Determination of Flying Objects Position

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ABSTRACT: This paper describes various methods of flying object positioning with the emphasis on their accuracy. Based on the accomplished analysis, we will select the most appropriate method to determine the position of a flying object for relative navigation purposes. The primary criterion for choosing a positioning method is the accuracy of distance measurement within users working in the aviation communications network. The results presented in the paper have been based on mathematical modelling and computer simulation performed in the Matlab programming environment. The results obtained can be used to navigate flying or non-flying objects that work in the air communications network.

1 INTRODUCTION

Airplane position is detected by air navigation systems, which consist of a combination of different types of air navigation devices. The position of the aircraft on the track is defined as the intersection of two or more position lines. The first indication of addressing the positioning problem was heavenly navigation. She used the knowledge of the mutual geometric arrangement of the stars and the measurement of the angles between them. Inability to measure distance to the stars has led to automatic positioning of flying objects (FO) using radio signals. The principle of radionavigation is based on measuring the time of transmission of the signal from the transmitter to the receiver. At the known velocity of propagation of electromagnetic waves (light velocity), it is possible to calculate the distance between the transmitter and the receiver. In this case, the emphasis is placed on the precision of the signal propagation time measurement [Dzunda, M. & Kotianova, N. 2016, Dzunda, M. & Kotianová, N. 2015, Dzunda, M. Kotianova N. & Holota, K. & Et Al. 2015, Dzunda, M. & Hrban, A. 1998], because a time error of 1 μs causes a 300 m error at the measured distance. For precise positioning, we require that the receiver be able to measure the propagation time of the signal to 1 ns accuracy. Ecosystem emits continuous radiation waves into the surrounding space.

2 POSITIONING POSITIONS WITH SATELLITE NAVIGATION SYSTEMS

Air Traffic Services of the SR are using communication, radio navigation and radar systems to control air traffic. Communication systems are operating at frequencies of 100-150 MHz. The output power of these systems is 5.0 to 20.0 W. The primary surveillance radar is operating at frequencies of 2-4 GHz. Transmitted power of these systems is 14-25 kW. The power of older types of radar is several hundred kW. Secondary radars operate at frequencies of 1030 MHz and 1090 MHz. Transmission power of a device is about 2 kW.
NDB navigation systems and ADF work on the frequency kHz 200-525. Transmission power of a device is 25-50 W. VOR operates on a frequency MHz 108-112. Transmission power has a 25-100 W. DME measures the distance and works on the frequency 960-1215 MHz. Transmission power of a device is 100 W.

The ILS precision approach:
- Localizer LLZ (device frequencies: 108-112 MHz, transmission power 2W)
- GP Glide path beacon (device frequencies: 328.6 to 335.4 MHz, transmission power 2W)
- VHF marker beacons (frequency 75 MHz, transmission power 3W) Using satellite navigation systems, we can determine the location in two basic ways:
  1. Point positioning - absolute positioning
  2. Relative positioning - relative positioning

In absolute positioning, it is the positioning of individual points to the used positional system that forms a network on the Earth's surface. The absolute positioning method is the basic GPS assignment. In addition to determining the instantaneous position, these methods also serve to determine the speed of the receiver's movement and to navigate it on the ground or in the space. With absolute positioning, we can determine the position in general with an accuracy within ± 10 m to ± 30 m (SA mode) and within ± 5 m to ± 10 m, (without SA mode). We can increase the accuracy of geocentric coordinates determination by applying differential corrections of measured pseudo-distances. Accuracy in coordinate determination is increased and ranges from ± 1 m to ± 5 m.

The relative positioning method determines the coordinates of the new points relative to the position of the reference point whose geocentric coordinates are known. Relative methods are of primary importance for geodetic applications, as they allow measurements whose results lead to coordinates with accuracy in mm. It proceeds from the measurement of the phase of the carrier wave, while the mathematical model does not directly use the phase measurements in the processing of the measurement results, but differentiates them in the appropriate way - simple, double and triple. Both methods are suitable for determining the position of the moving and immovable object. Several methods are used to determine location by satellites. According to the measured parameter, they are divided into:
- angle measuring method,
- long distance method,
- Doppler method,
- interferometric method,
- combined method.

The common feature of all methods is that we need to know the location of the satellites to determine the position. The different methods differ from one another in what the electromagnetic wave (propagation signal) parameter is measured. These methods can be used to create new navigation systems, whether satellite or alternate

2.1 Angle measuring method

The angle measuring method is one of the oldest and at least the least accurate methods. The principle of positioning lies in the measurement of the elevation angle to the satellite. The geometric point of the points with the constant elevation angle to the satellite is the cone with the top in the space of the satellite, as shown in Fig. 1 [Kotianova, N. 2017]. To determine the elevation angle accurately, it is necessary to use directional antennas with a narrow beam in the direction of the maximum radiating characteristic. This method did not spread any more, because it required very large antenna systems. If we make a measurement of the same satellite (at another time) or another satellite (at the same time), we will determine the second cone. Due to the rapid movement of the satellites and thus the rapid change of the azimuth and the altitude of the satellite it is possible to repeat the measurement after about 2 minutes. The cross member of the two cones with the Earth's surface, respectively, with the height at which the position of the point is located intersect with the measured point.

In the case of an angle method with increasing distance from the reference point, the measurement error is increasing. The positioning accuracy is given by the accuracy of the measurement of the elevation angular directional antennas, and this is not too high.

Figure 1. The principle of the angle measuring method of positioning [Kotianova, N. 2017].

2.2 Doppler method

The basis of this method is the Doppler effect when the transmitter and receiver move at different speeds. In this method, the satellites must not move on a geostationary path. If their angular velocity is the same as the angular velocity of a point on the Earth's surface, it would not move relative to the observer on Earth, and thus would not show a Doppler shift.

A satellite that moves over a non-geostationary orbit sends a signal with a stable frx frequency. The signal transmits in a suitable manner the time stamps transmitted at time points \( t_0, t_1, t_2 \) with a constant time interval \( \Delta T = t_1 - t_0 \). The frequency of the signal received by the user at the position measurement site is due to the Doppler effect equal to the frx value that differs from frx. The received signal is coupled to the oscillator signal with the frequency fed to the mixer. The output signal is a signal with the frequency difference fo - frx. Periods of this signal read the counter triggered and blocked by consecutively received time stamps. If the distance between the
satellite and the user would not change, the number of periods would be equal to:

$$N_i = \Delta T (f_o - f_{TX})$$  \hspace{1cm} (1)

However, the distance between the satellite and the user changes between two time stamps from $d_i$ to $d_{i+1}$ (Figure 2 [Kotianova, N. 2017]). In this case, the timestamp is received by the user at the time $t_i + \Delta t_i$, where $\Delta t_i = d_i/c$ represents the time required for signal transmission to distance $d_i$ between the satellite and the user at the velocity propagation velocity $c$. The frequency difference meter period counter essentially measures the phase change of the signal between two received time stamps.

$$N_i = \int_{t_i}^{t_i + \Delta t_i} (f_{LO} - f_{RX}) dt =$$

$$\Delta T f_o + (d_i + d_{i+1}) \frac{f_o}{c} - \Delta T f_{TX}$$  \hspace{1cm} (2)

The number of periods of the signal emitted between the two adjacent time stamps is the same as the number of periods of the signal received between the adjacent marks, because the doppler effect will of course also occur in the time domain:

$$f_{TX} f(t_{i+1} - t_i) = f_{TX} \Delta T =$$

$$= f_{RX} \left[ (t_i + \Delta t_i) - (t_i + \Delta t_{i+1}) \right]$$  \hspace{1cm} (3)

If we denote $F = f_o - f_{RX}$ and the coordinates of the satellite at the moment $t$ as an ordered triple $(x_i, y_i, z_i)$, respectively, at the moment $t_i + 1$ as the triple $(x_{i+1}, y_{i+1}, z_{i+1})$, and the user’s coordinates as $(x_u, y_u, z_u)$, we obtain the equation:

$$N_i = \Delta T F + \frac{f_{LO}}{c} \left[ \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \right]$$

$$- \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2}$$  \hspace{1cm} (4)

Figure 2. Doppler frequency shift\[5\].

If we perform a measurement of at least three periods between the four time stamps, we obtain three times of the difference signal period after the $N_i$, $N_{i+1}$, $N_{i+2}$ mixer. If we know the coordinates of the satellites in the moments of $t$, $t_i$, $t_{i+1}$, we can solve a system of three equations of three unknowns, which are the coordinates of the user at the location location $(x_u, y_u, z_u)$. The position of the satellite is determined from the current Keplerian parameters of its path, which will be transmitted by the satellite in the form of a navigation message so that the positioning error in the time points $t_i, t_{i+1}, t_{i+2}$ is as small as possible. The Doppler method has been applied to some navigation systems as the primary method, the Parus / Cycladic System and the US Navy Transit. Today, it is rather used as a support method for the telemetric method.

2.3 Angle measuring method

The long-distance method is one of the most widely used positioning methods. It is used by current systems, GPS, GLONASS and the future European GALILEO system. In the distance method, the position of the object is determined by its distance from the broadcast source (satellite). The distance $d_i$ between the receiver and the satellite can be calculated from the time of $\tau_{di}$, which elapses from the transmission of the signal from the satellite to its reception at the receiver in relation to:

$$d_i = \tau_{di} c$$  \hspace{1cm} (5)

where $c$ - the speed of light. If the coordinates of the satellite are known, it is possible to calculate the distance $d_i$ in relation to:

$$\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} = d_i,$$  \hspace{1cm} (6)

where:
- $x_i, y_i, z_i$ - the coordinates of the transmitter (satellites),
- $d_i$ - the distance between the receiver and the transmitter, $x, y, z$ - the coordinates of the receiver.
- $\tau_{di}$ according to equation (5) can be precisely determined only if the perfect time synchronization of the pair of satellite-receiver is ensured. With an electromagnetic wave propagation velocity $c = \text{of about } 3 \times 10^8 \text{ m} \cdot \text{s}^{-1}$, 1 m distance in the free space corresponds to a 3.3 ns time delay of the signal. Ensuring such synchronization of remote independent clock generators in two independent systems (on-board satellite and user receiver) is a very challenging task, leading in particular to the higher complexity of the whole system.

For this reason, the additional $b$ [Awnage, J.L. & Grafarend, E.W. 2004] is introduced into the calculation, which represents the time difference $\Delta t$ calculated over the distance. To calculate position in three-dimensional space, it is necessary to process a signal from at least four satellites.

Considering (6) the coordinates of the receiver we obtain by solving four equations in the form:

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 - (b - d_i)^2 = 0,$$  \hspace{1cm} (7)

$$(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2 - (b - d_j)^2 = 0$$  \hspace{1cm} (8)
\[(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 - (b-d_i)^2 = 0, \quad (9)\]

\[(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2 - (b-d_i)^2 = 0, \quad (10)\]

where \(x, y, z\) - coordinates of the receiver, \(x_i, y_i, z_i\) - satellite coordinates, \(d_i - d_i\) pseudo-distance between the receiver and the satellite, \(b\) - time delay calculated over distance.

We can divide the distant systems into:

1 **Passive-receiver** (user) receives and processes signals from the satellite. The receiver receives a copy of the satellite signal to which the distance is measured. This copy is synchronized with the receive signal in the user receiver to obtain a delay associated with the time base of the receiver. To measure the signals from the four satellites, we obtain a fourfold delay that corresponds to the pseudo-distance. Putting into a set of four equations (7-10) we calculate the user's coordinates.

2 **Active** - the receiver (the user) sends the signal to the satellite (to the transponder), which sends the signal with delay. The appropriate distance between the satellite and the user is then determined by the relationship:

\[d_i = \tau_{di} \cdot c = \frac{\tau_{ci} - \tau_{zi}}{2}, \quad (11)\]

where \(d_i\) is the distance between the satellite and the beacon, \(i\) represents the measured total delay of the signal from the user to the satellite and back. Then, \(\tau\) represents a delay in the transponder of the \(i\)-th satellite. Therefore, no time synchronization is required between system devices. \(\tau_{ci}\) is measured only from the time base of the user. The disadvantage of such a system is that the user must be an active component. This leads to higher energy demands of small navigation terminals. Another problem is limited user capacity when at a certain point in time the given transponder channel is available to only one user, which is economically disadvantageous. [Pavolová, H. & Tobisová, A. 2013, Rozenberg, R. & Szabo, S. & Šebeščáková, I. 2014, Sabo, J. & Korba, P. & Antoško, M. & Sekelová, M. & Rozenberg, R. 2017, Šebeščáková, I. & Melníková, L. & Socha, L. 2013]. The total effective error value is equal to the product of the standard deviation of the distance error \(\sigma_d\) and the DOP value (dilution of precision):

\[E_{rms} = \sigma_d \cdot DOP \quad (12)\]

The DOP coefficient depends only on the geometric configurations of satellites and receivers. It is defined as the ratio of the mean error of the coordinate, time, position, and mean error measured by the precision:

\[DOP = \frac{\sigma}{\sigma_o} \quad (13)\]

It is true that what the DOP is less, the more accurate the results. DOP consists of a horizontal and a vertical component. The total radial coefficient of deterioration of PDOP accuracy, positional DOP equals:

\[PDOP = \sqrt{HDOP^2 + VDOP^2} \quad (14)\]

The PDOP is calculated before the measurement, from the coordinates of satellites originating from the almanac and the coordinates of the receiver, which is sufficient to know with the accuracy per kilometer.

In real measurement with error \(\sigma_d\), the search position is located in a certain space, which is given by the intersection of spheres with a measured pseudo-radius. The accuracy of the measurement corresponds to the size of this space. The solution with a given error \(\sigma_d\) can be located anywhere in this space. From a geometric point of view, it is also advisable to select the satellites so that the space created is as small as possible. The reported accuracy for a normal civil GPS receiver that uses the principle of a long-distance method and operates only with a C/A code is 25 meters (2 drms, 95%) in the horizontal plane and is 43 meters (95%) in the vertical plane.

Accuracy in height is usually 2-3 times worse than horizontal line accuracy. The accuracy at the satellite system level in terms of timing synchronization is satisfactory because the satellites have precision atomic clocks and the entire synchronization is provided by the control segment that is bound to the world reference time norms. In addition, any deviation of satellite hours may be specified in the navigation report together with ephemeris. The clock uncertainty error in the receiver is addressed by the use of multiple satellite signals because the receiver can not contain atomic clocks. In Table no. 1 is an estimate of the impact of different error sources on positioning accuracy.

<table>
<thead>
<tr>
<th>Cause of error</th>
<th>Estimation of impact on positioning accuracy [m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time synchronization error</td>
<td>± 2</td>
</tr>
<tr>
<td>Multi-way signal propagation</td>
<td>± 1</td>
</tr>
<tr>
<td>Movement of orbit satellites</td>
<td>± 2,5</td>
</tr>
<tr>
<td>Tropospheric effects</td>
<td>± 0,5</td>
</tr>
<tr>
<td>Ionospheric effects</td>
<td>± 5</td>
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<tr>
<td>Calculation errors</td>
<td>± 1</td>
</tr>
</tbody>
</table>

3 **SIMULATION OF DETERMINATION POSITIONS OF FLIGHT OBJECTS BY A LONG DISTANCE METHOD**

We created simulation models based on conditions that are as close to real as possible. We have chosen an airspace that is analogous to actual flight radar24 traffic [Vagner, J. Pappová, E. 2014]. Flightradar24 is a flight tracking service that provides real-time information about thousands of aircraft worldwide. Based on observations of the movement of aircraft over the territory of the Slovak Republic on 28.06.2018 08:12. we chose 5 planes randomly and found their
coordinates at a given time. We assume LO is working in aviation communications network. Because the LO position data originates from the GPS, it is determined by the latitude and longitude according to the WCS-84. Figure 3 shows a random selection of 4 broadcast sources (aircrafts) of known position, and the red color is highlighted by the fifth aircraft whose position is calculated according to the algorithms 5 to 10. We hereby verify the suitability of the LO positioning algorithms 5 to 10 that work in the air communications network. Such a positioning system is called a relative navigation system. The information about their starting position:

**FO SAS777** with coordinates: latitude 0.85398056 rad, longitude 0.31139381 rad. Height above ellipsoid 10653 m and altitude 10613 m. Geoid curl 40 m. FO AIC131 with coordinates: latitude 0.84694871 rad, longitude 0.32232797 rad. Height above ellipsoid 12192 m and altitude 12152 m. Geoid curl 40 m. FO LOT224 with coordinates: latitude 0.84418725 rad, longitude 0.31173396 rad. Height above ellipsoid 8809 m and altitude 8769 m. Geoid curl 40 m. FO CLX856 with coordinates: latitude 0.84384361 rad, longitude 0.31586831 rad. Height above ellipsoid 10668 m and altitude 10628 m. Geoid curl 40 m. FO OHY805 with coordinates: latitude 0.84762904 rad, longitude 0.31724117 rad. Height above ellipsoid 10958 m and altitude 10918 m. Geoid curl 40 m. We designed 5 LO motion models to simulate movements of aviation objects. We have simulated situations where users have left each other. The distance between users is between 10 km and 100 km: De (10, 100). We modeled straight-line LO flights with a length of 375 seconds. At the same time, we assume that there is an error measuring distance from all four broadcast sources. For distance measurement errors:

$$
\Delta d_1 \neq 0 \wedge \Delta d_2 \neq 0 \wedge \Delta d_3 \neq 0 \wedge \Delta d_4 \neq 0,
$$

where:

- $\Delta d_1$ is the error of measuring the distance of an unknown user LO1 from LO2;
- $\Delta d_2$ is the error of measuring the distance of an unknown user LO2 from LO3;
- $\Delta d_3$ is the error of measuring the distance of an unknown user LO3 from LO4;
- $\Delta d_4$ is the error of measuring the distance of an unknown user LO4 from LO1;

We assume that during the simulation time synchronization has been compromised between all users. Therefore, the information about the coordinates of each user was loaded with an error. Since we do not know how to fetch a time error directly into our algorithms, we replaced it with the X, Y, Z for each LO against user error. Coordinate errors $\Delta x$, $\Delta y$, and $\Delta z$ were generated by a random number generator with a normal distribution and these parameters: mean value equal to zero, dispersion equal to one. The magnitude of the generated coordinate errors was gradually changed by multiplying them by the constant to the following:

$$
X_{LOi} = k \cdot [\Delta x_i, \Delta x_i, ..., \Delta x_i];
Y_{LOi} = k \cdot [\Delta y_i, \Delta y_i, ..., \Delta y_i];
Z_{LOi} = k \cdot [\Delta z_i, \Delta z_i, ..., \Delta z_i], \text{ where } k=0.1
$$

$$
X_{LOi} = k \cdot [\Delta x_i, \Delta x_i, ..., \Delta x_i];
Y_{LOi} = k \cdot [\Delta y_i, \Delta y_i, ..., \Delta y_i];
Z_{LOi} = k \cdot [\Delta z_i, \Delta z_i, ..., \Delta z_i], \text{ where } k=0.25
$$

$$
X_{LOi} = k \cdot [\Delta x_i, \Delta x_i, ..., \Delta x_i];
Y_{LOi} = k \cdot [\Delta y_i, \Delta y_i, ..., \Delta y_i];
Z_{LOi} = k \cdot [\Delta z_i, \Delta z_i, ..., \Delta z_i], \text{ where } k=0.5
$$

$$
X_{LOi} = k \cdot [\Delta x_i, \Delta x_i, ..., \Delta x_i];
Y_{LOi} = k \cdot [\Delta y_i, \Delta y_i, ..., \Delta y_i];
Z_{LOi} = k \cdot [\Delta z_i, \Delta z_i, ..., \Delta z_i], \text{ where } k=1
$$

$$
X_{LOi} = k \cdot [\Delta x_i, \Delta x_i, ..., \Delta x_i];
Y_{LOi} = k \cdot [\Delta y_i, \Delta y_i, ..., \Delta y_i];
Z_{LOi} = k \cdot [\Delta z_i, \Delta z_i, ..., \Delta z_i], \text{ where } k=2
$$

Some simulation results are shown in Fig. 4 and in Table No. 2. In Fig. 4 shows errors of determination of the coordinates x, y, z unknown FO 5. The last graph shows the position error of the unknown FO 5. Simulation parameters: $k = 0.1$ and $D = 20.0$ km. The simulation results show that the accuracy of the relative navigation system that operates in the air communications network depends on the geometry of the system. The accuracy is worsening from the increasing distance between network users. See the dispersion of the LO positioning error in table 2.

![Figure 3. Layout of aircraft in airspace from flightradar24.](image-url)
ensure the synchronization of this network with the accuracy of the nanosecond series. One possibility to ensure synchronization of the air communications network is to use the currently available atomic norms. It is clear from the simulation results that the accuracy of the relative navigation system also depends on the geometry of the air traffic network. At greater distances between FO, the positioning error substantially increases. Modeling has confirmed that at the distance of an unknown FO from other airline communications network users that is greater than 30.0 km, the precision of determining its position is inadequate.

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4 CONCLUSION

One of the basic requirements for positioning LO is accuracy. In this paper, we describe various LO positioning methods with emphasis on their accuracy. The different methods differ from one another in what the electromagnetic wave (propagation signal) parameter is measured. These methods can be used to create new navigation systems, whether satellite or alternate. The main focus is on the long-distance

FO positioning method that works in the air communications network. The analysis made shows that if we want to determine the FO position that works in the air communications network, we need to know the positions of at least four other FOS from that network. It is clear from the simulation results that the distance method is suitable for determining the FO position. It is used to determine absolute position as well as relative positioning.

The exact positioning of this method is to maintain the maximum time synchronization of time base individual users who work in the air communications network. The accuracy of the satellite-level synchronization is satisfactory because the satellites have precision atomic clocks and the entire synchronization is ensured by a control segment that is bound to world reference time norms. In the Aviation Communications Network, it is necessary to