Analysis of the Effect of Time Delay on the Integrated GNSS/INS Navigation Systems

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ABSTRACT: The performance of tightly coupled GNSS/INS integration is known to be better than that of loosely coupled GNSS/INS integration. However, if the time synchronization error occurs between the GNSS receiver and INS (Inertial Navigation System), the situation reverses. The performance of loosely coupled GNSS/INS integration and tightly coupled GNSS/INS integration is analyzed and compared due to time synchronization error by computer simulation.

1 INTRODUCTION

GNSS (Global Navigation Satellite System) and INS (Inertial Navigation System) are widely used as standalone navigation systems, respectively. The integration of GNSS and INS not only leads to high accuracy of vehicle’s position, velocity, and attitude, but also provides position error bound. To maximize the performance of GNSS/INS integrated systems, time synchronization between GNSS and INS is an important issue.

There has been some research for time synchronization of GNSS/INS measurements for loosely coupled GNSS/INS systems or tightly coupled GNSS/INS systems. However, there are few research to consider both. This paper compares the time delay effect of loosely coupled GNSS/INS systems and tightly coupled GNSS/INS systems. For loosely coupled GNSS/INS systems, we analyze the effect of time delay for the case when only position information is used as Kalman filter measurement. For tightly coupled GNSS/INS systems, we analyze the effect of time delay for the case when only pseudorange information is used as Kalman filter measurement.

To compare the performance of the two integration systems, it is analytically studied how the time delay between GNSS and INS has effect on the Kalman filter innovation for each integration method. Then based on the analysis results, computer simulations are performed to check how each integration method has effect on the navigation performance such as position, velocity, and attitude of the vehicle. The two GNSS/INS integration methods are reviewed briefly in section 2, and the effects of time delay are analyzed in section 3. Computer simulations are performed in section 4 and conclusions are given in section 5.

2 INTEGRATED GNSS/INS NAVIGATION SYSTEMS

Several approaches are possible for the integrated GNSS/INS systems depending on which information is shared between GNSS and INS. Loosely coupled and tightly coupled integrated systems are considered.
in this paper. The position and velocity calculated from GNSS are used to update the INS filter in the loosely coupled integration, while raw pseudorange is used in the tightly coupled integration for Kalman filter.

2.1 Loosely coupled GNSS/INS navigation system

The block diagram of the loosely coupled GNSS/INS integration is shown in Figure 1. The position and velocity is calculated in the GNSS receiver and then used as a measurement in the INS Kalman filter.

As seen in Figure 1 the structure is simple and modularization is possible. The computation time is reduced since the algorithm in the GNSS receiver can be used. The Kalman filter in the loosely coupled GNSS/INS integration has the error model with dimension of 15.

![Figure 1. Loosely coupled GNSS/INS integration](image)

2.2 Tightly coupled GNSS/INS navigation system

The block diagram of the tightly coupled GNSS/INS integration is shown in Figure 2. The GNSS pseudorange and pseudorange rate are used directly in the measurement of Kalman filter and the clock error of the GNSS receiver should be included in the state variable to be estimated. Thus the Kalman filter error model contains position error, velocity error, attitude error, accelerometer bias, gyroscope bias, clock bias and clock drift of GNSS receiver, resulting in 17 state variables as in (4).

\[
\begin{align*}
\dot{x}_{\text{INS}} &= F_{\text{INS}} x_{\text{INS}} + w_L, \quad w_L \sim N(0, Q_L) \\
 x_{\text{INS}} &= [\delta l, \delta \omega, \delta \delta h, \delta \nu, \Phi, B_{\text{acc}}, B_{\text{gyro}}]^T
\end{align*}
\]

The state variable contains the position error with latitude, longitude, and height \((\delta l, \delta \omega, \delta \delta h)\), and velocity error \((\delta \nu)\), attitude error \((\Phi)\) expressed in the navigation frame, and bias errors \((B_{\text{acc}}, B_{\text{gyro}})\) of accelerometer and gyroscopes(refer to Knight 1997 for detail error model). The measurement equation of the loosely coupled GNSS/INS integration can be denoted as follows:

\[
\begin{align*}
\dot{x}_{\text{INS}} &= F_{\text{INS}} x_{\text{INS}} + w_L, \quad w_L \sim N(0, Q_L) \\
 x_{\text{INS}} &= [\delta l, \delta \omega, \delta \delta h, \delta \nu, \Phi, B_{\text{acc}}, B_{\text{gyro}}]^T
\end{align*}
\]

(1)

(2)

Where \(\delta c_{\text{bias}}, \delta c_{\text{drift}}\) denotes for clock bias and clock drift of GNSS receiver and \(F_{\text{INS}}\) \& \(F_{\text{clock}}\) are described in Knight 1997.

The measurement is given as in (6).

\[
z = [p_{\text{INS}}] - [p_{\text{GPS}}] = H_L x_{\text{INS}} + v_L, \quad v_L \sim N(0, R_L)
\]

(3)

Where \(H_L = [I_{3x3}, 0_{3x9}].\)

In the case of loosely coupled integration there are some drawbacks. When there are short of visible satellites, navigation information cannot be calculated in the GNSS receiver. Under the large dynamic environment GNSS navigation solution becomes inaccurate, thus the integration performance becomes worse.

![Figure 2. Tightly coupled GNSS/INS integration](image)
3 EFFECTS OF THE TIME DELAY IN THE INTEGRATED GNSS/INS SYSTEM

Figure 3. Structure of time synchronization error Td(=Td2 - Td1) in the integrated GNSS/INS navigation system

For the GNSS/INS integration there exist time delay between GNSS receiver and INS since not only sampling time but also the signal processing time is different as in Figure 3. For simplified analysis, the GNSS receiver sampling period is assumed to be a multiple M of the IMU sampling period Td. Td1 and Td2 are processing time delays of the inertial navigation system(INS) and the GNSS receiver, respectively and Td=Td2 - Td1, i.e., the difference of the INS and GNSS processing time.

3.1 EKF algorithm

Consider the error model of INS and EKF equation to analyze the effects of time delay in measurement. The error model can be described as follows:

\[ x_{k+1} = \Psi_k x_k + G_k e_k \]  
\[ z_k = H_k x_k + w_k \]  

(7)  
(8)

Here, \( \Psi_k \) is the error state transition matrix, \( e_k \) is the process noise, \( G_k \) is the process noise gain, \( H_k \) is the measurement matrix, and \( w_k \) is the measurement noise. The subscript \( k \) implies \( k \)-th sampling sequence. The process noise \( e_k \) and the measurement noise \( w_k \) are considered as white noise, uncorrelated with each other and the covariance matrices are \( R = \text{E}\{w_k w_k^T\} \) and \( Q = \text{E}\{e_k e_k^T\} \), where, \( \text{E}\{ \} \) denotes the expectation operator.

The EKF algorithm for the GNSS/INS integration is given in Table 1.

| Table 1. EKF Algorithm for Integration between the GNSS and the INS |
|------------------|------------------|
| **Kalman gain update** | \( K_k = P_k H_k^T (H_k P_k H_k^T + R)^{-1} \) |
| **Difference of the two measurements** | \( z_k = \hat{p}_{k, \text{INS}} - \hat{p}_{k, \text{GNSS}} \) |
| **Estimation of the state errors** | \( \hat{x}_k = \delta \hat{r}_k \) |
| **Error correction** | \( \hat{r}_k \) \[\begin{bmatrix} \hat{r}_k \\ \hat{e}_k \end{bmatrix} = \hat{r}_k \left[ \begin{bmatrix} \delta \hat{r}_k \\ \delta \hat{e}_k \end{bmatrix} \right] + \hat{x}_k \) |
| **Covariance update** | \( P_k = (I - K_k H_k) P_k^{-1} \) |
| **Sensor error compensation** | \( \hat{u}_k = g(\hat{u}_k, \hat{e}_k) \) |
| **Navigation equation update** | \( \hat{r}_{k+1} = f(\hat{r}_k, \hat{u}_k) \) |
| **Sensor error update** | \( \hat{e}_{k+1} = \hat{e}_k \) |
| **Covariance prediction** | \( P_{k+1} = \Psi_k P_k \Psi_k^T + G_k Q G_k^T \) |

3.2 Effects of time synchronization error

Vectors \( \hat{r}_k \), \( \hat{e}_k \), and \( \hat{u}_k \) in Table 1 denote the estimated navigation state, the estimated sensor errors, and the IMU measurements, respectively. Effects of time synchronization error between GNSS receiver and INS can be analyzed as follows.

Let the time difference of sampling measurement between GNSS receiver and INS be \( T_d \) and suppose that INS sampling has time synchronization error less than 1sec. Then in the case that vehicle position is used in loosely coupled integration, the estimated position in INS is given as (9), where \( p(t) \) means true trajectory.

\[ \hat{p}_{k, \text{INS}} = p(kT_d - T_d) \equiv p_k - v_k T_d + 0.5 a_k T_d^2 + w_k \]  

(9)

where \( |T_d| \leq 1 \), \( v_k \) is the velocity of the vehicle and \( a_k \) is the acceleration.

Suppose that the pseudorange, which is the measurement of tightly coupled integration, has time synchronization error less than 1sec, then pseudorange estimated in INS can be described as (10), where \( \rho(t) \) is true pseudorange.

\[ \hat{p}_{k, \text{INS}} = \rho(kT_d - T_d) \equiv \rho_k - \rho_k T_d + 0.5 \rho_k T_d^2 + w_k \]  

(10)

where the pseudorange rate is \( \dot{\rho}_k = (v_k \rho_{k, \text{GNSS}} - v_k) l_k \), pseudorange acceleration is \( \ddot{\rho}_k \) \( = (a_k \rho_{k, \text{GNSS}} - a_k) l_k \), \( l_k \) is LOS(line of sight) vector between satellite and vehicle, and \( v_k \rho_{k, \text{GNSS}} \) and \( a_k \rho_{k, \text{GNSS}} \) are velocity and acceleration of satellite, respectively.

Thus the true observation \( z_k^a \) of two integration systems are obtained as (11).
For loosely coupled integration

\[ z^a_k = \hat{p}_{k,\text{INS}} - \hat{p}_{k,\text{GNSS}} \]  
\[ \approx p_k - v_k T_d + a_k \frac{T_d^2}{2} + w_k - \hat{p}_{k,\text{GNSS}} \]  
\[(11a)\]

For tightly coupled integration

\[ z^a_k = \hat{p}_{k,\text{INS}} - \hat{p}_{k,\text{GNSS}} \]  
\[ \approx p_k - \hat{p}_{k,\text{INS}} T_d + a_k \frac{T_d^2}{2} + w_k - \hat{p}_{k,\text{GNSS}} \]  
\[(11b)\]

Let \( H_k x_k = p_k - \hat{p}_{k,\text{GNSS}} \) or \( H_k x_k = p_k - \hat{p}_{k,\text{INS}} \), then (11) becomes (12).

\[ z^a_k = H_k x_k + w_k + d_k \]  
\[(12)\]

In (12) the residual error vector of position or the residual error vector of pseudorange is introduced and defined as

\[ d_k = -v_k T_d + a_k \frac{T_d^2}{2} \quad \text{or} \quad d_k = -\hat{p}_{k,\text{INS}} T_d + a_k \frac{T_d^2}{2} \]  
\[(13)\]

In Skog & Händel (2011), the errors of the closed-loop system are found to be

\[ x_{k+1} = \Psi_k (x_k - \hat{x}_k) G_k e_k \]  
\[ = \Psi_k x_k - \Psi_k K_z z_k + G_k e_k \]  
\[(14)\]

Let \( K_{p,k} = \Psi_k K_{z} \), then (14) becomes (15).

\[ x_{k+1} = (\Psi_k - K_{p,k} H_k)x_k - K_{p,k} w_k + G_k e_k \]  
\[(15)\]

Here, by substituting \( z_k \) with \( z^a_k \) in (14) and inserting (12) into this, the following difference equation for the error state can be obtained:

\[ x_{k+1} = (\Psi_k - K_{p,k} H_k)x_k - K_{p,k} w_k + G_k e_k + K_{p,k} d_k \]  
\[(16)\]

If we introduce \( \Phi_k = \Psi_k - K_{p,k} H_k \), then the MSE(mean square error) of the navigation solution can be expressed as in (17).

\[ P_{k+1} = E\{x_{k+1}(x_{k+1})^T\} \]  
\[ = \Phi_k P_k \Phi_k^T + K_{p,k} R K_{p,k}^T + G_k Q G_k^T \]  
\[ + K_{p,k} d_k d_k^T K_{p,k}^T - \Phi_k \bar{x}_k d_k^T K_{p,k}^T - K_{p,k} d_k \bar{x}_k^T \Phi_k^T \]  
\[(17)\]

where the mean error \( \bar{x}_k = E\{x_k\} \) can be calculated as in (18).

\[ \bar{x}_{k+1} = \Phi_k \bar{x}_k - K_{p,k} d_k \]  
\[(18)\]

Notice the residual error vector \( d_k \) in (13). For the loosely coupled integration, time delay \( T_d \) has little effect on \( d_k \) in the case of small velocity and acceleration, i.e., \( d_k \approx 0 \). Thus for small velocity \( d_k \)

can be neglected in the loosely coupled integration and MSE value in (17) becomes

\[ P_{k+1} = \Phi_k P_k \Phi_k^T + K_{p,k} R K_{p,k}^T + G_k Q G_k^T. \]

However, for the tightly coupled integration, the residual error vector \( d_k \) does not become zero even though the velocity of vehicle is almost zero because of high speed of GPS satellite like 3.9km/sec, i.e.,

\[ d_k \approx -(v_{k,\text{GNSS}} T_d + (\alpha_{k,\text{GNSS}} T_d) \frac{T_d^2}{2} \]

Thus we can notice that time delay in the tightly coupled integration has more effect on the navigation performance than in the loosely coupled integration when velocity and acceleration of vehicle are not large.

4 SIMULATIONS

In this section computer simulation is performed to verify the analysis result for the effects of time delay. Table 2 shows the specification of INS and GPS used in the simulation. The vehicle trajectory is assumed as circle and run time is 200sec. Vehicle speed is 10km/sec, 200km/sec, and 400km/sec and the time delay is assumed to be 1sec in INS measurement.

<table>
<thead>
<tr>
<th>Error Sources</th>
<th>1-σ value</th>
</tr>
</thead>
<tbody>
<tr>
<td>INS initial position error</td>
<td>10m</td>
</tr>
<tr>
<td>initial velocity error</td>
<td>1m/sec</td>
</tr>
<tr>
<td>initial horizontal attitude error</td>
<td>0.03 deg</td>
</tr>
<tr>
<td>initial vertical attitude error</td>
<td>5 deg</td>
</tr>
<tr>
<td>accelerometer bias</td>
<td>500 μg</td>
</tr>
<tr>
<td>gyro bias</td>
<td>3 deg/hr</td>
</tr>
<tr>
<td>clock bias</td>
<td>10m</td>
</tr>
<tr>
<td>clock drift</td>
<td>1m/sec</td>
</tr>
</tbody>
</table>

Table 2. Specification of error sources of the INS and the GPS

Figure 4 shows the position errors in north direction and pitch errors in the loosely coupled GPS/INS according to the various vehicle speeds. Generally the position error and attitude error become large as the vehicle speed increase. This happens because the residual error vector in (13) becomes large according to the vehicle speed and thus the optimality of Kalman filter gets worse. The oscillation of the position error comes from the circle trajectory.
5 CONCLUSIONS

The estimation performance of loosely coupled GNSS/INS integration and tightly coupled GNSS/INS integration is compared due to time synchronization error between GNSS receiver and INS. If the time synchronization error occurs between two sensor measurements, residual error exists. For loosely coupled integration the residual error vector increases according to vehicle speed while for tightly coupled integration the residual error vector increases according to satellite speed as well as vehicle speed. The GPS satellite speed is about 3.9 km/sec, which is much larger than the vehicle speed. Thus residual error vector in tightly coupled integration depends mainly on the satellite speed.

Simulations are performed to verify the analysis result for the two GPS/INS integration methods. The simulation shows that time synchronization error has effect more on the tightly coupled integration than the loosely coupled integration.
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