Safer and More Efficient Ship Handling with the Pivot Point Concept

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ABSTRACT: The concept of the pivot point of a turning ship has been in existence for more than two centuries. It was not, however, properly understood from the beginning, and thus some misconceptions developed. This in turn caused it to be viewed as something mystical, thus preventing ship handling from scientific approach. The concept is expounded in a fresh light, deriving an equation for the definition and others for the calculation of the pivot point location both in general and for specific examples in an idealized condition. The implications of the derived equations are discussed. The results of a verification experiment are presented, which proved centuries’ of teachings and learnings to have been incorrect. A number of exercises for both steady and unsteady cases have been suggested for the training of the practitioners in the light of these new findings.

1 INTRODUCTION

Ship handling has been viewed by many as an 'art', meaning that it cannot be performed by scientific calculations alone, but must also be relied upon one's own experience and intuition. One of the factors contributed to this view was the concept of 'pivot point'. It has been the central and important tool in ship handling, unfortunately, however, it has been a rather ambiguous entity, resulting in some confusion and misuse amongst ship handlers. Yet practitioners have been trying to understand ships' motion in terms of it.

In recent years a number of authors gave clearer expositions of it - Tseng C-Y (1998), Artyszuk J (2010), Seo S-G (2011-6), for example - and demonstrated what can be achieved with the correct understanding. These enabled the practitioners to have an unambiguous picture of the concept.

2 THE CONCEPT OF PIVOT POINT

The concept of pivot point has been an essential tool in ship handling. The knowledge about the position of the pivot point in a manoeuvring situation provides the ship handler with the information on the geometry of motion of the ship. Rowe R W (2000), Clark I (2005), Cauvier H (2008) and Baudu H (2014). Hence, it has been a requirement for the ship handler to understand how and why the ship behaves in each condition.

Ship’s motion in a confined area can be modelled as a planar rigid body motion assuming no vertical movement of any point of the ship. This is justified for the relatively calm free surface in such an area.

When sway and yaw occur simultaneously, a ship handler can only perceive the combined effect of drift and turn, which gives him a false impression that only a rotational motion happened about a certain point on the ship’s centreline. This apparent centre is called the Pivot Point of the ship. This is a simplification of
perception from two motions down to one motion, which is the very reason why the pivot point concept is so useful to ship handlers.

2.1 How is the Pivot Point brought about?

Water being a yielding material, any active force turning a ship will cause drifting motion at the same time. In the case when ‘drift’ happens before ‘yaw’, the two motions in sequence are shown in Figure 1. The thick red arrow indicates the drift and the thin arrows depicts the yaw motion.

![Figure 1: Actual Motion (Sway and Yaw)](image)

A few notes on Figure 1:
- The ship is depicted as turning about the centre of gravity, G2.
- The distance of G1 from P is not the same as that of G2 from P.
- P is not a fixed point in the ship but dependent on the sway and the yaw motion.

For a brief moment, G1 and G2 can be taken to be approximately the same distance from P. The two motions of the ship in Figure 1 can then be imagined as a turning motion about P, as shown in Figure 2. This imaginary motion, when taken over an infinitesimally small time interval, is correct as far as the kinematics of the ship is concerned. This is normally the way that a ship’s motion is perceived by ship handlers.

Point “P” is traditionally called the ‘pivot point’ of the ship. For practical purposes, this is an extremely useful concept because it reduces two motions down to one.

![Figure 2: Apparent Motion for a Small Time Interval (Yaw only)](image)

Figure 2 actually depicts a pure turning motion of a rigid body in which every point in the ship is moving along a concentric circle centred on P. Therefore, the planar motion of a ship in a confined area could be represented by the composition of a surge motion and a yaw motion about P only, by using the pivot point concept.

2.2 The Mathematical Definition of the Pivot Point

Among all the points in the ship in planar motion, there is only one point on the centreline at which the sway and yaw completely cancel each other, thus making this point seem to be stationary. All other points appear to be turning about this point. This point is the Pivot Point as explained in 2.1. If the sway speed and yaw speed are known, the position of the pivot point can be obtained as the distance from the centre of mass (GP) using Equation (1).

\[ v + (GP \times r) = 0 \]  

where, \( v(m/s) \) = sway speed of G; \( G \) = Centre of Gravity; \( P \) = Pivot Point; \( GP(m) \) = distance to P from G; \( r(rad/s) \) = yaw Speed.

2.3 The Pivot Point Position due to a Turning Force

Since the pivot point is defined on the centerline of the ship, only one dimensional coordinate system will suffice for our purpose. The vertical line through the centre of gravity is taken as the origin, one side of which is taken as positive direction, the other side negative direction. See Figure 3.

![Figure 3: 1-D Coordinate System](image)

When a force causes a ship to drift and turn, the centre of gravity will move due to the drift motion.

\[ GG_i = \frac{1}{2} \left( \frac{F}{\Delta} \right) t^2 \]  

where, \( F \) is the force; \( \Delta \) is the mass displacement of the ship; \( t \) is the time taken.

The arc drawn by G in an imaginary yaw motion with P as the pivotal point is:

\[ (\text{arc})GG_i = GP \times \frac{1}{2} \left( \frac{F \times (-GF)}{I} \right) t^2 \]  

where, \( GF \) is the distance from G to F, the negative sign indicating the other side of G (origin) from P; \( I \) is the second moment of mass of the ship about the origin.

For a minuscule change of heading, \( GG_i \) can be equated to \( (\text{arc})GG_i \) giving,

\[ GP = \frac{1}{\Delta \times (-GF)} \]
Equation (4) gives the position of the pivotal point in terms of GP. This pivotal point (P) is naturally called the 'Pivot Point' of the ship even though it is imaginary. Under the assumption of a solid ship of uniform density with multiple number of controlling forces, Equation (4) becomes:

$$ GP = \frac{1}{V \times GF_c} \int r^2 dV $$

(5)

where, \( V \) is the volume of the ship; \( GF_c \) is the longitudinal distance along the centreline between \( G \) and \( F_c \), the position of the resultant of all applied controlling forces; \( r \) in this equation is the radial distance of the infinitesimal volume from the origin.

Equation (5) reduces, for a box barge \((C_w = 1.0)\), to an elegantly simple equation.

$$ GP = \frac{L^2 + B^2}{12GF_c} $$

(6)

A wall-sided hull could be defined by:

$$ y = \frac{B}{2} \left[ \left( \frac{2x}{L} \right)^2 \right] $$

(7)

where, \( L \) is the length of ship; \( x \) is the longitudinal position of a point along \( L \) with the origin at midships; \( y \) is the half beam.

A Wigley Hull, of which the lines are shown in Figure 4, is defined by:

$$ y = \frac{B}{2} \left[ \left( \frac{2x}{L} \right)^2 \right] \left[ 1 - \left( \frac{z}{D} \right)^2 \right] $$

(8)

where, \( D \) is the depth of ship; \( z \) is the vertical position of a point assuming the ship is completely immersed in water.

Figure 4. Wigley Hull Form

Assuming \( F_c \) at rudder stock \((GF_c = 0.5L)\), \( B = L/7\), \( D = L/7\), some calculations are carried out using Equation 5, and the results are shown in Table 1.

<table>
<thead>
<tr>
<th>Relative Position of the Pivot Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Box Barge</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>L</td>
</tr>
<tr>
<td>GP</td>
</tr>
<tr>
<td>From Bow</td>
</tr>
</tbody>
</table>

By comparing the results, one can deduce that a bigger block coefficient will cause the pivot point to be closer to the bow. The calculation was carried out assuming an unresisting medium. The numerical values are not the same as reality. However, the qualitative deductions made would still be correct.

Any applied force on a ship in real fluid will set her into motion. The gradually increasing motion changes the aerodynamic and hydrodynamic environment. The reactive forces increase until they balance with the active forces. By then the ship will have gained some momentum. This momentum adds further movement in the pivot point position, which settles down as the motion becomes steady. In reality, therefore, dealing this way with the unsteady process accurately is very difficult, if not impossible, particularly when various forces are involved.

2.4 Interpretation of the formula for the Pivot Point Position

Two important aspects are noted from Equation (5).

Firstly, the minus (-) sign indicates that the pivot point appears on the other side of \( G \) from \( F_c \). Secondly, a bigger \( GF_c \) yields a smaller absolute \( GP \), which means that an external force farther away from \( G \) causes the pivot point to be closer to \( G \). These two findings are essential knowledge for the practitioners to proactively control the pivot point.

3 TRADITIONALLY HELD VIEW POINTS ON THE PIVOT POINT AND DISCUSSION

3.1 Traditional View of the Pivot Point

A few examples of traditional views found in the literature are that the pivot point:

- moves towards the bow or towards the stern depending on the sense of surge motion
- is the centre of rotation (yaw)
- has instantaneous movement
- is the fulcrum of the turning moment

All of the above four views are incorrect. The facts are that the pivot point:

- is independent of surge motion, not to mention the sense
- is only an imaginary point
- moves gradually
- is not a physical entity.

However, ship handling professionals, particularly the seasoned practitioners, find it very difficult to accept these findings. For example, drawings like the one below are commonly found in the literature.
The situation is described as (i) two equally-powered tugs are pulling the ship laterally with the same turning moment (ii) the engine starts making stern way (iii) then the ship turns bow to starboard. Supposedly, this is because the pivot point is on the aft side of the centre of gravity, thus giving a longer moment arm to the tug at bow. In other words, the drawing is used for two fold purposes. One, as the proof of the pivot point being near the stern when moving astern, and two, as the explanation of the heading change in terms of the pivot point.

The account is mistaken in two aspects. One is that the pivot point is treated as a physical entity (as the fulcrum). The other is that the pivot point is treated as a cause. The correct explanation is that when the ship starts moving astern, the centre of lateral resistance moves sternwards from midship. Thus the reactive hydrodynamic forces providing an extra turning force about the centre of gravity, which is the net force that actually turns the ship about its centre of gravity. The tugs are pulling the ship to starboard. The starboard sway causing the pivot point to appear between the centre of gravity and the stern as the result.

3.2 A Verification Experiment

Historically, it was said that the pivot point is located near the stern when a ship is moving astern. An example from a ship handling book says, “When making sternway, the pivot point moves aft and establishes itself approximately 1/4L from the stern.” The derived Equation 5, however, implies that the sense of surge motion is irrelevant with the pivot point location, but rather, if the propeller and rudder combination at the stern is used as the only propulsion system, the pivot point will always appear near the bow.

A verification experiment was conducted at Warsash Maritime Academy Ship Handling Centre on 11th February 2016. The ship, “Progress”, is a 1:25 scale model of Panamax, LOA 225 m, beam 32 m.

Figure 6 shows the starting point. The ship’s turning force was provided by setting the engine half astern. The propeller is right handed with fixed pitch. Figure 7 shows the final position.

<table>
<thead>
<tr>
<th>Interval</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Position</td>
<td>16.7</td>
<td>16.7</td>
<td>16.7</td>
<td>17.0</td>
<td>17.4</td>
<td>17.0</td>
<td>15.4</td>
<td>14.8</td>
</tr>
</tbody>
</table>

The table shows plainly that the pivot point was at around 17% of the ship length from the bow. Near the end of the experiment, it is obvious that the pier is interfering with the water flow being created by the propeller. This experiment conclusively proves that the traditional teachings and learnings about the pivot point for centuries are incorrect.

4 SOME BASIC EXERCISES TO ACTIVELY CONTROL THE PIVOT POINT

Some basic exercises to actively control the pivot point are suggested below. Some of them require additional means of applying active forces other than the conventional propeller and rudder combination, such as a bow thruster, a stern thruster, a pod, tugs, etc.

4.1 Keeping P coincided with G while yawing

This is the case when the ship is yawing about the centre of gravity, which is taken as the centre of rotational motion. The ship has no translational motion (no surge, no sway). The pivot point (P) coincides with the centre of gravity (G). The centre of
turning circle (C) would also be at the same point, if it were not a singular point – Figure 8.

The radius of swept area will be the minimum. This will allow ship’s heading turned in a smallest possible area.

4.2 Keeping P between G and Bow, while swaying and yawing

In the absence of any longitudinal movement (no surge), if the ship drifts at the same time as turning, and if the pivot point is between G and the bow, the motion shown in Figure 9 will result. The centre of turning circle (C) could have appeared at the same spot as P, if it were not a singular point.

4.3 Keeping P forward of Bow, while swaying and yawing

Figure 10 depicts the situation. Again, C could have appeared at the same location as P, if there had been any surge motion.

4.4 Keeping P coincided with G, while turning

If a ship makes headway while yawing but without any sway motion, the resulting movement will look like the one shown in Figure 9. In this case, the pivot point (P) will coincide with the centre of gravity (G).

This manoeuvre causes no swing out of the stern, thus it may be a necessary manoeuvre in a tightly restricted waters. The area of sweeping path is the minimum possible among the manoeuvres with the same turning circle radius by P – compare with Figure 12 and 13.

4.5 Keeping P between G and Bow, while making a general planar motion

When all the three motions (Surge, Sway, Yaw) are present, all the three distinctive points (P, G, C) will exist separately as shown in Figure 10. In this particular case, the stern swings out sweeping a bigger area, as all skilled ship handlers are most conscious of. Ship motions in general fall in this
category. The position of the pivot point is directly related to the amount of swing out.

Figure 12. General Planar Motion, P between G and Bow

4.6 Keeping P ahead of Bow, while making a general planar motion

The sweeping area for this manoeuvre is much bigger now than in Figure 10. However, this may be a necessary manoeuvre for some situations such as preparing to enter a narrow passage.

Figure 13. General Planar Motion, Pivot Point ahead of Bow

5 SOME PRACTICAL MANOEUVRES

Now the training could continue on practicing more routines in which the pivot point positions are continuously changing. Good candidates for the routines are turning around a sharp corner, entering a narrow cut and turning short round.

Figure 14. Short Round

1 Start manoeuvre at a slow speed, then hard to starboard with a kick ahead
2 Stop engines – rudder midships
3 Engine astern, transverse thrust continues to turn vessel.
4 Vessel at a stop over the ground, continue with engines astern. Transverse thrust still acting on vessel.
5 Engine still running astern
6 Engine still running astern, about to stop engine.
7 Rudder hard to starboard, engine ahead.
8 Vessel completed short round.

Note: This is a simplified version rather than what would be required in reality.

Figure 15. Entering a Cut

While preparing for the manoeuvre shown in Figure 14, the clearance from the jetty and the longitudinal position are crucially important so as not to come into contact with any port structure during the manoeuvre.

The pivot point will initially appear near the centre of gravity, not nearer to the bow as normally quoted in ship handling literature, and then gradually move forward as the ship gains drifting momentum.

5.3 Southampton Container Port

The following sequence of screen shots have been taken from the PPU on the departure of the “CMA
CGM Marco Polo” from Southampton. The pilots portable unit is an AD Navigation ADX-XR which includes RTK, giving a very precise position. The performance criteria are:

- Position Accuracy: 1-2 cm (RTK mode)
  - 0.8 m with EGNOS/WAAS
  - 2 m uncorrected GPS/GLONASS
- Bow and Stern Speed: 1 cm/sec (0.02 knots)
- Vertical/Squat: 2-3 cm (RTK mode)
- Heading: 0.01 degree (20m POD separation)
- Rate of Turn: 0.1 degree/min

The “CMA CGM Marco Polo” is clear of the berth, moving astern. The vectors for the bow and stern are indicated by the black arrow, whilst the predicted position of the vessel is outlined for 4 positions.

The gray fill outline is the actual position of the vessel (400m LOA). The vessel has 3 tugs in attendance, and needs to swing within the swinging ground depicted by the purple circle.

Figure 16.

Figure 17.

Figure 18.

The ship outlines with no fill, are the predicted positions of the vessel after a set time duration. This is set by the pilot / operator. The vector of the bow and stern can also be seen, indicated on the chart with the black arrow from the centre line, fore and aft respectively.

Figure 19.

This is the sort of manoeuvre as the exercise shown in Figure 8. The ship is using tugs, in combination with own engine, rudder and bow thruster so as to maintain a rotational movement about the vessels midship position. Bow velocity is depicted at the right hand side of the screen, in this instance 1.6 knots to port, and depicts that the vessel is moving 0.06 knots astern. The stern velocity is presently 1.76 knots to starboard.
This depicts the vessel having completed her swing 141m off 107 berth proceeding outwards at 3.7 knots.

Screen shots courtesy of ABP Southampton.

6 CONCLUSION

For two hundred years or more the pivot point location in ship handling has been a rather ambiguous entity. Yet practitioners were taught and practiced to understand ships’ motion in terms of it.

Logical reasoning and mathematical derivations brought the mystified pivot point into the light, rectifying a few mistaken concepts. This brought ship handling closer to science than art. With correct understanding of the concept, ship handlers can now make manoeuvres more efficient and safer.

All ship handling books, instruction manuals, lecture notes, etc. should have a major rewrite reflecting the correct understanding of the pivot point concept.

All deck officers need to be re-educated and retrained to the new paradigm that the pivot point, which is the most important theoretical element of ship handling, is now science rather than art.

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