ABSTRACT: A programmatic method of correcting errors of the measuring track is presented. Methods of determining transfer function of the measuring track are introduced and measurement results with and without the correction are compared.

1 INTRODUCTION

Elements of a typical measuring track include: measuring transducers, elements adapting the measurement signal to elements of the measurement system, analogue-digital transducers, filters, and elements processing and analysing measurement signals. A typical measuring track is shown in Fig.1.

The output signal of a measuring track may be a signal used to control equipment, e.g. abrupt braking system, vehicle suspension, etc. In such circumstances, the basic criterion for real time controlling of this device is the correct processing of the input signal by the measuring transducer and sending of this signal to a control device over a time-span that allows the device to respond to the given situation. This implies correct choice of a transducer adapted to the input signal (the range of frequencies and sensitivities measured). It must be borne in mind that each element in a measuring track introduces errors and delays in processing of the measuring signal, often amplifying the errors introduced by the function of upstream elements of the measurement track. The processing time of a measurement signal by the track is important in applications that require rapid measurements. With the number of elements processing a signal, the time may be extended till a point when correct operation of a control system will no longer be possible.

2 TESTING OF THE MEASURING TRACK IN LABORATORY CONDITIONS

Figure 2 illustrates a diagrammatic measurement system used to test characteristics of measuring track processing. An accelerometer including an inbuilt DeltaTron preamplifier of sensitivity 10.18 mV/ms² (A2) and the range of measured frequencies 0.3 Hz to 6 kHz, and a conditioner of the operating range from 1 Hz to 20 kHz is the sensor under testing. The reference value of acceleration is obtained from the accelerometer, whose sensitivity is much greater than that of the tested accelerometer and whose measuring signal can be read without the need for additional equipment. To this end, a piezoelectric...
accelerometer of sensitivity 317mV/ms\(^2\) (A1) and the range of measured frequencies from 50 to 200 Hz is used.

![Flow diagram of a measuring track testing system](image)

Figure 2. Flow diagram of a measuring track testing system

Figure 3 shows courses of signals from the accelerometers A1 and A2 for sinusoidal input function of frequency 100 Hz and acceleration amplitude of 3.15 m/s\(^2\), as read from the course of the reference accelerometer A1. The delay between the signal from the accelerometer including the preamplifier A2 and the signal from the piezoelectric accelerometer A1 is 8.9 ms. The determined relative error between values of computed acceleration amplitudes is 40%. Measuring of the acceleration with the computer, measurement card (of sampling frequency 40 kHz), and the software cause a further delay of 1.1 ms between the measurements. The delay between measurement of the signal from the transducer A2 and A1 total 10 ms.

![The courses of signals from the reference and measurement accelerometer](image)

Figure 3. The courses of signals from the reference and measurement accelerometer

Figure 4 illustrates the attenuation diagram of the acceleration of the vibration transducer A2 as determined in testing. To enhance reliability of the results, the measurements were made in the frequency range of the correct operation of the reference transducer A1: 50 Hz – 200 Hz. Technical specifications of A1 indicate that, in this range, the amplification of the output signal's amplitude in relation to the transducer's input signal is within ± 1 dB. This is usually the acceptable value for purposes of measurements. Technical specifications of A2 state that, in the frequency range 0.3 Hz - 6 kHz, the same value is ± 10 % and corresponds to ± 0.91 dB. It was assumed that A1 relays the amplitudes closer to the actual values, as it is designed to operate in a narrower frequency range, at more than 30 times greater sensitivity, and without additional elements that would process, and affect, the measurement signal.

Accepting the amplification of ± 1dB in the frequency range 50 Hz - 200 Hz, declared in the specification of A1, the resulting characteristic curve shows that the value of amplification of A2 is achieved in the range of frequencies above 125 Hz. This is different than the manufacturer's bottom value of 0.3 Hz.

To avoid problems of correct reading of the values measured by the transducers, particularly the reference transducer, assume the amplification range of correctly measured accelerations ± 2 dB. The characteristic curve presented in figure 4 indicates this frequency is in the range 125 Hz - 200 Hz.

![The attenuation diagram of the measuring sensor](image)

Figure 4. The attenuation diagram of the measuring sensor

3 CORRECTION ALGORITHM

Determination of a dynamic correction algorithm of a real measuring transducer requires knowledge of its dynamics in the form of differential equations and of the dynamics of the entire measurement system. Manufacturers' attempts at maintenance of transducers' reproduction properties in the manufacturing process have been a failure. Depending on the transducer class, the particular pieces and production lots suffer from some errors of reproduction, sensitivity, and processing range. Therefore, correction algorithms must always be determined for specific measuring transducers and measurement systems,
and the resultant equations must only be applied to the particular sensors (Cioc 2006).

In measurements requiring highly accurate results, other measuring track elements (amplifiers, measuring cards, signal processing elements, etc.) should be taken into account when defining dynamic correction algorithms.

The need to consider elements other than the sensor itself when describing system dynamics is an initial difficulty with attempts to determine the correction algorithm. The signal from the reference transducer and from the measurement system need to be compared to describe the system’s dynamics. ARX (AutoRegressive with eXternal input) method (Luft 2005) was employed to identify dynamic parameters of the measuring track. The voltage signal from the end of A2’s measuring track is the identified signal, and voltage (S1), which is the response of the accelerometer A1 to sinusoidal input of 200 Hz, is the comparative (reference) signal. Acceleration values are determined on the basis of voltage signals from the sensors and the latter’s sensitivity. To limit the calculations and the processing time, the correction algorithm was assumed to estimate the voltage signal from the measuring transducer according to the algorithm parameters as determined by comparison of signals of the reference and measuring transducers.

The flow diagram of the correction algorithm described with (3) and illustrated in Figure 6 suggests that its correct function depends on correct determination of the factors of the discrete transducer model φ and ψ. These are constant for a specific digitisation time.

\[
\hat{x}(k-1) = \frac{1}{0.03215} [y(k-1) - \hat{u}_1(k-1)] \\
\hat{u}_1(k) = 0.0893 \cdot \hat{u}_1(k-1) + 1.914 \cdot 10^5 \cdot \hat{u}_2(k-1) - 0.0031 \cdot \hat{x}(k-1) \\
\hat{u}_2(k) = -441.97 \cdot \hat{u}_1(k-1) + 0.9849 \cdot \hat{u}_2(k-1) + 32.376 \cdot \hat{x}(k-1)
\]

where \(\hat{x}(k-1)\) = estimate of input quantity at moment \(k-1\); \(y(k-1)\) = measurement result at moment \(k-1\); \(\hat{u}_1(k)\), \(\hat{u}_2(k)\) = variable at moment \(k\).

The flow diagram of the correction algorithm described with (3) and illustrated in Figure 6 suggests that its correct function depends on correct determination of the factors of the discrete transducer model \(\phi\) and \(\psi\). These are constant for a specific digitisation time.

Application of ARX produces the transfer function \(G(s)\) which describes processing dynamics of the system: accelerometer – conditioner – measuring card – data acquisition software:

\[
G(s) = \frac{0.03215s^2 + 1319.6s + 1,338 \cdot 10^6}{s^2 + 4.678 \cdot 10^4s + 2,309 \cdot 10^7}
\]  

(1)

Magnitude and phase characteristic curves of the transmittance system (1) are shown in Figure 5. A great amplification can be seen in the magnitude characteristic curve as sensors of varying sensitivities are applied to testing of the voltage signals. The reference amplification level is expressed as a relation of the measuring transducer’s sensitivity to the sensitivity of the reference transducer on the logarithmic scale: \(W_r = -29.87\) dB. Given this value, the relation of acceleration determined on the basis of the measuring sensor’s voltage signal to the acceleration determined using the reference sensor is \(a_{A1}/a_{A2} = 1\). The magnitude characteristic curve at the adopted boundary value of the amplification ± 2dB \(W_r\) is in the frequency range over 117 Hz.

The dynamic correction algorithm for the transfer function (1) is expressed in a differential equation:

\[
d^2 y + 4.678 \cdot 10^4 \frac{d}{dt} y + 2.309 \cdot 10^7 y = \\
0.03215 \frac{d^2}{dt^2} x + 1319.6 \frac{dt}{dt} x + 1.338 \cdot 10^6 x
\]  

(2)

where \(x\) = input quantity; \(y\) = output quantity.

The correction algorithm (Jakubiec 2000) for the measuring track’s transfer function (2) and received digitisation time \(T_d=0.05\ ms\) becomes a set of equations:

Figure 5. Frequency characteristic curves of a measuring track including an accelerometer
assumed to equal 0. In effect, the algorithm will op-
erate correctly only after a correct variable $\hat{u}_1(k-1)$ is
automatically determined. This takes several digitisation steps. Given the accelerometer manufacturers’ recommendations to take measurements for several dozen seconds to several minutes after the system start-up, determination of an initial optimum value of $\hat{u}_1(k-1)$ is not necessary.

4 CORRECTION RESULTS

Figure 7 shows the waveforms of voltages and the resultant accelerations from the accelerometers in the measurement system illustrated in Fig. 4.2. The measurements were conducted at sinusoidal input of frequency 200 Hz – which is within the reading range of both the transducers (according to calibration cards of their manufacturers) and for which the input error, according to the magnitude characteristic curve in Fig. 4.4, becomes the lowest.

In this case, the delay between the waveforms obtained from the measuring (A2) and reference (A1) accelerometers is 5.3 ms. The acceleration magnitude, calculated as the mean value of absolute magnitudes in 2000 measurement samples, equals 7.01 m/s² for A1 and 9.05 m/s² for A2. This corresponds to a relative error in measurement of acceleration magnitude equal to 29.1 %.

The delays resulting from computer mathematical calculations are negligible. They are not significant given the current computer capacities and the calculation simplicity of an algorithm which consists of five additions and seven multiplications only. An attempted measurement of the time taken for the algorithm calculations in MATLAB environment, Windows XP, the processor Intel Celeron 1.5 Ghz, and 512MB of memory, produced results below 0.1 μs.

The mean magnitude of estimated acceleration of A2 for 2000 samples is 7.59 m/s². Compared to the
magnitude of the reference accelerometer, the measurement relative error is 8.3 % - which constitutes a three and half times reduction compared to the same error without applying the correction algorithm. Figure 9 illustrates the course of relative error values in respect of A2 accelerations using and not using the correction algorithm. The peak values in the characteristic curves result from the reference signal values approximating zero. Table 1 presents values of the relative error for an accelerometer employing and without employing the dynamic correction algorithm for a selected time range.

Table 1. Relative error values of an accelerometer without and with the correction

<table>
<thead>
<tr>
<th>Time [s]</th>
<th>Relative error of accelerometer A2 [%]</th>
<th>Relative error of accelerometer with the correction algorithm [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0020</td>
<td>-102.13 -98.74 -97.37 -96.37</td>
<td>-66.09 -18.61 -2.61 9.03</td>
</tr>
<tr>
<td>0.0025</td>
<td>-102.13 -98.74 -97.37 -96.37</td>
<td>-66.09 -18.61 -2.61 9.03</td>
</tr>
<tr>
<td>0.0030</td>
<td>-102.13 -98.74 -97.37 -96.37</td>
<td>-66.09 -18.61 -2.61 9.03</td>
</tr>
<tr>
<td>0.0035</td>
<td>-102.13 -98.74 -97.37 -96.37</td>
<td>-66.09 -18.61 -2.61 9.03</td>
</tr>
<tr>
<td>0.0040</td>
<td>-95.40 -93.86 -86.98 -100.08</td>
<td>17.2432.06111.86 -35.81</td>
</tr>
<tr>
<td>0.0045</td>
<td>-95.40 -93.86 -86.98 -100.08</td>
<td>17.2432.06111.86 -35.81</td>
</tr>
<tr>
<td>0.0050</td>
<td>-95.40 -93.86 -86.98 -100.08</td>
<td>17.2432.06111.86 -35.81</td>
</tr>
<tr>
<td>0.0060</td>
<td>-95.40 -93.86 -86.98 -100.08</td>
<td>17.2432.06111.86 -35.81</td>
</tr>
</tbody>
</table>

The mean relative errors at the waveform magnitudes are: -96.4 % for the signal from the measuring transducer, and 23.1% for the estimated signal.

Figure 10 presents the characteristic curve of A2's absolute errors with and without the correction. The mean absolute error of A2's magnitude, determined as the absolute mean value of the magnitudes in 2000 samples, is 5.11 m/s². The same error in respect of measurements including the dynamic correction algorithm diminishes to 1.94 m/s². The great value of the absolute error, in the case of measurements both with and without the correction, is a result of the transducer's dynamic properties, i.e. a phase shift of measurands. The successive measurement values change too fast for the transducer's capability of reproducing the input magnitude. When the absolute error is determined, a measurand's waveform is shifted in relation to the actual value by a value determined by the transducer's frequency characteristic curve. The correction reduced the resultant absolute error by more than two and a half times.

Figure 11 plots the course of the absolute error of A2's acceleration magnitudes prior to and post the correction relative to the frequency of the sinusoidal input signal. At 200 Hz, where parameters of the correction algorithm were defined, the post-correction relative error of the acceleration reduces to a minimum, to rise as it diverges from this value. The minimum post-correction relative error of the acceleration is 1.2 %, compared to 4.7 % without the correction.

5 CONCLUSION

Accurate and fast measurements require corrections to be applied to the measuring track in order to reduce the measurement error. The programmatic correction method proposed by the authors significantly reduces errors and enables the measurement system to operate 'on-line'.

REFERENCES