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A Marine Traffic Flow Model

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ABSTRACT: A model is developed for studying marine traffic flow through classical traffic flow theories, which can provide us with a better understanding of the phenomenon of traffic flow of ships. On one hand, marine traffic has its special features and is fundamentally different from highway, air and pedestrian traffic. The existing traffic models cannot be simply extended to marine traffic without addressing marine traffic features. On the other hand, existing literature on marine traffic focuses on one ship or two ships but does not address the issues in marine traffic flow.

1 INTRODUCTION

Different types of traffic flow exist in busy ports around the world (Figure 1). Marine traffic is special and different from other types. Ships are not unlimited in their navigation, and deep waterways exist across the port. These deep waterways are generally associated with the principal water flow path of the tidal system. In some cases, the position of the principal waterway varies significantly, season to season, and migrates across the port region. Marine traffic within ports is frequently diverse as the shallow, relatively sheltered waters permit the safe navigation of small craft (leisure vessels, fishing boats, fast launches) across the port, while larger vessels (coastal cargo vessels and ocean-going carriers) inhabit the deeper waterways. The continued growth of port traffic increases the congestion of waterways.

This conceptual paper represents our first attempts to investigate the traffic characteristics of marine traffic flow in order to develop a marine traffic model. As macroscopic models have been advanced in different traffic disciplines, the proposed establishment of marine traffic flow models is expected to complement the existing literature of land traffic.

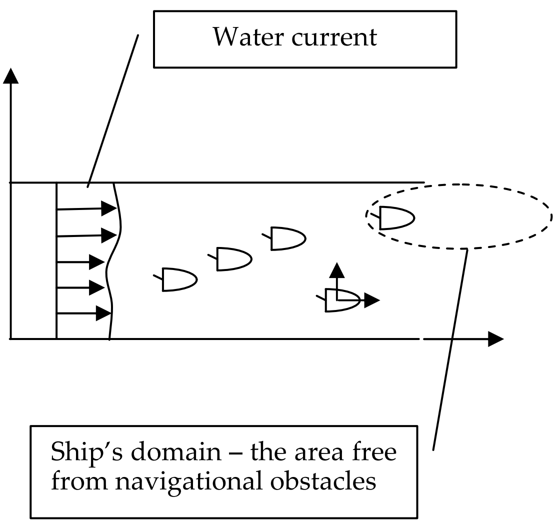

Figure 1. Marine traffic flow.

The paper is organized as follows. Section One provides the background of ship navigation and summarise characteristics of marine traffic. Section Two reviews previous studies on the traffic theory and ship navigation modelling. Section Three develops a new model for marine traffic and especially marine traffic characteristics are incorporated in the model. Section Four analyzes and understands the marine traffic model. Section Five concludes the present study.

2 LITERATURE REVIEW

Systematic studies of traffic flow have been conducted for more than five decades. A basic building block is the kinematic waves in traffic (Lighthill and Whitham 1955; Richards 1956), which relates the continuum traffic flow, the traffic speed and the traffic density. The focus of the present study is the models of marine traffic flow.

Highway traffic has attracted considerable attention for decades, for example, Gazis (2002). Many highway traffic models assume the homogenous vehicles are not applicable to marine traffic. The heterogeneity is recently considered in highway traffic research, for example, Wong & Wong (2002), Park et al., (2010). They however did not consider as a whole where the existing models can be used as marine traffic.

In the air traffic control, Andersson et al. (2003) proposed a novel optimisation approach to analyse collaborative airport arrival planning. Ship manoeuvring simulators are common in many maritime countries and generally operate in the time domain. Their use ranges from the full mission bridge simulator to PC-based simulator. Existing Traffic Alert and Collision Avoidance System (known as TACS II) is used to detect the altitudes of aircraft and then resolve (altitude crossing) encounters in the vertical domain. If an encounter is identified, TACS II will command one aircraft to climb and the other to descend. However, ships can only manoeuvre horizontally and ships have different manoeuvrability. Different from air traffic control, VTS is only an advisory service for ships; ship masters are responsible for a ship’s course, speed and safety.

On the track of pedestrian flow, some models have been developed (see Hughes, 2003). The models of pedestrian flow have three common assumptions. First the speed of pedestrian walk is determined solely by the density of surrounding pedestrians. Second, all pedestrians are the same, similar to a fluid particle in flows. Third, pedestrians avoid extreme densities, and so the model is mathematically convenient.

Previous research may not be applicable to marine traffic flow, as existing studies do not take into consideration the differences between ships. Traffic in previous models is considered as continuous flow and not as single ships with their individual characteristics of type, dimensions and velocity. Marine traffic is over moving water current. Real marine traffic is not consisted of ships of equal size moving with equal manoeuvrability. The depth of water has considerable influence on the rate of ship’s turn which may be obtained at a given rudder angle. If navigation in confined to waters that require large alternations of course, the turning manoeuvres must be commenced in due time with the knowledge of how much room the ship needs to carry out the alteration of course. This will, especially with regard to large ships, necessitate longer response time, larger reaction zone ahead, and technically a higher relaxation effect.

Ship-ship collision models have been developed on the basis of geometrical distribution and/or encounter-to-collision. Pedersen (2002; 2010), Montewka et al. (2010), Debnath & Chin (2010), Tan & Otay (1999), Seong et al. (2012) developed geometrical collision probability models that describe the geometrical probability model of collision. Fowler & Sorgard (2000) estimated the collision based on encounters by assuming the traffic is independent or uncorrelated. USCG (1999) found different types of encounters have different relative significance, with crossings more hazardous than head-on encounters, which are in turn more risk prone than over-takings. These assumptions are applicable only when the traffic density is low. In reality, ships may change speed or direction so as to avoid possible collisions, e.g. see Merrick et al. (2002). The crossing traffic models cover only a crossing situation of two vessels. In particular, in heavily trafficked ports, like Hong Kong, three or even more ships may approach an area at the same time. In this kind of situation, a collision is more difficult to avoid when the actions of several other vessels need to be observed. Hu et al. (2010) used AIS to determine the congestion level of marine traffic in restricted waters. Their findings are useful to develop macroscopic marine traffic models.

3 TRAFFIC MODEL

3.1 *Macroscopic model*

Let us estimate how the water current manifests itself in the marine traffic flow problem (Figure 1). The lack of experimental data does not allow the marine traffic flow to be formulated mathematically. Based on some analogies between marine traffic and land traffic (e.g. Payne, 1971), the one-dimensional marine traffic model is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left(\rho v_g\right) = 0\,, \tag{1}$$

$$\frac{\partial v_g}{\partial t} + v_g \frac{\partial v_g}{\partial x} = \frac{V_w(\rho) - v_w}{\tau} - \frac{C_0^2}{\rho}\frac{\partial \rho}{\partial x}, \tag{2}$$

where t is time, x is horizontal coordinate, ρ is traffic density, v_g is average traffic velocity over the ground in the $x-$ direction, v_w is average traffic velocity through water. $V_w(\rho)$ is the characteristic through-water speed determined by speed-density relationship (should be determined from field survey). We have to emphasise that both the relaxation term and anticipation term in Eq. (2) depend on the average velocity (speed) through water, rather than mean velocity (speed) over the ground.

Assuming the ship draft is deep, ship's velocity over the ground v_g is the velocity through water v_w plus velocity of water current v_c, such that

$$v_g = v_w + v_c . \tag{3}$$

Consider a uniform water current v_c constant of t and x, Eq. (1) and (2) will become:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_w) = 0, \tag{4}$$

$$\frac{\partial v_w}{\partial t} + v_w \frac{\partial v_w}{\partial x} + v_c \frac{\partial v_w}{\partial x} = \frac{V_w(\rho) - v_w}{\tau} - \frac{C_0^2}{\rho}\frac{\partial \rho}{\partial x}. \tag{5}$$

Comparing Eq. (4) and (5) against the known Payne traffic flow model (Payne, 1971), the presence of water current adds an extra acceleration term $v_c \frac{\partial v_w}{\partial x}$ in the dynamic equation. The effect of extra acceleration term increases the apparent acceleration of marine traffic in the magnitude of v_c, if the ships navigate along the direction of water current.

Similarly for the two-dimensional marine traffic, the governing equations (4) and (5) become:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_w) + \frac{\partial}{\partial y}(\rho v_w) = 0, \tag{6}$$

$$\frac{\partial v_w}{\partial t} + v_w \frac{\partial v_w}{\partial x} + v_w \frac{\partial u_w}{\partial y} + v_c\left(\frac{\partial v_w}{\partial x} + \frac{\partial u_w}{\partial y}\right) = \frac{V_w(\rho) - v_w}{\tau} - \frac{C_0^2}{\rho}\left(\frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial y}\right). \tag{7}$$

However, several major components should be integrated in the marine traffic models. As the manoeuvrability of a ship is related to its length and water current, the relaxation

$$C_0 = f(v_w, L_s, T_s), \tag{8}$$

in which L_s, T_s are average ship length and ship type, respectively. The water current v_c varies across the waterway,

$$v_c = v_c(y) \quad (0 \le y \le W), \tag{9}$$

and thus most variables vary across the waterway.

3.2 *Ship domain*

A critical step is to determine the density-speed relationship of ships, which can be estimated by ship domains. The ship domain that the navigator wants to keep clear of other ships is defined as the ship domain (Goodwin, 1975).

As shown in Figure 2, the ship domain occupied by stationery ship can be calculated in a rectangular form *B x L*, where *B* and *L* are the beam and length overall of the ship. When a ship is navigating, a greater ship domain is required, that is *W x D*. Both the terms *W* and *D* can be expressed as a function of the navigating speed through water v_w.

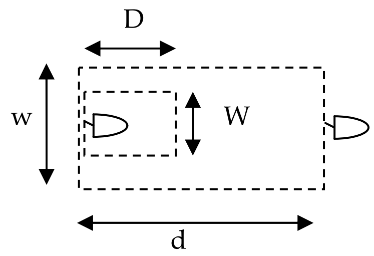

Figure 2. Rectangular ship domain.

Different from other traffic flows, the *W* and *D* are also dependent of water flow, since ships tend to navigate with more clearance when the water speed v_c is large. The length of ship domain of ship moving over water, *d*, can be expressed as the sum of two terms: the navigating length (which is a function of the navigating velocity) and the watch distance (which is a function of velocity, visibility, traffic and local and psychological factors). The navigating length is the distance of the ship navigating over a certain unit of time (e.g. 5 seconds). The watch distance is the distance required by the ship for steering and reaction.

4 ANALYSIS

The asymptotic method of homogenisation will be applied to deduce the traffic flow equations for different mechanisms. The homogenisation method is based on the asymptotic technique of multiple scales, e.g. Ng & Yip (2001). Consider three time scales are associated with the marine traffic flow in a horizontal waterway (Figure 1): t_0 for longitudinal navigation; and t_1 for longitudinal collision-avoidance movements. The model assumes "no conflict" or minimum space is preserved between ships. As the width of waterway is shorter than the length, $t_0 << t_1$. When ships navigate downstream, ships of different speeds, manoeuvrability, etc. will spread out along the length of the waterway. By and large, we expect that the time for ships to spread is longer than the time for ships to remove conflicts. With these assumptions of two time scale $t = t_0 + \varepsilon t_1$, the traffic flow problem can be decomposed into two simpler subproblems (t_0 subproblem; t_1 subproblem), Ng & Yip (2001) refers.

The original derivative becomes, according to the chain rule:

$$\frac{\partial}{\partial t} \to \frac{\partial}{\partial t_0} + \varepsilon \frac{\partial}{\partial t_1} \tag{10}$$

The traffic density

$$\rho \to \rho_0 + \varepsilon\rho_1 \tag{11}$$

The velocity is:

$$v = V_w(\rho_0 + \varepsilon\rho_1) \to V_w(\rho_0) + \varepsilon V_w(\rho_1) \tag{12}$$

The term in Eq. (1) becomes:

$$\begin{aligned} &\rho V_w(\rho) = (\rho_0 + \varepsilon\rho_1)V_w(\rho_0 + \varepsilon\rho_1) \\ &\to \rho_0 V_w(\rho_0) + \varepsilon\rho_0 V_w(\rho_1) + \varepsilon\rho_1 V_w(\rho_0) \end{aligned} \tag{13}$$

With Eq. (10)-(13), Eq. (4) becomes:

$$\left(\frac{\partial}{\partial t_0} + \varepsilon \frac{\partial}{\partial t_1}\right)\left(\rho_0 + \varepsilon\rho_1\right) + \frac{\partial}{\partial x}\left(\rho v\right) = 0 \tag{14}$$

At the zero-th order $O(\varepsilon^0)$,

$$\frac{\partial\rho_0}{\partial t_0} + \frac{\partial}{\partial x}\rho_0 V_w(\rho_0) = 0 \tag{15}$$

At the first order $O(\varepsilon)$,

$$\frac{\partial\rho_1}{\partial t_0} + \frac{\partial\rho_0}{\partial t_1} + \frac{\partial}{\partial x}\left[\rho_0 V_w(\rho_1) + \rho_1 V_w(\rho_0)\right] = 0 \tag{16}$$

Similarly, Eq. (2) at $O(\varepsilon^0)$ becomes:

$$\frac{\partial v_0}{\partial t_0} + v_0 \frac{\partial v_0}{\partial x} = \frac{V_w(\rho_0)}{\tau} - \frac{C_0^2}{\rho_0}\frac{\partial\rho_0}{\partial x} \tag{17}$$

Eq. (2) at the first order $O(\varepsilon)$ is:

$$\frac{\partial v_1}{\partial t_0} + \frac{\partial v_0}{\partial t_1} + v_0 \frac{\partial v_1}{\partial x} + v_1 \frac{\partial v_0}{\partial x}$$

$$= \frac{V_w(\rho_1)}{\tau} - \frac{C_0^2}{\rho_0}\frac{\partial\rho_1}{\partial x} \tag{18}$$

The model, Eq. (15)-(18), is then written in two vector forms:

$$\frac{\partial Q_0}{\partial t_0} + A_0(Q_0)\frac{\partial Q_0}{\partial x} = S_0 \tag{19}$$

$$\frac{\partial Q_1}{\partial t_0} + A_1(Q_1)\frac{\partial Q_1}{\partial x} = S_1(v_0, \rho_0) \tag{20}$$

where Q_0, Q_1 are the conservative variables, A_0, A_1 are the fluxes, while S_0, S_1 are the source terms, such as:

$$Q_0 = \begin{bmatrix} \rho_0 \\ v_0 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} \rho_1 \\ v_1 \end{bmatrix}, \text{ etc.} \tag{21}$$

Eq. (19) and (20) are in the quasi-linear matrix form of the governing equations and can be solved numerically by the method of characteristics.

The two dimensional problem, Eq. (6)-(9), can be solved similarly.

5 CONCLUSIONS AND DISCUSSION

This conceptual paper is the first attempt to develop a marine traffic for studying the dynamic behaviour of ships. The study uses the classical traffic model to consider two special marine traffic characteristics (a) the water current, and (2) the ship domain concept.

Since the marine traffic is more sophisticated than the classical (land) traffic, the marine traffic model will enable a new and richer insight in traffic behaviour in general. In the marine traffic engineering, the marine traffic flow theory and traffic control schemes are under development using different approaches based on classical traffic flow theory and ship manoeuvring characteristics. They can provide a supportable foundation for vessel traffic control.

Future work of this research is to conduct computational experiments and to develop control strategies of marine traffic for different scenarios. Computational experiments can be further conducted to evaluate the overall control strategies applied to a combination of traffic mix.

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REFERENCES

Andersson, K., Hall, W., Atkins, S. & Feron E. 2003. Optimization-based analysis of collaborative airport arrival planning. *Transportation Science*, 37(4), 422-433.

Debnath, A.K. & Chin, H.C. 2010. Navigational traffic conflict technique: A proactive approach to quantitative measurement of collision risks in port waters. *The Journal of Navigation*, 63(1): 137-152.

Fowler, T.G. & Sorgard, E. 2000. Modeling ship transportation risk. *Risk Analysis*, 20(2): 225-244.

Gazis, D.C. 2002. *Traffic Theory*, Kluwer Academic Publishers.

Goodwin, E.M. 1975. A statistical study of ship domains. *The Journal of Navigation*, 28(3), 328-344.

Hu, Q., Yong, J., Shi, C., & Chen, G. 2010. Evaluation of main traffic congestion degree for restricted waters with AIS reports. *International Journal on Marine Navigation and Safety of Sea Transportation*, 4(1), 55-58.
Hughes, R.L. 2003. The flow of human crowds. *Annual Review of Fluid Mechanics*, 35: 169-182.
Lighthill, M.J. & Whitham, G.B. 1955. On kinematic waves II: A theory of traffic on long crowded roads. *Proceedings of the Royal Society of London*, 229A: 317-345.
Merrick, J.R.W., van Dorp, J.R., Mazzuchi, T., Harrald, J.R., Saphn, J.E. & Grabowski, M. 2002. The Prince William Sound risk assessment. *Interface*, 23(6): 25-40.
Montewka, J., Hinz, T., Jujala, P. & Matusiak, J. 2010. Probability modelling of vessel collisions. *Reliability Engineering and System Safety*, 95(5): 573-589.
Ng, C.O. & Yip, T.L. 2001. Effects of kinetic sorptive exchange on solute transport in open-channel flow. *Journal of Fluid Mechanics*, 446(1), 321-345.
Park, B.J., Zhang, Y. & Lord, D. 2010. Bayesian mixture modelling approach to account for heterogeneity in speed data. *Transportation Research Part B*, 44(5): 662-673.
Payne, H.J. 1971. Models of freeway traffic and control. Simulation Councils Proceedings. *Mathematical Models of Public Systems*, 51-61.
Pedersen, P.T. 2002. Collision risk for fixed offshore structures close to high-density shipping lanes. *Journal of Engineering for the Maritime Environment*, 216(1): 29-44.
Pederson, P.T. 2010. Review and application of ship collision and grounding analysis procedures. *Marine Structures*, 23 (3): 241-262.
Pietrzykowski, Z. & Uriasz, J. 2009. The ship domain – A criterion of navigational safety assessment in an open sea area. *The Journal of Navigation*, 62(1), 93-108.
Richards, P.I. 1956. Shock waves on the highways. *Operations Research*, 4: 42-51.
Seong, Y.C., Jeong, J.S. & Park, G.K. 2012. The relation with width of fairway and marine traffic flow. *International Journal on Marine Navigation and Safety of Sea Transportation*, 6(3), 317-321.
Tan, B., & Otay, E.N. 1999. Modelling and analysis of vessel casualties resulting from tanker traffic through narrow waterways. *Naval Research Logistics*, 46(8): 871-892.
USCG 1999. *Regulatory Assessment Use of Tugs to Protect Against Oil Spills in the Puget Sound Area*, Report No. 9522-002, United States Coast Guard.
Wong, G.C.K. & Wong, S.C. 2002. A multi-class traffic flow model – an extension of LWR model with heterogeneous drivers. Transportation Research Part A, 36(9): 827-841.