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An Invariance of the Performance of Noise-Resistance of Spread Spetrum Signals

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ABSTRACT: The paper is a study on the invariance of the performance of noise-resistance with respect to the quasi-determined spectrum-concentrated interferences with transmitting of spread spectrum signals with a fixed algorithm of processing.

1 INTRODUCTION

One of the main characteristics determining the effectiveness of a radio communication system is the stability against disturbances [1,2]. It is characterized with the dependency of the fidelity of received communications on the line energy parameters, algorithms used to transmit information and statistical characteristics of interferences [1,2]. With discrete systems of connections, the error probability of distinguishing signals is used for fidelity assessment [3,4].

The modern radio communication systems with safety responsibility are required to guarantee that the error-probability given will not exceed the preliminary specified permissible value independently of the variability activity of the channel. In essence it means that the given quantity of the system functioning is achieved thanks to the independency (partial or complete) of the performance of the noise-resistance from reasons causing the non-stationary state of the channel of connection. In the theory of automatic control this ability of the system to oppose resist against the disturbing actions is known as invariance. In the systems of connection, the part of disturbing effects is played by different disturbances and noise-resistance is the feature of the system that is invariant to them.

2 DEFINING THE CONDITIONS OF INVARIANCE

For a radio channel, the typical situation is the one where the performance of noise-resistance is determined by the presence of disturbances of several classes (fluctuating, spectrum-concentrated, impulse). The functional kind of the expression of the error probability with receiving by elements depends on the sets of signal parameters, the disturbances and the interaction between them:

$$P = \left[\left\{\overline{h}_i^2\right\}, \left\{\overline{h}_\zeta^2\right\}, \overline{G}_{ij}^2\right], \tag{1}$$

where $\overline{h}_i^2$ and $\overline{h}_\zeta^2$ express the mean statistical properties of the ratios between the energies of the *i-th* signal variant and the *j-th* disturbance variant and the white noise spectral density.

The part of parameters of interaction between the signal and disturbances is played by set $\{\overline{G}_{ij}^2\}$, $i=1 \div n$, $j=1 \div n_\zeta$, average statistical values of the coefficients of the reciprocal differences in the frequency-and-time area of their structures.

As the degree of the interaction between the useful signal and the disturbance on the frequency-and-time plane is analogous to their mutual correlation function, it is suitable to assume the average statistical value of the mutual difference coefficient in the position of interaction between them. This value is expressed in the kind of:

$$\overline{G}_{ij}^{2} = \left[\frac{K_0 K_\zeta}{2 P_i T} \int_0^T \dot{S}_i(t) \Sigma_{\zeta_j}^{*}(t) dt \right]^2 , \qquad (2)$$

where K_0 , K_ζ , are the amplitude coefficients of the signal and disturbance, $T=\tau_0 N$ is the signal length, $\dot{S}_i(t)$ and $\Sigma_{\zeta_j}^{*}(t)$ are the complex functions of the *i-th* signal and *j-th* disturbance,

$P_i = \frac{K_0^2}{T} \int_0^T s_i^2(t) dt$ is the average power of the *i-th* signal variant.

The conditions of the invariance of the connection system are expressed in relation to a certain class of disturbances and in dependence with the metrics selected on the signal space.

If $n(t)$ and $\zeta(t)$ are random realizations of fluctuation noise {N} and quasi-determined interferences {Ξ} respectively, then the performance of noise-resistance is a function of both interferences:

$$P = P(N, \Xi,). \qquad (3)$$

The system of connection is absolutely invariant to $\zeta(t)$, if :

$$P(n,\zeta) = P(n,0) = P(n) \qquad (4)$$

is fulfilled.

When the noise-resistance characterization depends on interferences Ξ to a certain extent, e.g.:

$$|P(n,0) - P(n,\zeta)| \le \varepsilon , \qquad (5)$$

then the system is relatively invariant (invariant to ε), where ε presents the given distance between $P(n,\zeta)$ and $P(n,0)$:

$$\varepsilon = \max_{\zeta} |P(n,\zeta) - P(n,0)| \qquad (6)$$

3 STUDY ON THE INVARIANCE OF THE PERFORMANCE OF NOISE-RESISTANCE IN REGARD TO SPECTRUM-CONCENTRATED INTERFERENCES WITH COHERENT RECEIVING SSS.

Under the condition of the effect only of fluctuating white noise {N} the noise-resistance of the system is determined by the ratio of signal energy power W_1 to the spectrum density of white noise power ν_0^2 :

$$h^2 = \frac{W_1}{\nu_0^2} . \qquad (7)$$

With transmitting opposed signals and fixed ratio signal/noise, the optimal operator of the receiver that is to ensure maximum noise-resistance against the interference {N} is the algorithm of coherent receiving with probability of error [3]:

$$P = \frac{1}{2}\left[1 - \Phi\left(h\sqrt{2}\right)\right] , \qquad (8)$$

where Φ(.) is the integral function of distribution of Cramp.

With complicating the noise situation in the channel, when besides the fluctuating noise there are also effects caused by spectrum-concentrated interferences $\zeta(t)$, the probability of error is determined with independence [2]:

$$P = \frac{1}{2}\left[1 - \Phi\left(\sqrt{2} h_e\right)\right], \qquad (9)$$

where h_e considerably depends on the type of receiver and the frequency- and time properties of the signals processed and the influencing disturbances. For a receiver optimal for channels with fluctuation noise and working with the influence of spectrum-concentrated disturbances:

$$h_e^2 = \frac{h^2}{1 + \overline{h}_\zeta^2 G_{ij}^2} \qquad (10)$$

It is function $P(h)$ of the probability of error from parameter *h* that appears in the capacity of invariant of the system in relation to disturbance $\zeta(t)$. Taking into consideration dependencies (9) and (10), it is obtained for it:

$$P = \frac{1}{2}\left\{1 - \Phi\left[\sqrt{2} h \left(1 + \overline{h}_\zeta^2 G_{ij}^2\right)^{-\frac{1}{2}}\right]\right\}. \qquad (11)$$

The characteristic of noise-resistance obtained depends of the effect of $\zeta(t)$ and $P = f\left(\overline{h}_\zeta^2\right)$. Therefore:

$$P(h,\zeta) \ne P(h,0) , \; P(h) \ne in\,\mathrm{var}\,\zeta(t) ,$$

i.e. the condition of absolute invariance (4) to spectrum-concentrated disturbances has not been satisfied.

The Table 1 gives the values of the probability of error calculated according to dependency (10) for the cases when there are no concentrated disturbances $(\bar{h}_\zeta^2 G_{ij}^2 = 0)$ and when a disturbance of $\bar{h}_\zeta^2 = 10^2$ и $G_{ij}^2 = 10^{-3}$ is influencing.

From the analysis of the data in Table 1 it follows that the maximum increase of the probability of error in area $P \le 10^{-2}$ is $6{,}259.10^{-3}$. According to the condition of invariance (3) it follows that the relative invariance feature of noise-resistance up to $\varepsilon = 6{,}259.10^{-3}$ is available in respect to spectrum-concentrated interferences .

When the concentrated interferences are of uniform spectrums, the coefficient of reciprocal difference G_{ij}^2 from the signal basis can be expressed as [2]:

$$G_{ij}^2 = \frac{\rho}{F_i T} \qquad i=1,2, \tag{12}$$

where, with given $S_i(t)$ и $\Sigma_j(t)$, ρ is a constant quantity located in the interval:

$1 \le \rho \le F_\zeta T \le F_i T$,

as the left limit of the interval is valid for a sinus-like shape of disturbance. With $F_i = F = const$, dependency (10) takes the kind of:

$$h_e^2 = \frac{h^2}{1 + \frac{\rho \bar{h}_\zeta^2}{FT}} \tag{13}$$

Table 1. Values of the probability of error

P(h)	
$\bar{h}_\zeta^2 G_{ij}^2 = 0$	$\bar{h}_\zeta^2 G_{ij}^2 = 0.1$
0.5	0.5
0.079	0.089
0.023	0.028
$7.153\cdot10^{-3}$	$9.759\cdot10^{-3}$
$2.339\cdot10^{-3}$	$3.5\cdot10^{-3}$
$7.827\cdot10^{-4}$	$1.284\cdot10^{-3}$
$2.66\cdot10^{-4}$	$4.785\cdot10^{-4}$
$9.141\cdot10^{-5}$	$1.802\cdot10^{-4}$
$3.167\cdot10^{-5}$	$6.841\cdot10^{-5}$
$1.105\cdot10^{-5}$	$2.614\cdot10^{-5}$
$3.872\cdot10^{-6}$	$1.004\cdot10^{-5}$
$1.363\cdot10^{-6}$	$3.872\cdot10^{-6}$
$4.817\cdot10^{-7}$	$1.499\cdot10^{-6}$
$1.707\cdot10^{-7}$	$5.818\cdot10^{-7}$
$6.066\cdot10^{-8}$	$2.265\cdot10^{-7}$
$2.16\cdot10^{-8}$	$8.834\cdot10^{-8}$

It is in Fig. 1 where the dependency of the probability of error $P = f(h^2)$ determined by dependency (9) for different values of $\frac{\rho \bar{h}_\zeta^2}{FT}$ has been studied. Under the conditions of influence of only a fluctuating noise (Curve 1), the probability of error is determined only from h^2 regardless of the shape of signals that are transmitted. With $\frac{\rho \bar{h}_\zeta^2}{FT} \ge 10$ (curves 3 and 4), the efficiency of the coherent receiver under examination has been reducing considerably.

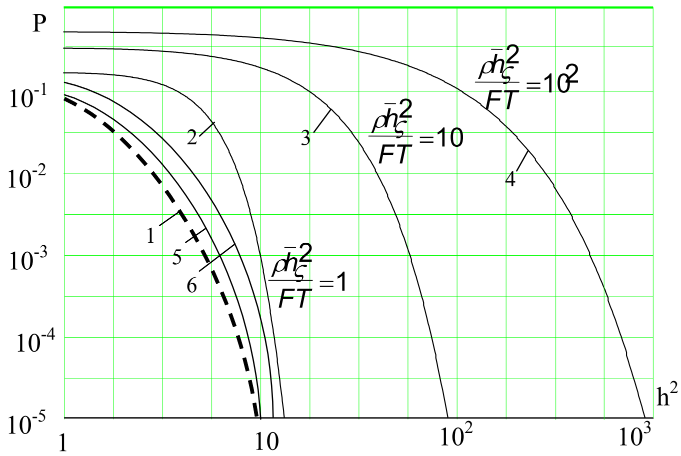

Fig.1. The dependency $P = f(h^2)$ for different cases

For the radio channels of decimeter range, the intensity of the concentrated interferences s is characterized by $\bar{h}_\zeta^2 = 10 - 10^4$. Hence, to provide invariance in respect to concentrated interferences and to guarantee the given noise-resistance level, it is necessary to use complex signals with a basis size depending on the ratio $\bar{h}_\zeta^2$.

The complicated noise background requires an optimization of the circuit of the receiver and adaptation of its structure depending on the interfering effects. In all known cases of systems designed with considering the effect of fluctuating noise and spectrum- concentrated interferences [2], the expressions of the probability of error depend monotonously on the value of the product for random j:

$$\delta_j = \frac{G_{0j}^2 \bar{h}_{\zeta_j}^2}{1 + \bar{h}_{\zeta_j}^2}, \tag{14}$$

so that

$$h_e^2 = h^2 \left(1 - \delta_j\right). \tag{15}$$

Taking into account (14) and (15), the probability of error can be expressed as:

$$P = \frac{1}{2}\left\{1 - \Phi\left[\sqrt{2}h\left(1 - \frac{\overline{h}_{\zeta}^{2} G_{0j}^{2}}{1 + \overline{h}_{\zeta}^{2}}\right)^{\frac{1}{2}}\right]\right\} \qquad (16)$$

With an intensity of the spectrum-concentrated interferences , such as $\overline{h}_{\zeta}^{2} \geq 10$, it is obtained that :

$$\frac{h_{\zeta}^{2}}{\left(1 + h_{\zeta}^{2}\right)} \approx 1$$

and $\delta_j \leq G_{0j}^{2} = const\left(h_{\zeta}^{2}\right)$.

Hence h_e^2 does not depend on $\overline{h}_{\zeta}^{2}$ and the optimized receiver is invariant with regard to the influencing spectrum-concentrated interferences unlike the receiver of nature (8). Besides that, for signals of sufficiently big bases $\delta_j \leq \frac{\rho}{FT} << 1$ and $h_e^2 \approx h^2$ has been provided, i.e. in practice, the noise-resistance of that receiver does not differ from the noise-resistance in channels only of fluctuating noise. In Fig.1 what has been studied is the dependency of the probability of error from h^2 with two values of coefficient δ_j - curve 5 (with $\delta_j = \frac{1}{4}$) and curve 6 (with $\delta_j = \frac{1}{10}$).

4 CONCLUSIONS

The paper presents the obtained analytical dependencies between the size of the basis of the transmitted complex signals and the possibilities of coherent demodulators to compensate the effect of interferences. A coherent demodulator optimal in respect to white noise and an optimized demodulator operating under the conditions of the effect of white noise spectrum-concentrated interferences have been compared. The results obtained have shown that the optimized receiver can keep a fixed level of probability of error with considerably smaller signal bases providing an absolute invariance of the nature of noise-resistance against spectrum-concentrated interferences.

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