively.

As a result of the linearization performed in the whole range of the nominal control signals linear state-space models of the container ship are obtained:

\[
\begin{align*}
\dot{x}_2(t) &= A_2[x_2(t) - x_{2n}] + B_2[u(t) - u_n] \\
y(t) - y_n &= C_2[x_2(t) - x_{2n}],
\end{align*}
\] (18)

where:

\[
A_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]

\[
B_2 = \begin{bmatrix}
b_{11} b_{21} b_{31} b_{41} 0 0 \\
b_{12} b_{22} b_{32} b_{42} 0 0 \\
b_{13} b_{23} b_{33} b_{43} 0 0 \\
b_{14} b_{24} b_{34} b_{44} 0 0 \\
b_{15} b_{25} b_{35} b_{45} 0 0 \\
b_{16} b_{26} b_{36} b_{46} 0 0 \\
\end{bmatrix},
\]

\[
C_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix},
\]
with the entries \(a_{ij}\) and \(b_{ij}\) depending on the values of surge velocity \(u_s\), sway velocity \(v_s\), yaw angular velocity \(r_s\) and control signals \(u_c = [\delta, n_c]^T\), the nominal operating points of the container vessel. Now, the full state vector \(x(t)\) of the vessel can be taken as:

\[ x(t) = [\delta, n, u, v, p, r, \phi, \psi]^T. \]  

Finally, the full linearized model of the container vessel is described by the matrices:

\[
A = \begin{bmatrix} A_1 & 0 \\ B_2 & A_2 \end{bmatrix}, B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} \theta & C_2 \end{bmatrix}.
\]  

The obtained linear models (20) with known parameters are the starting point for applying many known methods for linear multivariable control system design. When the linear MIMO systems are considered multivariable modal (or possibly optimal LQG/LQR) controllers are usually designed.

In the case of non-measurable state variables, modal controllers used in the proposed control system structure are multivariable dynamic systems with parameters defined in time domain by:

\[
\dot{x}_r(t) = A_r x_r(t) + B_r e(t)
\]

\[
u(t) = C_r x_r(t) + D_r e(t),
\]  

where:

\[
A_r = A - BF - LC, B_r = L, C_r = -F, D_r = 0.
\]  

Here, \(F\) is the state feedback matrix related to the state vector components of the plant models, and \(L\) is the gain matrix of full-order Luenberger observers, which reconstruct the state vector of the plant linear models (20). Synthesis of modal controllers is based on the use of the various techniques of pole placement in stable regions of the s-plane. As it was shown in (Bańka et al. 2013) the designed modal controllers may be calculated using four methods: Eigenvalues Method (EM), Eigenvectors Method (EVM), Polynomial Method (PM) and Polynomial Matrix Equations Method (PME) which in case of MIMO plant yield different results for the same data taken for calculations. The method we finally choose should depend on the numerical conditions for the plant given, its local linear models as well as the result of calculations we need. In the EM, EVM, and PM methods the synthesis of MIMO modal controllers base on separately finding the matrices \(F\) and \(L\) for which, according to (22), their “standard” state-space equations have been formulated. In the PME method, instead of separately calculating the matrices \(F\) and \(L\), the controller transfer function matrix is directly obtained at one go by solving the Diophantine left polynomial matrix equation. More particular details on this subject may be found in (Bańka et al. 2013).

The controller presented in the paper has been synthesized with the use of EVM method.

If strictly causal modal controllers based on the full-order Luenberger observers are selected then designing performed directly in the time domain as well as in s-domain (without solving polynomial matrix equations) leads to calculating the feedback matrix \(F\) which places the closed-loop system matrix eigenvalues in the desired locations on the s-plane and the weight matrix \(L\) of the full-order Luenberger observer for appropriately desired observer poles. In the case of measurable state variables it is sufficient to determine the state feedback gain matrix \(F\) in order to synthesize modal control system in time domain. If the plant model is described by matrices (20) the vector of commanded control signals is as follows:

\[
u_c(t) = F(x_{ref} - x(t)) + u_c,
\]  

which shifts the poles of a linear plant model to desired locations, which in our case are as follows: [-0.11, -0.12, -0.13, -0.14, -0.15, -0.16, -0.17, -0.18]. These experimentally assumed values allow us to achieve sufficiently fast dynamics of control system and reduce output signals overshoots and control signals saturation. The reference state vector \(x_{ref}\) is defined as:

\[
\begin{bmatrix} 0 & n_{ref} & u_{ref} & 0 & 0 & 0 & 0 & \psi_{ref} \end{bmatrix}^T.
\]  

Here \(n_{ref}\) is the reference shaft speed corresponding to the reference surge velocity of the ship in a steady-state for \(\delta = 0\). The resulting set of 992 local controllers has been used to create multivariable adaptive controller with stepwise varying parameters. This controller is tuned with four measured auxiliary signals including: surge and sway speed components of the ship with respect to water as well as yaw rate and roll angle of the ship, which are shown in Figures 2, 3, 4 and 5. The current nominal operating point is determined by minimization of a quadratic functional \(J_n:\)

\[
J_n = \left( \frac{\Delta u}{u_{max}} \right)^2 + \left( \frac{\Delta v}{v_{max}} \right)^2 + \left( \frac{\Delta r}{r_{max}} \right)^2 + \left( \frac{\Delta \phi}{\phi_{max}} \right)^2,
\]  

where \(\Delta u, \Delta v, \Delta r, \Delta \phi\) are auxiliary signals deviations from the values in the nominal operating points, whereas \(u_{max}, v_{max}, r_{max}, \phi_{max}\) are the maximum values of auxiliary signals in the whole range of nominal operating points.

The block diagram of the proposed multivariable adaptive control system is shown in Fig. 6. It consists of the state feedback matrix \(F\) whose entries are switched in a stepwise manner according to the current operating point of the ship.
If the state vector of the ship model (1) is not measurable the state feedback matrix should be replaced by an adaptive modal controller (21) based on the Luenberger observer or the Kalman filter (Bańka et al. 2013).

The stability of the above described closed loop system with modal (gain-scheduled) controller has been proved by the use of the stability theory of the nonsmooth system given in (Shevitz and Paden, 1994), used successfully e.g. in (Lee et al., 2001).

4 SIMULATION TESTS

The usability of the propose control system is illustrated with a multivariable adaptive control system for the nonlinear MIMO model of a container vessel (1). The goal of the presented control system was a simultaneous control of the course angle and forward speed of the container ship. Results of simulations carried out in the MATLAB/Simulink environment are presented in Fig. 7 and 8. The initial state vector of the ship was:

\[
\mathbf{x}(0) = \begin{bmatrix} 0 & 40 & 8.14 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T.
\]  

which means that the ship goes forward with the speed of 8.14 knots. The first maneuver at t=100 s was the change of the desired forward speed to 25.44 knots. Then after 200 s the desired course angle was changed to 20° with keeping the ship forward speed at 25.44 knots. Both changes have been done according to the assumed ship dynamics and all maneuvers have been done with acceptable values of the control signals: rudder angle and shaft speed, presented in Fig. 8.

Figure 9 presents values of indices i and j which denote the current operating point. Changes of their values show moments in which the feedback matrix \( F \) entries are modified (switched).

5 CONCLUSIONS

In the paper an adaptive control system for the nonlinear MIMO plant was proposed and tested. The utilized adaptive gain scheduling modal controller allows one to control a strongly nonlinear process, here the model of a container vessel. The synthesis of the controller is based on the linearization of a nonlinear ship model in operating points corresponding to the set of 992 typical operating regimes. The adaptive controller parameters vary in a stepwise way on the basis of auxiliary signals measured during ship operation. The presented example of multivariable control of the ship, shows efficiency of this method and the appropriateness of its use to the direct control or as a part of more complex control systems, e.g. a model loop in the MFC control structure (Dworak et al. 2012b).

Figure 6. Block diagram of the proposed control system structure.

Figure 7. The course angle and speed of the ship.

Figure 8. Rudder angle and shaft speed.

Figure 9. Moments of switching of the feedback matrix \( F \).
REFERENCES


