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Consideration of a General Model of Stochastic Service Curve for Communication Networks

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ABSTRACT: In this paper, a general framework for modelling stochastic service curves for communication networks is presented. It connects with each other two approaches to the traffic analysis and performance evaluation of communication systems, namely, the one which is called a deterministic network calculus with its stochastic counterpart. Thereby, it enables to treat any communication traffic in a consistent way. Further, as we show here, it enables also achieving new results. Derivation of the above model is presented in details in this paper. Finally, some hints are given how the method presented in this article could be applied in maritime telecommunications area.

1 INTRODUCTION

The so-called network calculus as defined and developed by the authors of publications cited in References at the end of this paper is a theoretical framework for analysis of performance guarantees in communication networks. And performance is a key criterion for any system. Further, system designers tend to produce systems with the highest performance at the lowest cost. Communication systems share resources and are very hard to forecasting.

The network calculus framework (Le Boudec J.-Y. & Thiran P. 2012) offers an environment for consideration of computer networks in terms of the worst case analysis. This method is based on the so-called min-plus algebra (Le Boudec J.-Y. & Thiran P. 2012). And in real-time systems, the worst case analysis proves to be more useful than the calculations that are based on averaging probabilistic parameters characterizing queues. This concerns mostly the cases where we want to know the boundaries for the so-called Quality of Service (QoS) parameters of a network in order to predict its critical behavior. However, there are also cases where a more sophisticated and detailed evaluation of a network is needed. Then, the usual network calculus as mentioned above, which is also called a deterministic network calculus, must be evolved and widened. This is done within a framework that is called a stochastic network calculus (Jiang Y. 2006). And a stochastic service curve plays a fundamental role in it, similarly as a deterministic service curve in the deterministic network calculus. This paper presents a general model of the stochastic service curve for communication networks in a new perspective.

The remainder of the paper is organized as follows. Section 2 introduces the concept of a stochastic service curve. In Section 3, we discuss a method of evaluation of the available bandwidth in a network through the service curve estimation. In the next section, a new approach to modelling of stochastic service curves for communication networks is presented. Finally, Section 5 concludes the paper as well as provides some hints on how the method presented in this article could be applied in maritime telecommunications area.

2 STOCHASTIC SERVICE CURVE

As already mentioned, a fundamental tool in the network calculus framework is a service curve. Further, note that in computer networks the traffic service in a node (as well as on an end-to-end path) is a stochastic process. The randomness of flows results from the fact that nodes simultaneously support different kinds of traffic derived from different networks as well as from different nodes of a network considered. From the reason of traffic aggregation, nodes experience a cross traffic, which depends on time and is transmitted at different speeds. In addition, a technology (for example radio communication), random actions and preferences of users influence the randomness. And, see that a stochastic service curve models very well the phenomena mentioned above. Furthermore, it describes the bounds for traffic streams in a system and is able to take into account the events and behavior described above.

A stochastic service curve can be defined as a non-random function with an error function describing the probability of violating performance boundaries or as a random process.

In the literature, there are a few definitions of the stochastic service curve. The definition of Cruz (Cruz R. L. 1996), which describes the stochastic service curve as a non-random function, is considered to be a basic one.

DEF1: Definition of the stochastic service curve after Cruz is the following. Let denote $A(t)$ an arrival function applied to a system and $D(t)$ the related departure function. And we assume that they represent the corresponding random processes. Further, we say that the system offers a *stochastic service curve* $S(t)$ if for $t \geq 0$ and $k \geq 0$

$$P\left\{D(t) < A \otimes S(t) - k\right\} \leq \varepsilon(k), \qquad (1)$$

where $\varepsilon(k)$ is called a deficit profile (or an error function) that represents the probability of violating restrictions imposed on the service. In the above expression, the symbol $\otimes$ means the convolution operator in the min-plus algebra, $P\{event\}$ stands for the probability of an event indicated, t is a time variable, and k denotes a modelling parameter of the error function $\varepsilon(k)$ mentioned above

It can be shown that all the other published definitions of stochastic service curves for systems governed by a single traffic stream (single-input systems) - as non-random functions - can, in principle, be derived from the Cruz's definition given above. They can be obtained through bigger or smaller modifications of this definition. And then, they are called differently as, for example, a statistical or an effective service curve (Burchard A., Liebeherr J., Patek S. D. 2002), (Burchard A. & Liebeherr J. & Patek S. 2006). In these works, a probabilistic version of the service curve (that is a stochastic one) is defined as follows:

DEF2: A non-decreasing and non-negative function S is named an *effective service curve* with the probability of violation ε if for $t \geq 0$ we have

$$P\left\{D(t) \geq A \otimes S(t)\right\} \geq 1 - \varepsilon, \qquad (2)$$

where an arrival curve $A(t)$ and the related departure curve $D(t)$ in the time period $[0,t]$ are the representatives of the corresponding random processes.

The so-called *leftover service curve* (Fidler M. 2010) is used in the traffic analysis and calculations, when a traffic system possesses two or more inputs. That is in cases of consideration of multi-input traffic systems.

The leftover service curve plays an important role in many performance analyses of networks as shown, for example, in (Fidler M. 2010) and other references referred to as in this paper. However, it provides a rather pessimistic estimation of the service available to a stream in a system that is also crossed by some other streams (that is such a one that experiences the so-called cross traffic). Also, it is worth noting here that there occur in the literature a few variants of this curve (Ciucu F., Burchard A., Liebeherr J. 2005), (Li C., Zhang S., Wang W. 2013), (Burchard A., Liebeherr J., Patek S. D. 2002), (Li C., Burchard A., Liebeherr J. 2003), and (Burchard A., Liebeherr J., Patek S. D. 2006).

Let us now illustrate, in more detail, the concept of the above curve exploiting an example. To this end, we use Fig. 1 that shows two competing streams: F_1 and F_2, where a server named here S_2 schedules the aggregated traffic, while an another server named S_1 supports only the flow F_1. In order to calculate a rate of the flow F_1 on the path of the connected nodes in Fig. 1, we need to know how much service received the F_1 flow on the S_2 server.

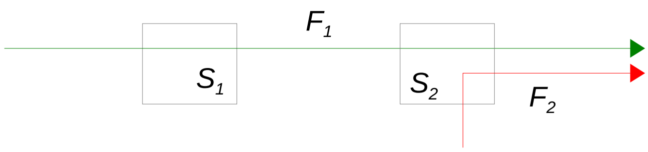

Figure 1. The traffic flowing through a network of interconnected n nodes (here n=2).

Finally at this point, we note that for the calculations indicated above we can use one of the theorems published in the literature that regard the leftover service curve. For example, we can utilize a theorem that was formulated and proved in (Xie J. 2011).

Note now that to make the leftover service curve a more optimistic performance measure in analyses of multi-input networks we can proceed similarly as in the case just discussed of the usual service curve. We can simply do this by expressing the leftover service curve in terms of the probability theory applied along the lines of DEF1 and DEF2.

Without going into details of partly troublesome and sophisticated considerations related with the notion, definition, and derivation of the corresponding expressions describing the so-called *stochastic leftover service curve*, presented in (Ciucu F., Burchard A, Liebeherr J. 2006) and (Ciucu F. 2007), we say about, here, only what follows.

DEF3: Let the cross-traffic to a given traffic system can be bounded by the so-called path envelope denoted here as $E^c(t)$ with an overflow profile $\varepsilon(k)$. Then, this system offers a stochastic leftover service curve $S^l(t)=\left[S(t)-E^c(t)\right]^+$ to its through-traffic with $\varepsilon(k)$ meaning the deficit profile $\varepsilon(k)$ occurring in (1) that is applied to the through-traffic of the system considered. Moreover, the symbol $\left[x(t)\right]^+$ in the expression above denotes the operation of finding a maximum value in the set $\{x(t),0\}$ for each time instant. Furthermore, with regard to the topic of traffic envelopes, note that a detailed material on this stuff is presented, for example, in (Le Boudec J.-Y., Thiran P. 2012), (Fidler M. 2010), and (Jiang Y. 2006).

Example definition of the stochastic service curve assumed to be a random process can be found in (Ciucu F. 2007). And for its formulation, we need to define first a process that is called an almost surely ordering of random varables.

DEF4: After (Ciucu F. 2007), we say that a random variable X is almost surely smaller than a random variable Y , and we write this as $X \le Y$ *a.s.*, if $P(X>Y)=0$.

Second, we must use another definition of the convolution operation in the definition of the stochastic service curve. It is different from the standard one that is used in the min-plus algebra, i.e. different from the one which was denoted by $\otimes$ in the considerations above. It can be defined as follows (Ciucu F. 2007).

DEF5: Let us denote representatives of two random doubly-indexed processes $B(t_1,t_2)$ and $G(t_1,t_2)$ as $b(t_1,t_2)$ and $g(t_1,t_2)$, respectively. Then, their convolution, named an "indexed" one to distinguish it from the previous one, is defined as

$$b\otimes_i g(u,t)=\inf_{u\le s\le t}\left(b(u,s)\ +\ g(s,t)\right) \tag{3}$$

for $0\le u\le t$. Note also that because of the reasons mentioned above the convolution symbol used in (3) is slightly different than before, namely $\otimes_i$.

And finally now, we are able to define the *stochastic service curve as a random process*. It is called also the *statistical service curve with a.s. ordering* and its definition after (Ciucu F. 2007) is the following.

DEF6: A nonnegative, doubly-indexed random process $S(t_1=s,t_2=t)$ can be regarded as a service curve (in the sense of a random process), when for an arrival process $A(t)$ (which needs to be re-indexed to $A(t_1=u,t_2=s)$), the corresponding departure process $D(t)$ satisfies

$$D(t)\ge A\otimes_i S(t)\quad a.s. \tag{4}$$

for all $t\ge 0$ and after applying $u=0$ there.

It is also worth noting at the end of this section that using the concept of the *a.s.* ordering *the leftover service curve as a random process* can be also formulated. This was done Fidler in (Fidler M. 2006).

3 AVAILABLE BANDWIDTH ESTIMATION

One of the key problems of ensuring high quality of services provided in packet networks is the estimation of bandwidth available between the sender and the recipient. Formally, the available bandwidth *B* on a route at time *t* means that unused bandwidth, which can be utilized by an application without any influence of the transmission quality of flows occurring on this route. The available bandwidth depends on time *t.* If there occur *n* nodes on an end-to-end path, then the available bandwidth *B* on this path at a time *t* is given by

$$B(t)=\min_{i=1,\ldots,n}\left\{B_i(t)\right\}, \tag{5}$$

where B_i means an available bandwidth in the *i*-th node.

Knowledge about available bandwidth on an end-to-end path may be useful for the correct operation of many network applications as, for example, VoIP, Audio/Video, P2P, network games or services on demand. Estimation of the available bandwidth can be also very useful in the process of selecting a route in overloaded networks or verifying the s-called Service Level Agreements (SLAs). Transmission conditions on an end-to-end path can change dynamically and in an unpredictable way, so estimating the available bandwidth a priori is not an easy task. There are two classes of methods for estimating the available bandwidth: active and passive ones (Liebeherr J., Fidler M., Valaee S. 2010). Active methods involve sending traffic samples and analyzing their statistics after reaching the destination. Among many tools available for this purpose are, among others, the following ones: IGI, Pathload, pathChirp - description of these methods can be found in (Strauss J., Katabi D., Kaashoek F. 2003). However, all these methods just mentioned require performing an installation on the both sides of a tested path, which is not always possible. Passive measurements, on the other hand, involve capturing traffic in a working network and then analyzing it. Although it is a fast and light-weight method while conducting the analysis one should remember about the changing network conditions and the influence of buffers, control mechanisms, and cross flows on the analyzed flow.

In (Liebeherr J., Fidler M., Valaee S. 2010), an available bandwidth estimation method that utilizes a service curve estimator $\tilde{S}$ was conceived. And the service curve estimator $\tilde{S}$ derived in (Liebeherr J., Fidler M., Valaee S. 2010) is given by the following formula:

$$\tilde{S}=D^P \oslash A^P\ , \tag{6}$$

where $D^P(t)$ means a traffic departure function and $A^P(t)$ is a traffic arrival function. The latter is understood a stream that comes from a traffic trace of one or more flows. Both the functions describe a cumulative traffic that is they are sums of bits in the outgoing and incoming traffic, respectively, in the time period $[0,t]$. Note also the use of the superscript "*P*" in the symbols $D^P(t)$ and $A^P(t)$ to indicate that these are network data probes. Moreover, the symbol $\oslash$ in (6) stands for a deconvolution operator that is defined in the sense of the min-plus algebra as

$$f \oslash g(t) = \sup_{\tau \geq 0}\left\{f(t+\tau) - g(\tau)\right\} \tag{7}$$

for functions *f* and *g*. Liebeherr J., Fidler M., and Valaee S. in their publication (Liebeherr J., Fidler M., Valaee S. 2010) shown that the estimator $\tilde{S}$ given by (6) can be viewed as the best possible estimate of an actual service curve that can be obtained from the available measurements of A^P and D^P.

4 GENERAL FRAMEWORK FOR STOCHASTIC SERVICE CURVE MODELLING

The Internet environment is changing dynamically. Because of this reason and also due to traffic aggregation and multiplexing, any deterministic type of estimation gives weak results. Furthermore, the cross traffic is always present in real networks. Therefore, because of the reasons mentioned above, when we calculate a service curve for any node and/or any traffic system working in a real environment, it will be and should be considered as a one understood in the sense of the stochastic leftover service curve or a related one. And once again, we remind here that just this type of the service curve shows how much bandwidth actually is left for the through traffic in a network.

Furthermore, the randomness of the service curve follows clearly from what was said at the beginning of this section. That is it must be considered as a stochastic process. And, in this context, the definition DEF6 referred to as above only supports this fact.

Novelty of our approach to modelling the stochastic service curve, which we present in this paper, relies, first of all, upon an assumption that a service curve as a random process possesses generally two parts. One of them is deterministic and the second strictly stochastic.

Second, we treat here representatives of the service curve random process as time series and utilize widely tools that are available in the literature for their analyses. Our observations of time series related with service curves show that principally the following components: (a) a deterministic trend, (b) a strictly stochastic part, and (c) random disorders can be recognized in them. In what follows, we present a general model of the stochastic service curve which is based on the above findings.

Suppose $S_s(t)$ and $S_b(t)$ are random processes, and $S_d(t)$ is a (deterministic) function. Then, a stochastic service curve as a random process $S_I(t)$, which combines - through an addition operation - the two processes mentioned above, assumes the following form:

$$S_I(t) = S_d(t) + S_s(t) + S_b(t), \tag{8}$$

where $S_d(t)$ stands for a deterministic part of the service curve, $S_s(t)$ means its stochastic part, and $S_b(t)$ represents a random errors, noise etc.

Note that it is also possible to model the stochastic service curve in a multiplicative way, when the processes $S_s(t)$ and $S_b(t)$ are multiplied with each other. Then, the form of the "multiplicative" stochastic service curve $S_{II}(t)$ assumes the following form:

$$S_{II}(t) = S_d(t) \cdot S_s(t) + S_b(t). \tag{9}$$

Observe also that expressions (8) and (9) simplify to

$$S_I(t) = S_d(t) + S_b(t) \tag{10}$$

and

$$S_{II}(t) = S_d(t) \cdot const + S_b(t), \tag{11}$$

respectively, in the absence of a strictly stochastic component in a service curve. In (11), *const* means a constant.

Further, note that in publications regarding time series analyses the components $S_d(t)$ and $S_s(t)$ occurring in formulas (8) and (9) are called a deterministic trend and a stochastic trend, respectively.

As well known, any stochastic process (in our case a one-dimensional one) is characterized by giving the corresponding probability distributions (which can change with the passing of time). Equivalently, other parameters as, for example, moments or the characteristic function of a random variable (which in our case is dependent upon time) can be provided. However, this means of a stochastic process characterization is oft too complicated from the practical point of view, and therefore rather not used in the network calculus.

Sometimes, however, it is enough to know only one representative of a given stochastic process. That is a one time series that characterizes its behavior. It can, for example, be obtained by performing measurements. In what follows, we describe how to obtain such a time series for the stochastic service curve $S_I(t)$. Note however that this is not an obvious task because in fact this series is hidden in measured data. That what is available are measured data regarding the input and output flows of bits to a system (or node). And just to obtain the time series of $S_I(t)$, we need to carry out a processing these data. To this end, we will use the Liebeherr, Fidler and Valaee method (called here shortly the LFV method), which was already mentioned and cited above (Liebeherr J., Fidler M., Valaee S. 2010).

In the setting considered in this section, the input and output traffic to a node or a system, which are represented in (6) by the cumulative arrival $A^P(t)$ and departure $D^P(t)$ functions, respectively, are considered to be time series associated with (representatives of) the stochastic processes $A^P(t)$ and $D^P(t)$. And note that just this understanding and interpretation allows us to apply (6) to obtain from it a time series of $S_I(t)$. Then simply, (6) describes a relation existing between the three time series indicated above.

Now, in more detail, we calculate an estimator of the time series of $S_I(t)$ (being an representative of the stochastic process $S_I(t)$) using (6). That is the following:

$$\tilde{S}_I(t) = D^P \oslash A^P(t). \tag{12}$$

The question how reliable is the estimator $\tilde{S}_I(t)$ given by (12) in evaluation of the available and/or utilized bandwidth in a traffic node or a traffic system is not trivial. This was noted and also discussed in (Wasielewska K. 2014) and (Wasielewska K, Borys A. 2019). Here, however, we do not consider this topic.

It has been shown in (Liebeherr J., Fidler M., Valaee S. 2010) that for linear systems (that is for those in which $D^P = A^P \otimes S$ is satisfied) the following relation:

$$\tilde{S}(t) \le S(t) \tag{13}$$

holds for each value of *t*. In (13), S means the system's service curve. So, after applying this result in our case, we get

$$\tilde{S}_I(t) \le S_I(t). \tag{14}$$

It follows from (14) that $\tilde{S}_I(t)$ is a lower service curve in the sense of a lower time series for a one of the unknown time series which describe the system's service curve understood as a random process.

It can be shown (Liebeherr J., Fidler M., Valaee S. 2010) that when a traffic node or a traffic system behaves nonlinearly, then determining of $\tilde{S} = D^P \oslash A^P$ gives only a lower bound for an upper service curve $\bar{S}$ defined by the following relation:

$$D \le A \otimes \bar{S} \quad . \tag{15}$$

So, in this case, we will have

$$\tilde{S}(t) \le \bar{S}(t) \tag{16}$$

satisfied for each value of *t*. And after applying the latter result in our considerations regarding the notion of a service curve represented by one of its time series, we obtain

$$\tilde{S}_I(t) \le \bar{S}_I(t). \tag{17}$$

See that according to (17) $\tilde{S}_I(t)$ represents a lower bound for a one of the unknown upper bound time series which describe (indirectly) the system's service curve understood as a random process.

Let us now return to consideration of a traffic system that behaves linearly. And, let ΔS mean the difference between the system's real service curve and its estimator. That is

$$\Delta S(t) = S_I(t) - \tilde{S}_I(t) \ge 0. \tag{18}$$

for each value of *t*. Further, note that then $S_e(t) = \Delta S(t)$ can be considered as a systematic error related to the approximation operation. Therefore, we can write

$$S_I(t) = \tilde{S}_I(t) + S_e(t). \tag{19}$$

And substituting (19) into (8) gives

$$\tilde{S}_I(t) = S_d(t) + S_s(t) + S_b(t) - S_e(t). \tag{20}$$

The components $S_b(t)$ and $S_e(t)$ are random, therefore we rewrite (20) as

$$\tilde{S}_I(t) = S_d(t) + S_s(t) + S_{be}(t), \tag{21}$$

where $S_{be}(t) = S_b(t) - S_e(t)$. And this result means that $\tilde{S}_I(t)$ is an estimator of $S_d(t) + S_s(t)$. So, let us rewrite (21) as

$$\tilde{S}_I(t) \cong S_d(t) + S_s(t). \tag{22}$$

Finally, (22) shows that after calculating the service curve estimator $\tilde{S}_I(t)$ according to the formula (12), one should separate, from each other, the deterministic trend $S_d(t)$ from the stochastic trend $S_s(t)$ in the series obtained (that is in $\tilde{S}_I(t)$). In fact, they will form their estimates $\tilde{S}_d(t)$ and $\tilde{S}_s(t)$, respectively.

Note now that we can proceed similarly in the case of considering a nonlinear traffic system. Therefore, the results and interpretations will be then also similar. Only difference will regard the estimator $\tilde{S}_s(t)$. Here, it will be interpreted as the estimator of the stochastic part of the time series $\bar{S}_I(t)$.

Observe further that we do not know whether a traffic node or a traffic system which has a service curve that is a random process behaves linearly or nonlinearly. In other words, we do not know which of the following three relations: $D^P(t) = A^P \otimes S_I(t)$, $D^P(t) < A^P \otimes S_I(t)$, and $D^P(t) > A^P \otimes S_I(t)$ actually holds for an actually analyzed representative (time series) of the random process $S_I(t)$, in a given moment *t*. And obviously, the changes between these two working regimes of the system analyzed, which are mentioned above, will be random. That is we will have periods of time in which the system will behave as a linear one and also such ones in which it will behave nonlinearly. We also emphasize here the fact that even the same representative of the random process $S_I(t)$ will show the linear as well as the nonlinear behavior.

Of course, the behavior of all the representatives of the random process $S_I(t)$ will be similar. So, for the same traffic node or a traffic system, they will show different periods of linear and nonlinear behavior. That is in the sense that their lengths and times of occurrence will be random.

Note now that it follows from the above remarks and the previous considerations that when we calculate the estimators $\tilde{S}_I(t)$ using the formula (6), we in fact do not know whether it is in the sense of the estimator of the process $S_I(t)$ or of the process $\bar{S}_I(t)$. So, it follows from the above that the estimator $\tilde{S}_I(t)$ for a given traffic system achieved in our calculations will be partly more accurate and partly less accurate. An obviously, we will not know in which periods the former parts will occur and where the latter ones. Simply because their occurrences will be random.

Finally in this section, we would like to say that the model presented here has been intensively exploited and validated in (Wasielewska K. 2014). There have been presented many results of simulations which prove its usefulness.

5 CONCLUSIONS

A general novel framework for modelling stochastic service curves for communication networks has been proposed in this paper. It relies upon exploiting some basic tools of the theory of stochastic processes as well as the tools developed in the literature for analysis of time series. All the definitions of stochastic curves, which have been formulated and discussed in the literature, and which are also referred to as in this paper, can be derived along the lines of our novel approach presented in this article.

We will continue the theme of this paper. Mainly because of the fact that the methods of the stochastic network calculus are very promising in solving many traffic problems we are interested in as, for example, those which occur in the areas of the so-called advanced autonomous waterborne applications and remotely controlled ships. In our view, they are also very important for optimization of remote control systems for vehicles.

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